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Resonant scattering of ultracold atoms in low dimensions

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Low energy scattering amplitudes for two atoms in one- and two-dimensional atomic wave guides are derived for short range isotropic and resonant interactions in high partial wave channels. Taking into account the finite width of the resonances which cannot be set arbitrarily to zero is shown to have important implications for the confinement induced resonances. For spin polarized fermions in a linear wave guide in the limit of infinitely strong odd-wave scattering length it imposes a strong constraint for achieving the Fermi Tonks Girardeau gas. For a planar wave guide, the p - and d -wave scattering resonances are analyzed in terms of the appearance of a low energy quasi-bound state.

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Recent experimental progress in degenerate atomic gases make possible accurate studies of quasi one- (1D) [1, 2, 3, 4, 5, 6, 7] and quasi two-dimensionnal (2D) [6, 8, 9] configurations. These systems are interesting in view of future applications involving coherent manipulation of matter waves [7, 10] and can be used also for studying generic phenomena in low dimensions [1, 9, 10]. One major interest of atomic gases is the precise knowledge and experimental control of the low energy interatomic collisions: the effective two-body interaction can be tuned with a Feshbach resonance [11] by applying an external magnetic field and/or with a Confined Induced Resonance (CIR) by varying the extension of the trap in the tight transverse direction [10, 12, 13, 14]. These techniques open possibilities for achieving new types of strongly correlated quantum systems. For example, thanks to s -wave collisional properties in quasi-1D wave guides [12], it has been possible to observe the so-called Tonks Girardeau (TG) gas [15] where 1D hard-core bosons can be mapped onto a system of non interacting fermions [3, 4]. For quasi-1D spin polarized fermions an analogous regime: the Fermionic Tonks Girardeau (FTG) gas is subject to intensive studies [16, 17, 18]. In this case, as a consequence of the Pauli exclusion principle, atoms interact predominantly in the p -wave channel and it has been predicted that the strongly interacting 1D polarized Fermi gas can be mapped onto a non interacting Bose gas.

In this letter, it is shown that the finite width of the p -wave CIR, introduces a strong constraint for achieving the FTG regime. At resonance in the linear wave guide, the scaling invariance is not in general verified so that the Fermi-Bose mapping theorem in Ref.[19] can be applied to specific configurations only. More generally, this regime corresponds to a narrow BEC-BCS crossover for the 1D polarized Fermi gas. Moreover, in view of future prospects in low dimensional superfluidity, the two-body p - and d -wave scattering amplitudes are derived in quasi-2D atomic wave guides. These results show that the high partial waves CIR depend strongly on the width of the 3D scattering resonance.

In the following, the true interatomic forces are sup-

posed to be short range and isotropic. Moreover, they are considered in the neighborhood of a resonance. The 3D low energy collisional properties of two atoms are then parametrized in each partial wave ($l \geq 0$) by the following phase shift $\delta_l(q)$ [20]:

$$q^{2l+1} \cot \delta_l(q) = -\frac{1}{w_l} - \alpha_l q^2 + O(q^4), \quad (1)$$

in Eq.(1) q is the relative momentum of the two colliding atoms, and the resonant regime in the l -partial wave is achieved for $|w_l| \gg R^{2l+1}$, where R is the characteristic radius of the pairwise potential (in the standard terminology, w_0 is the usual scattering length and w_1 is the scattering volume). The parameter α_l is linked to the width of the resonance: a large (small) value of $\alpha_l R^{2l-1}$ corresponds to a narrow (relatively broad) resonance. In the neighborhood of the resonance and for $l \geq 1$, using the analyticity of the scattering amplitude, it has been shown recently that for a finite range potential [21, 22, 23]:

$$\alpha_l R^{2l-1} \gtrsim (2l-3)!!(2l-1)!! \quad (2)$$

Consequently, $\alpha_l |w_l| \gg R^2$, and this inequality reveals the existence of a low energy scale in the system. This feature is linked to the presence of a shallow bound state for large and positive values of w_l (and of a quasi-bound state for large and negative w_l). Hence, unlike s -wave broad resonances where $\alpha_0 = O(R)$ can be neglected in the low energy limit, for $l > 0$ the parameter α_l which depends on the specific resonance considered, *cannot* be set arbitrarily to zero. For example, in experiments the resonant regime can be achieved by using the concept of Feshbach resonance [11] on the spin degree of freedom. An external magnetic field B modifies the detuning between an open and a closed channel of the two-body system, and in the vicinity of the resonance in the partial wave l , $w_l^{-1} \propto -(B - B_0)$ (where $B \sim B_0$) while the parameter α_l is almost constant. In the subsequent analysis, the pairwise interaction is modeled by a zero range interatomic potential (it is assumed that the smallest characteristic length of the trapping potential is $\gg R$). In this approximation, for non vanishing relative

coordinates ($\mathbf{r} \neq 0$), the wave function is solution of the free Schrödinger equation (this is an exact assumption for $r > R$). The interatomic potential is replaced by a source term of vanishing range $\epsilon \rightarrow 0$ and for each partial wave a contact condition is imposed on the wave function, such that without external potential the phase shifts obtained in this approach coincide *exactly* with the first two terms in the right hand side of Eq.(1) [21]. In other words, the effect of interatomic forces for $r > R$ are obtained by imposing a formal singular behavior of the wave function in the region $r < R$. The trapping potentials considered in this letter are always harmonic, so that the center of mass and relative coordinates are decoupled each from the others. For a binary collision in the center of mass frame at energy $E = \hbar^2 q^2 / \mu$ (μ is the reduced mass), the wave function $|\Psi\rangle$ verifies the Lippman-Schwinger equation:

$$|\Psi\rangle = |\Psi_0\rangle + \frac{2\pi\hbar^2}{\mu} \sum_{l \geq 0} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{k^l \delta_\epsilon(k) [\mathcal{R}_l \cdot \mathcal{S}_{l,\mathbf{k}}]}{\mathcal{H}_0 - E - i0^+} |\mathbf{k}\rangle, \quad (3)$$

where \mathcal{H}_0 is the free Hamiltonian which includes the external potential, and $|\Psi_0\rangle$ belongs to the kernel of $\mathcal{H}_0 - E$. In Eq.(3), the source term is introduced in the momentum representation: $\delta_\epsilon(k) = \exp(-k^2 \epsilon^2 / 4)$ is the Fourier transform of a normalized Gaussian weight having a vanishing range ϵ [24], \mathcal{R}_l and $\mathcal{S}_{l,\mathbf{k}}$ are Symmetric Trace Free (STF) tensors of rank l , and the notation $[\mathcal{R}_l \cdot \mathcal{S}_{l,\mathbf{k}}]$ means a contraction between the two tensors. The tensors \mathcal{R}_l characterize the behavior of the wave function at short interatomic distance, and the tensors $\mathcal{S}_{l,\mathbf{k}}$ are reminiscent of a standard multipolar expansion; they are defined by [21]: $\mathcal{S}_{l,\mathbf{k}}^{[\alpha\beta\dots]} = (-1)^l k^{l+1} (\partial_{k_\alpha} \partial_{k_\beta} \dots) k^{-1} / (2l-1)!!$, where $\{k_\alpha\}$ are the Cartesian components of the vector \mathbf{k} ($\alpha \in \{x, y, z\}$). In the momentum representation, the contact condition can be written as [26]:

$$\text{Reg}_{\epsilon \rightarrow 0} \int \frac{d^3\mathbf{k}}{(2\pi)^3} k^l \langle \mathbf{k} | \Psi \rangle \mathcal{S}_{l,\mathbf{k}} = - \frac{l! \mathcal{R}_l}{a_l(q) (2l+1)!!}, \quad (4)$$

where $a_l^{-1}(q) = (w_l^{-1} + \alpha_l q^2)$. In Eq.(4), $\text{Reg}_{\epsilon \rightarrow 0}$ means the regular part of the integral obtained when the formal range ϵ is set to zero. This way, the source term in Eq.(3) is itself a functional of $|\Psi\rangle$, and a closed equation for \mathcal{R}_l is obtained by combination of Eqs.(3,4). In the next parts, the scattering problem for p - or d -wave channels is solved successively in 1D and 2D harmonic wave guides.

Linear atomic wave guide. — Two atoms are confined in a two-dimensional harmonic trap while they move freely along the third direction (z). In the center of mass frame the non interacting Hamiltonian is:

$$\mathcal{H}_0 = -\frac{\hbar^2}{2\mu} \Delta_{\mathbf{r}} + \frac{1}{2} \mu \omega_{\perp}^2 \rho^2 - \hbar \omega_{\perp}, \quad (5)$$

where $\mathbf{r} = z \hat{\mathbf{e}}_z + \boldsymbol{\rho}$ are the relative coordinates. The state $|\Psi_0\rangle$ in Eq.(3) which is associated with a scattering process at energy $E = \hbar^2 q^2 / 2\mu$ is given by: $\langle \mathbf{k} | \Psi_0 \rangle = (2\pi) \delta(k_z - q) \langle \mathbf{k}_{2D} | \phi_{00} \rangle$, where $|\phi_{00}\rangle$ is the

ground state of the 2D harmonic oscillator in Eq.(5), and $\mathbf{k} = \mathbf{k}_{2D} + k_z \hat{\mathbf{e}}_z$. Hereafter, the system is in the monomode regime ($E < 2\hbar\omega_{\perp}$), so that at large interatomic distances $|z| \gg a_{\perp}$ (where $a_{\perp} = \sqrt{\hbar/\mu\omega_{\perp}} \gg R$) the wave function factorizes as $\langle \mathbf{r} | \Psi \rangle = \langle \boldsymbol{\rho} | \phi_{00} \rangle \psi_{1D}(z)$, and for $E > 0$:

$$\psi_{1D}(z) = \exp(iqz) + \exp(iq|z|) \left[f^{\text{even}} + \text{sign}(z) f^{\text{odd}} \right]. \quad (6)$$

In this situation, the system which is frozen along the transverse direction, can be considered as quasi-1D. The even (odd) scattering amplitude f^{even} (f^{odd}) results from the asymptotic contributions ($z \gg a_{\perp}$) in the different even (odd) 3D partial waves. I consider first two spin polarized fermions at low collisional energy. Consequently, one can restrict the source term in Eq.(3) to the p -wave contribution. In this channel, $\mathcal{S}_{1,\mathbf{k}} = \mathbf{k}/k$ and the homogeneous solution $|\Psi_0\rangle$ imposes that $\mathcal{R}_1 = P_{1D} \hat{\mathbf{e}}_z$. Computation of the regular part of the integral in Eq.(4) is much more simpler in the domain of negative energy $E < 0$ and from Eq.(3) one can show that:

$$P_{1D} = \frac{-3q\phi_{00}(0)}{\frac{1}{a_1(q)} + \frac{6}{\sqrt{\pi}a_{\perp}^3} \text{P.f.} \int_0^{\infty} \frac{du}{u^{3/2}} \frac{\exp(\tau u)}{1 - \exp(-u)}}. \quad (7)$$

where $\tau = E/2\hbar\omega_{\perp} < 0$ and P.f. denotes the ‘‘partie finie de Hadamard’’ of the integral [28]. Interestingly, one recognizes the Riemann-Hurwitz Zeta function $\zeta_H(-1/2, -\tau)$ in the regularized integral of Eq.(7). Hence, by analytic continuation in the domain $\tau > 0$, one obtains a simple expression of the scattering amplitude, and in the low energy limit $qa_{\perp} \ll 1$:

$$f_p^{\text{odd}} = 2i\pi P_{1D} \phi_{00}^*(0) \simeq -iq \left(\frac{1}{l_p} + iq + q^2 \xi_p \right)^{-1}, \quad (8)$$

with the odd-wave scattering length l_p and the effective range ξ_p given respectively by:

$$l_p = 6a_{\perp} \left[\frac{a_{\perp}^3}{w_1} - 12 \zeta\left(-\frac{1}{2}\right) \right]^{-1} \quad \text{and} \quad \xi_p = \frac{\alpha_1 a_{\perp}^2}{6}. \quad (9)$$

The expression for l_p in Eq.(9) coincides with the result found using the K-matrix formulation [14]. Eq.(9) shows also that the effective range ξ_p is in general not negligible for all $q \ll a_{\perp}^{-1}$. Indeed, the condition $\xi_p \sim a_{\perp}$ is not easy to reach as a consequence of Eq.(2) ($\alpha_1 R \gtrsim 1$) and also from the inequality $a_{\perp} \gg R$ which is needed from the zero range approximation. For example taking $a_{\perp} > 10R$ and $\xi_p = a_{\perp}$, one obtains a value of $\alpha_1 R$ close to unity. Previous studies of the resonant polarized fermionic gas have neglected the effective range term (a valid hypothesis for a Fermi momentum $k_F \ll \xi_p^{-1}$), therefore the system was always supposed to be in the FTG regime for $|l_p| = \infty$ [16, 17, 18]. In actual experiments two p -wave resonances are used: the resonance for ${}^6\text{Li}$ atoms ($R \simeq 2.8$ nm) at $B_0 \simeq 215$ G with $\alpha_1 R \simeq 5.5$ [33], which

is then not a very good candidate for a clear achievement of the FTG regime, and the one at $B_0 \simeq 198.8$ G for ^{40}K atoms ($R \simeq 6.8$ nm) where $\alpha_1 R \simeq 2.8$ [32], and which is more favorable. Interestingly, for $\xi_p \neq 0$ (meaning that $\xi_p \gg a_\perp$), the scale invariance in the linear wave guide is broken at low energy and, in presence of a longitudinal harmonic trap the many-body ansatz proposed in Ref.[18] which is exact at the FTG limit, is not a solution of the time dependent problem. At the two-body level and for $l_p > 0$, one finds a positive pole in Eq.(8) at $\kappa = (-1 + \sqrt{1 + 4\xi_p/l_p})/2\xi_p$ (where $\kappa = -iq$) which gives the energy ($-\hbar^2\kappa^2/2\mu$) of a shallow bound state: a composite boson. In the regime where the scattering volume w_1 is positive and such that $R^3 \ll w_1 \ll a_\perp^3$, then $l_p \ll a_\perp \ll \xi_p$ and $\kappa \simeq 1/\sqrt{\alpha_1 w_1}$ which coincides with the 3D result. Moreover, for large and positive values of $l_p \gg \xi_p$, then $\kappa = l_p^{-1}$ and the bound state has a vanishing energy (*i.e.* $\ll \hbar\omega_\perp$). Consequently, extension of the formalism in Ref.[18] for $\xi_p \neq 0$ amounts to the characterization of a narrow cross-over from a BEC molecular phase ($l_p > 0$) to a BCS phase ($l_p < 0$) associated to a strongly interacting quasi-1D gas of polarized fermions. In view of future computation of few- and many-body properties, the scattering amplitude in Eq.(8) may be obtained from a 1D effective theory where the wave function ψ_{1D} in Eq.(6) solves the non interacting Schrödinger equation and satisfies the following contact condition:

$$\lim_{z \rightarrow 0^+} \left(\frac{1}{l_p} + \partial_z - \xi_p \partial_z^2 \right) \psi_{1D}^{\text{odd}}(z) = 0, \quad (10)$$

where $\psi_{1D}^{\text{odd}}(z) = [\psi_{1D}(z) - \psi_{1D}(-z)]/2$, is the projection of the wave function onto its odd component [29]. As an example, one can consider a weak harmonic confinement along the z -direction which allows a 1D treatment (atomic frequency $\omega_z \ll \omega_\perp$ and length $a_z = \sqrt{\hbar/\mu\omega_z} \gg a_\perp$). The 1D two-body wave function in the center of mass frame is $\psi_\lambda(z) = \mathcal{N} \text{sign}(z) D_\lambda(\sqrt{2}|z|/a_z)$, where D_λ is the Weber parabolic cylinder function and $\lambda = E/\hbar\omega_z - 1/2$. From Eq.(10), one obtains the eigenvalue equation:

$$\Gamma\left(\frac{1-\lambda}{2}\right)/\Gamma\left(-\frac{\lambda}{2}\right) = \frac{a_z}{2l_p} + \frac{\xi_p}{a_z} \left(\lambda + \frac{1}{2}\right). \quad (11)$$

Example of parameters for a highly anisotropic quasi-1D trap are given by $\omega_\perp = 2\pi \times 69$ kHz and $\omega_z = 2\pi \times 10$ Hz. In this situation and for ^{40}K atoms at $B \sim 198.8$ G, then $\xi_p/a_z = 0.1$ which is a relatively small ratio, while for ^6Li atoms at $B \sim 215$ G one finds $\xi_p/a_z \simeq 3.2$. At resonance ($|l_p| = \infty$) the eigenenergies E_n depend crucially on the ratio ξ_p/a_z : in the extreme case $\xi_p/a_z \gg 1$ then $E_0 \simeq 0$ and $E_n \simeq (2n - 1/2)\hbar\omega_z$, $n \in \mathbb{N}^*$; this contrasts with the ‘‘bosonic’’ result for $\xi_p/a_z \ll 1$ where the first derivative of the wave function vanishes at $z = 0$ and $E_n = (2n + 1/2)\hbar\omega_z$, $n \in \mathbb{N}$. The mapping between a many-body fermionic wave function Ψ_F (with atomic coordinates z_i) and a bosonic wave function Ψ_B is obtained by introducing a phase factor:

$\Psi_F = \prod_{i < j} \text{sign}(z_{ij}) \Psi_B$ where $z_{ij} = z_i - z_j$ and for each pair of particles (ij), Ψ_B verifies the contact condition in Eq.(10) applied on its even part [from the Pauli principle: $\psi_B^{\text{even}}(z_{ij}; \dots) = \psi_B(z_{ij}; \dots)$]. However for $\xi_p \neq 0$, the mapping to a non interacting Bose gas is not possible and is then less interesting than the theorem in Ref.[19].

Planar atomic wave guide. — High partial wave superfluidity in quasi-2D geometries is interesting for its links with condensed matter like for example high- T_c superconductivity [34] where the order parameter has a d -wave symmetry. Moreover, in the p -wave channel a recent proposal shows a possible application for quantum computing [35]. These studies motivate a close investigation of quasi-2D scattering properties for ultra-cold atoms. In this geometry, the two colliding atoms are confined in a planar harmonic trap along the z -direction and move freely in the (xy) plane. The free Hamiltonian reads:

$$\mathcal{H}_0 = -\frac{\hbar^2}{2\mu} \Delta_{\mathbf{r}} + \frac{1}{2} m \omega_z^2 z^2 - \frac{\hbar\omega_z}{2}. \quad (12)$$

The homogeneous solution corresponding to a scattering process at energy $E = \hbar^2 q^2/2\mu$ is: $\langle \mathbf{k} | \Psi_0 \rangle = (2\pi)^2 \delta(\mathbf{k}_{2D} - \mathbf{q}) \langle k_z | \phi_0 \rangle$, where $|\phi_0\rangle$ is the ground state of the 1D harmonic oscillator in Eq.(12). In the monomode regime ($E < \hbar\omega_z$), the system can be considered as quasi-2D, and for large interparticle separation ($\rho \gg a_z$), the wave function factorizes: $\Psi(\mathbf{r}) = \phi_0(z) \psi_{2D}(\rho)$. The 2D partial scattering amplitudes $f^{[m]} = f^{[-m]}$ can be defined by the following expansion [36]:

$$\psi_{2D}(\rho) \underset{\rho \gg a_z}{=} \exp(i\mathbf{q}\cdot\rho) - \frac{i}{4} \sum_{m=-\infty}^{m=\infty} f^{[m]} H_m^{(1)}(q\rho) \exp(im\theta), \quad (13)$$

where $\theta = \pi/2 + \angle(\rho, \mathbf{q})$ and $H_m^{(1)}$ is the Hankel’s function of order m . Using Eqs.(3,4,13) the scattering amplitude for two atoms interacting in the s -wave channel is: $f^{[0]} = 4\pi[a_z \sqrt{\pi}/a_0(q) + J_0(\tau + i0^+)]^{-1}$, where $\tau = E/2\hbar\omega_z$, $J_0(\tau) = \ln(-B/(2\pi\tau)) + \sum_{n=1}^{\infty} \ln(n/(n-\tau))(2n-1)!/(2n)!!$ [37], and $B \simeq 0.9049$ [38]. In the p -wave channel, Eq.(2) implies the existence of a small parameter $\eta_1 = (\alpha_1 a_z)^{-1} \ll 1$ which plays a central role in the quasi-2D scattering properties. As example, if one considers ^{40}K atoms in a trap with $\omega_z = 2\pi \times 30$ kHz, for the resonance at 198.8 G then $\eta_1 \simeq 2.6 \times 10^{-2}$, while for ^6Li atoms at ~ 215 G one obtains $\eta_1 \simeq 2.1 \times 10^{-3}$. For $\mathbf{q} = q\hat{\mathbf{e}}_x$, the p -wave regular part in Eq.(3) is given by the dipolar term $\mathcal{R}_1 = P_{2D}\hat{\mathbf{e}}_x$ and $\mathcal{S}_{1,\mathbf{k}} = \mathbf{k}/k$. The expression of the dipole P_{2D} is derived from Eq.(4):

$$P_{2D} = -3q\phi_0(0) \left[\frac{1}{a_1(q)} + \frac{6J_1(\tau + i0^+)}{\sqrt{\pi}a_z^3} \right]^{-1}, \quad (14)$$

and the 2D p -wave scattering amplitude is $f^{[1]}(q) = -2\pi q \phi_0^*(0) P_{2D}$. In the low energy limit $|\tau| \rightarrow 0$ then $J_1(\tau) = J_1(0) + \tau \ln(-eB/2\pi\tau) + O(\tau^2)$, where $J_1(0) \simeq -5.4722 \times 10^{-2}$ [37, 39]. For $\kappa = -iq$, there is a positive pole in P_{2D} when the scattering volume w_1 is positive and $w_1 < w_1^* = -\sqrt{\pi}a_z^3/[6J_1(0)] > 0$. Therefore, by

modifying the trap frequency and/or the external magnetic field it is possible to drive the system from a regime without 2D p -wave shallow bound state ($w_1^{-1} < w_1^{*-1}$) to a regime with a quasi-bound state ($w_1 \lesssim w_1^*$). Taking into account the fact that $\eta_1 \ll 1$, the bound state energy is $E_b \simeq -\hbar^2(w_1^{-1} - w_1^{*-1})/2\mu\alpha_1$. The resonance in the scattering cross section $\sigma = |f_1|^2/4q$ occurs only for *positive* values of E_b , i.e. in presence of a quasi-bound state: this is a narrow resonance which appears for a colliding energy $E \sim E_b > 0$ (this feature is reminiscent of the 3D p -wave resonance [20]) [41]. The 2D p -wave resonance has been already observed in Ref.[6] and the results confirm the shift ($1/w_1^* > 0$) which grows as the confinement increases. In what follows, atoms interact resonantly in the 3D d -wave channel and the small parameter related to the width of the resonance is $\eta_2 = \alpha_2^{-1}a_z^{-3} \ll 1$. For $\mathbf{q} = q\hat{\mathbf{e}}_x$, the $l = 2$ regular part is given by: $k^2[\mathcal{R}_2 \cdot \mathcal{S}_{2,\mathbf{k}}] = S_{2D}(2k_z^2 - k_{2D}^2) + D_{2D}(k_x^2 - k_y^2)$, where S_{2D} contributes to the $m = 0$ 2D partial wave channel and D_{2D} contributes to the $|m| = 2$ channel (2D- d -wave). In the following, I focus on the $|m| = 2$ channel

only. The contact condition in Eq.(4) leads to two decoupled equations for S_{2D} and D_{2D} , and one obtains:

$$D_{2D} = -\frac{15q^2\phi_0(0)}{4} \left[\frac{1}{a_2(q)} + \frac{60J_2(\tau + i0^+)}{a_z^5\sqrt{\pi}} \right]^{-1}, \quad (15)$$

and for $\tau \rightarrow 0$, $J_2(\tau) = J_2(0) + \tau J_1(0) + O(\tau^2)$ where $J_2(0) \simeq -2.2752 \times 10^{-2}$ [37]. Using the expression of the partial scattering amplitude: $f^{[2]}(q) = -2\pi q^2 \phi_0^*(0) D_{2D}$, one deduces that there is a shallow bound state for $w_2 \lesssim w_2^* = -a_z^5\sqrt{\pi}/[60J_2(0)] > 0$, and from the inequality $\eta_2 \ll 1$ the bound state energy is $\simeq -\hbar^2(w_2^{-1} - w_2^{*-1})/2\mu\alpha_2$. For a gas composed of two spin-components fermions, this result opens the possibility of observing a BEC-BCS crossover for a d -wave order parameter by varying $(w_2^{-1} - w_2^{*-1})$ from positive to negative values.

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[36] In terms of the 2D phase-shifts δ_m^{2D} , the scattering amplitudes are defined by $f^{[m]} = -4i(1 + i \cot \delta_m^{2D})^{-1}$.
[37] The functions $J_m(\tau)$ are defined in the domain $\tau < 0$ by: $J_m(\tau) = \text{P.f.} \int_0^\infty du u^{-(m+1)} \exp(\tau u) [1 - \exp(-u)]^{-1/2}$, and appear in the computation of $f^{[m]}$. The expression of $J_m(\tau + i0^+)$ with $0 < \tau < 1$ can be obtained by analytical continuation from the series $J_{-1}(\tau) = \sum_{n=0}^\infty (n - \tau)^{-1} (2n - 1)!! / (2n)!!$ together with the recursive relation $dJ_m(\tau)/d\tau = J_{m-1}(\tau)$.
[38] This numerical result slightly differs from the one in Ref.[13]. However, a close investigation shows that the constant B which is obtained from $\ln(B/2\pi) = -\gamma + \int_0^\infty du \{u^{-1} [1 - \exp(-u)]^{-1/2} - u^{-3/2} - (1+u)^{-1}\}$, where γ is the Euler's constant, coincides with the evaluation of the slowly converging series in Ref.[13].
[39] In Ref.[40], the number $-6J_1(0)$ is evaluated with a series and equals $\simeq 0.325$, while in the present letter the integral representation of $J_1(q)$ [37] gives $-6J_1(0) \simeq 0.32833$.
[40] Z. Idziaszek *et al.*, Phys. Rev. Lett. **96**, 013201 (2006).
[41] The parameter η_1 is of crucial importance: by neglecting α_1 in Ref.[40] ($\eta_1 = \infty$) the resonance in the scattering cross section is found to occur in the opposite regime: $0 < w_1 < w_1^*$ and at collisional energy $E \equiv O(\hbar w_z)$.