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Nonquadratic stabilization of Takagi-Sugeno models: a piecewise quadratic approach

Mohammed CHADLI, Didier MAQUIN, José RAGOT

Centre de Recherche en Automatique de Nancy, CNRS, UMR 7039
2, Avenue de la forêt de Haye, 54516 Vandœuvre les Nancy – France.
Phone: (33) 3 83 59 57 02, Fax: (33) 3 83 59 56 44
Email: {Mohamed.Chadli, Didier.Maquin, Jose.Ragot}@ensem.inpl-nancy.fr

Abstract

This paper deals with the stabilization of Takagi-Sugeno models (T-S) using state feedback controllers. New sufficient stability conditions are given for both continuous and discrete T-S models. The stability conditions, formulated in term of bilinear matrix inequalities (BMIs), are based on a piecewise quadratic Lyapunov function and the use of the so-called S-procedure. A method of linearisation, applied on the obtained BMIs conditions, is presented.

Keywords: T-S model, multiple models, controller synthesis, Lyapunov methods, S-procedure, BMIs.

1 Introduction

The issue of stability, the design of state feedback controller as well as the design of state observer for nonlinear systems described by multiple models [17] have been considered actively during the last decade. This approach includes the T-S models [5][15] and can be seen also as Polytopic Linear Differential Inclusions (PLDI) [16].

Many works have been carried out to investigate the stability analysis of T-S models using a quadratic Lyapunov function and sufficient conditions for the stability and stabilization have been established [2][3][11][12][22]. The stability mainly depends on the existence of a common positive definite matrix guarantying the stability of all local submodels. These stability conditions may be expressed in linear matrix inequalities (LMIs) form [16]. The obtaining of a solution is then facilitated by using numerical toolboxes for solving such problems. Nevertheless, restriction to the class of quadratic Lyapunov function candidate may lead to significant conservativeness. To overcome this limitation, stability conditions relaxing previous constraints have been established using a piecewise quadratic Lyapunov function and the S-procedure [1][13][14]. While in [13], the continuity of Lyapunov function is carried out by requiring additional constraints, in [1][14] the Lyapunov function can be discontinuous. Another class of Lyapunov function candidate called polyquadratic is also studied for both continuous and discrete T-S systems [4][6][7][18][19]. For Linear Parameter Varying (LPV) systems, to reduce the conservativeness, quadratic parameter dependant Lyapunov functions are used [8][9][20][21]. However, as it is known, embedding nonlinear systems, and by the same way the T-S model, into LPV framework will lead to conservative results [1][4].

In this paper, the design of state feedback controller for T-S models is considered. New sufficient conditions for global asymptotic stabilization are obtained using a nonquadratic Lyapunov function and the so-called S-procedure. The proposed method is proved to be less conservative compared to those derived via quadratic stability analysis.

The paper is organized as follows. The section 2 is dedicated to the description of the continuous T-S model.

In section 3, firstly, previous stability conditions are recalled and, secondly, we establish the main results for which a method of linearisation of the obtained BMIs conditions is proposed. In section 4, the proposed synthesis is extended to discrete T-S models.

Throughout the paper, the following useful notation is used: X^T denotes the transpose of the matrix X, $X > 0$ $(X \geq 0)$ denotes symmetric positive definite (semidefinite) matrix, r is the number of submodels simultaneously activated, $\sum_{i<j}^{n}(.) = \sum_{i=1}^{n}\sum_{j=1,i<j}^{n}(.)$ and $I_n = \{1,2,..,n\}$

2 T-S continuous model

A continuous T-S model is based on the interpolation between several LTI local models as follows:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t))(A_i x(t) + B_i u(t)) \tag{1}$$

where n is the number of submodels, $x(t) \in \mathbb{R}^p$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $A_i \in \mathbb{R}^{p.p}$, $B_i \in \mathbb{R}^{p.m}$ and $z(t) \in \mathbb{R}^q$ is the decision variable vector.

The choice of the variable $z(t)$ leads to different classes of models. It can depends on the measurable state variables, be a function of the measurable outputs of the system and possibly on the input. In this case, the system (1) describes a nonlinear system. It can also be an unknown constant value, system (1) then representing a PLDI.

The normalized activation function $\mu_i(z(t))$ in relation with the i^{th} submodel is such that:

$$\begin{cases} \sum_{i=1}^{n} \mu_i(z(t)) = 1 \\ \mu_i(z(t)) \geq 0 \quad \forall\, i \in I_n \end{cases} \tag{2}$$

The global output of T-S model is interpolated as follows:

$$y(t) = \sum_{i=1}^{n} \mu_i(z(t)) C_i x(t) \tag{3}$$

where $y(t) \in \mathbb{R}^l$ is the output vector and $C_i \in \mathbb{R}^{l.p}$. More detail about this type of representation can be found in [2][19].

3. Stabilization of T-S models: Previous results

3.1. Analysis

Consider the following unforced continuous T-S model:

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t)) A_i x(t) \tag{4}$$

Sometimes, it is possible to prove the stability of a T-S model (4) using a quadratic Lyapunov function

$$V(x(t)) = x^T(t) P x(t),\ P > 0 \tag{5}$$

So, if there exists a common symmetric positive definite matrix P such that

$$A_i^T P + P A_i < 0 \qquad \forall i \in I_n \tag{6}$$

then the T-S model (4) is globally asymptotically stable.

Inequalities (6) give a sufficient condition for ensuring stability of (4). However, it is well known that in a lot of cases, a common positive definite matrix P does not exist whereas the T-S model is stable. To reduce the conservatism of (6), a sufficient condition for global asymptotic stability of T-S model (4) is established by using the S-procedure lemma (see Annex) and a nonquadratic Lyapunov function candidate of the form [1]:

$$V(x(t)) = \max\left(V_1(x(t)),..,V_i(x(t)),..,V_n(x(t))\right) \tag{7}$$

where $V_i(x(t)) = x(t)^T P_i x(t), \quad P_i > 0, \forall\, i \in I_n$

Theorem 1 : Suppose that there exists symmetric matrices $P_i > 0$ and scalars $\tau_{ijk} \geq 0$ such that $\forall\, (i,j) \in I_n^2$:

$$A_i^T P_j + P_j A_i + \sum_{k=1}^{n} \tau_{ijk}\left(P_j - P_k\right) < 0 \tag{8}$$

Then the T-S model (4) is globally asymptotically stable.

Proof: see [1].

3.2. *Synthesis*

With the PDC (Parallel Distributed Compensation) control law:

$$u(t) = -\sum_{i=1}^{n} \mu_i(z(t)) K_i x(t) \tag{9}$$

the closed loop continuous T-S model obtained from (1) becomes

$$\dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(t))\mu_j(z(t))G_{ij}x(t) \tag{10}$$

where

$$G_{ij} = A_i - B_i K_j \tag{11}$$

Using a quadratic Lyapunov function (5), sufficient stability conditions for (10) are given by theorem 2 and for more relaxation by theorem 3.

Theorem 2 : Suppose that there exist symmetric matrices $P > 0$ and $Q \geq 0$ such that $\forall\,(i,j) \in I_n^2, i < j$:

$$G_{ii}^T P + PG_{ii} + (r-1)Q < 0 \tag{12a}$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) - Q \leq 0 \tag{12b}$$

and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the closed-loop T-S model (10) is globally asymptotically stable.

Proof: see [2].

Theorem 3 : Suppose that there exists symmetric matrices $P > 0$ and Q_{ij} such that $\forall\,(i,j) \in I_n^2, i < j$:

$$G_{ii}^T P + PG_{ii} + Q_{ii} < 0 \tag{13a}$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) + Q_{ij} \leq 0 \tag{13b}$$

$$\begin{pmatrix} Q_{11} & Q_{12}\cdots & Q_{1n} \\ Q_{12} & \ddots & \vdots \\ \vdots & & \\ Q_{1n} & \cdots & Q_{nn} \end{pmatrix} > 0 \tag{13c}$$

and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the closed-loop T-S model (10) is globally asymptotically stable.

Proof: see [3].

The control design problem is to find the feedback gains K_i such that the closed loop system (10) is stable. The conditions (12) and (13) are not convex in P and K_i. In order to convert them into an LMI problem, these inequalities are multiplied in the left and the right by P^{-1} with the variable change $Y_i = P^{-1}K_i$.

However, it is well known that in a lot of cases, a common positive definite matrix P does not exist whereas the T-S model (10) is stable. In the following section, we propose to relax the quadratic stabilization conditions (12) and (13) by using the piecewise quadratic Lyapunov function (7) and the S-procedure lemma.

4. Nonquadratic stabilization of T-S models

4.1. Continuous case

Knowing that the nonquadratic Lyapunov function (7) decreases the conservativeness of the result with regard to the quadratic Lyapunov function (5), this type of Lyapunov function candidate already used to derive relaxed stability conditions [1], can also be considered for the synthesis of control laws.

It is possible to substitute the matrices $A_i, i \in I_n$ by $G_{ij}, (i,j) \in I_n^2$ directly in the theorem 1. However for more relaxation, the idea is to exploit the results of theorem 2 and theorem 3. The following theorems give sufficient stability conditions of (10).

Theorem 4 : Suppose that there exists symmetric matrices $P_i > 0$, $Q \geq 0$ and scalars $\tau_{ijkl} \geq 0$ such that $\forall\,(i,j,k) \in I_n^3, i < j$:

$$G_{ii}^T P_k + P_k G_{ii} + (r-1)Q + \sum_{l=1}^{n} \tau_{iikl}(P_k - P_l) < 0 \tag{14a}$$

$$(G_{ij}+G_{ji})^T P_k + P_k(G_{ij}+G_{ji}) - 2Q + \sum_{l=1}^{n} \tau_{ijkl}(P_k - P_l) < 0 \tag{14b}$$

and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the T-S model (10) is globally asymptotically stable.

Proof: Considering the Lyapunov function candidate (7), it follows that

$$V(x) = V_k(x) \quad \text{if } V_k(x) \geq V_l(x), \quad \forall\,(k,l) \in I_n^2 \tag{15}$$

Consequently if $V_k(x) \geq V_l(x), \forall\,(k,l) \in I_n^2$:

$$\frac{dV_k(x(t))}{dt} = x(t)^T\left(\sum_{i=1}^{n} \mu_i(z(t))\left(G_{ii}^T P_k + P_k G_{ii}\right)\right)x(t) +$$

$$x(t)^T\left(\sum_{i<j=1}^{n} \mu_i(z(t))\mu_j(z(t))\left((G_{ij}+G_{ji})^T P_k + P_k(G_{ij}+G_{ji})\right)\right)x(t) \tag{16}$$

So, if the following conditions are satisfied

$$\begin{cases} \bullet\, x(t)^T\left(\left(G_{ij}+G_{ji}\right)^T P_k + P_k\left(G_{ij}+G_{ji}\right) - 2Q\right)x(t) < 0 \\ \quad \text{when } \forall x(t):\ V_k(x) \geq V_l(x),\ (k,l) \in I_n^2 \\ \bullet\, x(t)^T\left(G_{ii}^T P_k + P_k G_{ii} + (r-1)Q\right)x(t) < 0 \\ \quad \text{when } \forall x(t):\ V_k(x) \geq V_l(x),\ (k,l) \in I_n^2 \end{cases} \tag{17}$$

then from (16) we ensure $\frac{dV_k(x(t))}{dt} < 0, k \in I_n \ \ \forall\ x(t) \neq 0$ and the system (10) is globally asymptotically stable. Finally, constraints (14) are obtained by applying the S-procedure lemma to (17). ■

Theorem 5 : Suppose that there exists symmetric matrices $P_i > 0$ and Q_{ij} and scalars $\tau_{ijkl} \geq 0$ such that $\forall\ (i,j,k) \in I_n^3, i < j$:

$$G_{ii}^T P_k + P_k G_{ii} + Q_{ii} + \sum_{l=1}^{n} \tau_{iikl}\left(P_k - P_l\right) < 0 \tag{18a}$$

$$\left(G_{ij}+G_{ji}\right)^T P_k + P_k\left(G_{ij}+G_{ji}\right) + 2Q_{ij} + \sum_{l=1}^{n} \tau_{ijkl}\left(P_k - P_l\right) < 0 \tag{18b}$$

$$\begin{pmatrix} Q_{11} & Q_{12} \cdots & Q_{1n} \\ Q_{12} & \ddots & \vdots \\ \vdots & & \\ Q_{1n} & \cdots & Q_{nn} \end{pmatrix} > 0 \tag{18c}$$

and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the T-S model (10) is globally asymptotically stable.

Proof: The proof is obtained as in theorem 4, by using the Lyapunov function (7) and the S-procedure lemma. ■

In the particular case where the input matrices are linearly dependent (i.e. $\exists\ B \in \mathbb{R}^{p.m}$ and $\alpha_i > 0, i \in I_n$ such that $B_i = \alpha_i B$), the following control law could be considered instead of the PDC controller (9):

$$u(t) = -\frac{\sum_{i=1}^{n} \mu_i(z(t))\alpha_i K_i}{\sum_{i=1}^{n} \mu_i(z(t))\alpha_i} x(t) \tag{19}$$

Consequently, by substituting (19) into (1), the closed-loop T-S system (1) becomes

$$\dot{x}(t) = \sum_{i=1}^{n} \mu_i(z(t)) G_{ii} x(t) \tag{20}$$

where $G_{ii}, i \in I_n$ is defined in (11). The following theorem gives sufficient conditions in LMIs form to ensure asymptotic stability of (20).

Theorem 6 : Suppose that there exists symmetric matrices $P_i > 0$ and scalars $\tau_{ij} \geq 0$ such that $\forall\ (i,j) \in I_n^2$:

$$G_{ii}^T P_j + P_j G_{ii} + \sum_{k=1}^{n} \tau_{ijk}\left(P_j - P_k\right) < 0 \tag{21}$$

Then the T-S model (20) is globally asymptotically stable.

Proof: The proof is obtained directly from theorem 1, by substituting A_i by G_{ii}. ■

Remark 1 : It should be noted that the quadratic conditions (12) and (13) are included in conditions derived in (14) and (18). So when $P_i = P, \forall\ i \in I_n$ we have $P_j - P_k = 0$ and $V(x(t)) = \max_{i \in I_n}\left(V_i(x(t))\right) = x(t)^T P x(t)$. Then conditions (14) and (18) become those of the quadratic case, i.e. (12) and (13).

Remark 2 : The same result can be obtained by using the nonquadratic Lyapunov function

$$V(x(t)) = \min_{i \in I_n}\left(V_i(x(t))\right) \tag{22}$$

where $V_i(x(t))$ is defined in (8).

Remark 3 : The use of the S-procedure lemma and the nonquadratic Lyapunov function (8) leads to a non convex problem (14) with $n^2\frac{(n+1)}{2}$ BMIs to satisfy.

4.2. BMIs linearisation

We know that BMI problem is not convex and may have multiple local solutions. However, many control problems that require the solution of BMIs can be reformulated as LMIs, which may be solved very efficiently. Unfortunately, the LMI formulation is very difficult in our case.

In this paper, we use the path-following method, developed in [10], for solving BMI problem. This method utilises a first order perturbation approximation to linearize the BMI problem. Hence, the BMIs (14) and (18) are converted into a series of LMIs iteratively solved until a desired performance is achieved.

Let P_{k0} and K_{j0} be initial values of the unknown P and K such that

$$P_k = P_{k0} + \delta P_k,\ K_j = K_{j0} + \delta K_j \tag{23}$$

then

$$P_k G_{ij} = \left(P_{k0} + \delta P_k\right)\left(A_i - B_i\left(K_{j0} + \delta K_j\right)\right) \tag{24}$$

Thus, by neglecting the second order terms $\delta P_k B_i \delta K_j$, the matricial product (24) can be linearised with regard to the increment variables δP_k and δK_j.

$$P_k G_{ij} \approx \left(P_{k0} + \delta P_k\right)A_i - \left(P_{k0} + \delta P_k\right)B_i K_{j0} - P_{k0} B_i \delta K_j$$

Using this approximation, the BMIs (18) can be transformed into LMIs that can be solved simultaneously with the following constraints: $\left\|\delta P_k\right\| < \zeta \left\|P_{k0}\right\|$ and $\left\|\delta K_j\right\| < \zeta \left\|K_{j0}\right\|$, $0 < \zeta << 1$, necessary to ensure the validity of the linear approximation.

Next, an iterative method modifying the τ_{ijkl} parameters can be applied. The major weakness of this method is the choice of initial values guaranteeing the existence of a solution, if any.

4.3. Extension to discrete T-S models

A discrete T-S model is based on the interpolation between several LTI local discrete models as follows:

$$x(k+1) = \sum_{i=1}^{n} \mu_i(z(k))(A_i x(k) + B_i u(k)) \tag{25}$$

where n is the number of submodels, $x(k) \in \mathbb{R}^p$ is the state vector, $u(k) \in \mathbb{R}^m$ is the input vector, $A_i \in \mathbb{R}^{p.p}$, $B_i \in \mathbb{R}^{p.m}$ and $z(k) \in \mathbb{R}^q$ is the decision variable vector.

The closed loop model of (25) with the control law $u(k) = -\sum_{i=1}^{n} \mu_i\left(z(k)\right)K_i x(k)$ is :

$$x(k+1) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i(z(k))\mu_j(z(k)) G_{ij} x(k) \tag{26}$$

where G_{ij} is defined in (11).

To prove the stability of the unforced T-S model (26), sufficient conditions are derived using a quadratic Lyapunov function (5) [2]. To reduce the conservativness of the quadratic method, a sufficient condition for the computation of the piecewise Lyapunov function of the form (7) is given in the following part.

Theorem 7 : Suppose that there exists symmetric matrices $P_i > 0$ and $Q \geq 0$ and scalars $\tau_{ijkl} \geq 0$ such that $\forall\ (i,j,k) \in I_n^3,\ i < j$:

$$G_{ii}^T P_k G_{ii} - P_k + (r-1)Q + \sum_{l=1}^{n} \tau_{ijkl}\left(P_k - P_l\right) < 0 \tag{27a}$$

$$\left(G_{ij} + G_{ji}\right)^T P_k \left(G_{ij} + G_{ji}\right) - 4P_k - 4Q + \sum_{l=1}^{n} \tau_{ijkl}\left(P_k - P_l\right) < 0 \tag{27b}$$

and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then the T-S model (26) is globally asymptotically stable.

Proof : The proof is obtained as in theorem 1, by using the nonquadratic Lyapunov function (7) and the S-procedure lemma. ■

We can prove easily that the quadratic conditions are included in the derived conditions by substituting $P_i, \forall\ i \in I_n$, by P.

The conditions (27) are non convex and difficult to linearise. However the method presented in section 4.2 can be used after applying the Schur complement [19].

5. Conclusion

In this paper, the stabilization of nonlinear model described by T-S model is considered. Using the S-procedure and a piecewise quadratic Lyapunov function candidate, sufficient conditions for the global asymptotic stability are derived. Despite the fact that the obtained conditions are not convex, it is proved that the derived stability conditions allow to improve the results obtained by the quadratic method. A method of linearisation is proposed for the derived BMIs conditions.

Annex

Lemma 1 (S-*Procedure,* [16]):

Let $F_0\left(x(t)\right),\ldots,F_q\left(x(t)\right)$ be quadratic functions of the variable $x(t) \in \mathbb{R}^p$.

If there exists scalars $\tau_1 \geq 0,..,\tau_q \geq 0$ such that

$$F_0\left(x(t)\right) - \sum_{i=1}^{q} \tau_i F_i\left(x(t)\right) \leq 0$$

then $F_0\left(x(t)\right) \leq 0$ for all $x(t)$ such that $F_i\left(x(t)\right) \leq 0,\ \forall\ i \in I_q$. ■

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