Data validation of uncertain dynamic systems

Moustapha Alhaj-Dibo, Didier Maquin, José Ragot

Data validation of uncertain dynamic systems

ALHAJ DIBO Moustapha, MAQUIN Didier, RAGOT José [1]

Abstract. The methods of data validation which were developed these last years largely call for the redundancy resulting from models. The case of models with certain parameters (static and/or dynamic) was analyzed and received many solutions. However, there is relatively few work concerning the data validation in the presence of model uncertainties. The aim of this communication is to present a method of data validation for dynamic linear systems, which is able to take into account the uncertainties of the model parameters. Firstly we represent the dynamic model of the system in a static form by piling the state and measurement vectors on an observation window. Secondly, the elementary operations relating to the intervals make it possible to propose a state estimation of the system taking into account the parameters uncertainties. As the uncertainties are supposed to be bounded, the estimation's result is provided in an interval form. A sequential algorithm is used to obtain the state estimation by carrying out the intersection between the estimation resulting from three methods (Gauss elimination, Gauss-Seidel iteration and Krawczyk iteration). By analyzing this estimation, we can detect and isolate the data which are affected by gross errors as biases and propose a correction to make these data coherent with the model of the system.

1 INTRODUCTION

One of the most important tasks in process control is the extraction of information concerning the state of the system, in order to be able to evaluate its performances and, if necessary, to adjust the control strategy. If this information is erroneous, any decision will provide an unsuitable command. Thus, there is a need for data validation which takes place between the phase of acquisition of the data and those of decision-making. So "data validation" is the action of generating coherent information, which will be considered as usable for the following treatments. This estimation can be carried out either from the sensors measuring the process variables or from the process model. The data validation can be realized "on line" or "off line", according to the procedure of treatment carried out. In a "off line" procedure, the data of the process are validated overall a temporal horizon. In the case of the "on line" approach, the data of the process are validated at every sampling instant and allow an "instantaneous" monitoring of the process operation.

The majority of the methods developed in this field use techniques based on statistical considerations, where the noise affecting the measurement devices is often characterized by a statistical distribution whose parameters are a priori known. The results obtained by such methods are valid if the nature of the noise and the model of the system are perfectly known. However in main situations, this knowledge is very difficult to obtain or is only partially available. Thus, using uncertain model with bounded variables may constitute a suitable approach.

Uncertainties which affect a system can be structural or parametric. In the first case, a way of taking them into account consists in using a multiple-models representation obtained by the interpolation of several local models, each local model being associated to a particular regime of the process. In the second case, we can use a model made up of vague behavioral relations. Let us note that the uncertainties can affect the system itself and/or the measurement system. In this article, we are interested with the case where the uncertainties affect the system and the measurements simultaneously.

Taking into account the model uncertainties was the subject of a certain number of work as well for the problems of data validation as, for example, for those concerning the development of state observer [10]. Often, these uncertainties are modeled by using stochastic variables [8]. Another approach, called bounded approach, represents these uncertainties by a set of possible values for which we know only their bounds [1, 5, 11].

The interval analysis was initially developed to hold account on the inaccuracies of the numbers. These inaccuracies come from data resulting from a chain of instrumentation or from computing tool. One of the first work in the diagnosis field is presented by [4]. Later [12] proposes an interval technique for the detection and the isolation of sensor faults in the case of a static linear model while [3] treats the dynamic case. The reference [6] proposes a method for detecting fault affecting the sensors of a Turbofan jet engine, while [7] treats the problem of data validation in the case of certain systems with uncertain measurements. We developed a method of data validation for dynamic uncertain linear systems, based on a bounded approach, where all the state variables of the system are measured [2]. Here the proposed method treats the case where the state variables of the system are partially measured. Section 2 is devoted to the presentation of the model of the studied systems. Section 3 describes the state estimation method. Section 4 shows the way to detect, isolate and correct the data affected by faults. An academic example, illustrating the method suggested, is presented in section 5.

2 MODEL OF THE SYSTEM

We consider the class of dynamic systems represented by linear discrete state equations. The uncertainties are described by bounded and normalized variables, which modify the values of the coefficients of

[1] Institut National Polytechnique de Lorraine, Centre de Recherche en Automatique de Nancy, UMR 7039, 2, Avenue de la forêt de Haye 54516 Vandœuvre-lès-Nancy Cedex, FRANCE email : {malhajdi, dmaquin, jragot}@ensem.inpl-nancy.fr

matrices A, B and C used in the model :

$$\begin{array}{c} x(k+1) = A(\eta_A(k))x(k) + B(\eta_B(k))u(k) \\ y(k) = C(\eta_C(k))x(k) \end{array} \quad (1)$$

where $x \in R^n$ is the state vector of the system, $u \in R^q$ is the input vector and $y \in R^m$ is the output vector. $\eta_A(k), \eta_B(k)$ and $\eta_C(k)$ are bounded and normalized variables. The state transition matrix characterizing the dynamic behavior of the system $A(\eta_A(k)) \in \mathbb{R}^{n,n}$, the control matrix $B(\eta_B(k)) \in \mathbb{R}^{n,q}$ and the measurement matrix $C(\eta_C(k)) \in \mathbb{R}^{m,n}$ may be expressed as:

$$A(\eta_A(k)) = A_c + \Delta A \eta_A(k) \in \left[\underline{A}, \quad \overline{A}\right], \quad |\eta_A(k)| \leq 1$$

$$B(\eta_B(k)) = B_c + \Delta B \eta_B(k) \in \left[\underline{B}, \quad \overline{B}\right], \quad |\eta_B(k)| \leq 1$$

$$C(\eta_C(k)) = C_c + \Delta C \eta_C(k) \in \left[\underline{C}, \quad \overline{C}\right], \quad |\eta_C(k)| \leq 1$$

$A_c \in R^{n,n}$, $B_c \in R^{n,q}$ and $C_c \in R^{m,n}$ are respectively the center matrices (the center of an interval $\varphi = [\underline{\varphi}, \overline{\varphi}]$ is defined by $\varphi_c = \frac{\overline{\varphi}+\underline{\varphi}}{2}$) of interval matrices $[\underline{A}, \overline{A}]$, $[\underline{B}, \overline{B}]$ and $[\underline{C}, \overline{C}]$. $\Delta A \in R^{n,n}$, $\Delta B \in R^{n,q}$ and $\Delta C \in R^{m,n}$ characterize the uncertainty magnitudes of the matrices $[\underline{A}, \overline{A}]$, $[\underline{B}, \overline{B}]$ and $[\underline{C}, \overline{C}]$. It is supposed that the observability matrix (2) is invariant on the field of variation of $A(\eta_A(k))$ and $C(\eta_C(k))$.

$$\mathrm{O}(k) = \begin{pmatrix} C(\eta_C(k)) \\ C(\eta_C(k))A(\eta_A(k)) \\ \vdots \\ C(\eta_C(k))A^{n-1}(\eta_A(k)) \end{pmatrix} \quad (2)$$

Thus, the system (1) is observable at each sampling time.

For writing the dynamic model in a static form [1], the first equation of model (1) is formulated as follows:

$$\begin{array}{c} \left[\begin{array}{cc} A(\eta_A(k+j-1)) & -I_n \end{array}\right] \left[\begin{array}{c} x(k+j-1) \\ x(k+j) \end{array}\right] = \\ -B(\eta_B(k+j-1))\, u(k+j-1) \;\; , \; j \in N^* \end{array} \quad (3)$$

Starting from this relation, by piling up the state and the input vectors on an observation window $[k, k+s]$, we obtain the expression:

$$\tilde{A}(\eta_A(k,s))\, x(k,s) = \tilde{B}(\eta_B(k,s-1))\, u(k,s-1) \quad (4)$$

with

$$\tilde{A}(\eta_A(k,s-1)) = \begin{pmatrix} A(\eta_A(k)) & -I_n & \cdots & 0 & 0 \\ 0 & A(\eta_A(k+1)) & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & A(\eta_A(k+s-1)) & -I_n \end{pmatrix}$$

$$\tilde{B}(\eta_B(k,s-1)) = \begin{pmatrix} -B(\eta_B(k)) & 0 & \cdots & 0 \\ 0 & -B(\eta_B(k+1)) & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & -B(\eta_B(k+s-1)) \end{pmatrix}$$

and $x(k,s) = \left[x(k)^T \cdots\cdots x(k+s)^T\right]^T$, $x(k,s) \in \mathbb{R}^{s_n}$
$\tilde{A}(\eta_A(k,s-1)) \in \mathbb{R}^{s.n,s_n}$, $\tilde{B}(\eta_B(k,s-1)) \in \mathbb{R}^{s.n,s.q}$,
$s_n = (s+1)n$

The measurement vector $y(k)$ on the observation window $[k, k+s]$ can be written as a function of $x(k,s)$:

$$y(k,s) = \tilde{C}(\eta_C(k,s))x(k,s) \quad (5)$$

with $y(k,s) = [y(k)^T ... y(k+s)^T]^T, y(k,s) \in R^{s_t}$,
$\tilde{C}(\eta_C(k,s)) \in \mathbb{R}^{s_t,s_n}$, $s_t = (s+1)m$

$$\tilde{C}(\eta_C(k,s)) = \begin{pmatrix} C(\eta_C(k)) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & C(\eta_C(k+s)) \end{pmatrix}$$

By aggregation of the relations (4) and (5), the dynamic model (1) is written in the following static form :

$$M(\eta_A(k,s-1), \eta_C(k,s))x(k,s) = H(\eta_B(k,s-1)) \quad (6)$$

with

$$M(\eta_A(k,s-1), \eta_C(k,s)) = \left[\begin{array}{c} \tilde{A}(\eta_A(k,s-1)) \\ \tilde{C}(\eta_C(k,s)) \end{array}\right]$$

$$H(\eta_B(k,s-1)) = \left[\begin{array}{cc} \tilde{B}(\eta_B(k,s-1)) & 0 \\ 0 & I_{s_t} \end{array}\right] \left[\begin{array}{c} u(k,s-1) \\ y(k,s) \end{array}\right]$$

$M(\eta_A(k,s-1), \eta_C(k,s)) \in \mathbb{R}^{(s.n+s_t),s_n}$,
$H(\eta_B(k,s-1)) \in \mathbb{R}^{s.n+s_t}$

Based on (6), where the inputs and the outputs are known, the objective is to estimate the state vector $x(k,s)$. To simplify the notations, the equation (6) is written as follows:

$$[M]x(k,s) = [H] \quad (7)$$

where $M(\eta_A(k,s-1), \eta_C(k,s)) \in [M]$ with $[M] = [\underline{M}, \quad \overline{M}]$ and $H(\eta_B(k,s-1)) \in [H]$ with $[H] = [\underline{H}, \quad \overline{H}]$.

The brackets around the matrices M and H indicate that we are concerned with time varying bounded matrices. It is supposed that $m(s+1) > n$ which means that the number of rows of $[M]$ is larger than its number of columns, in order to obtain a redundancy of data.

3 STATE ESTIMATION

Equation (7) expresses clearly the dependance of the state in respect to the measurements collected on the observation window. Estimating the state for systems with certain parameters is a classical problem. Here, the situation is more difficult according to the presence of uncertainties.

There are several methods to calculate, by using the equation (7), an estimation of $x(k,s)$ [9] and [5]. Unfortunately, no method can be considered better than the others. In fact, the better method is

those which gives the smallest width of interval estimation.

Among the candidate methods, we choose the three following: Gauss elimination, Gauss-Seidel iteration and Krawczyk iteration [9]. These three methods, briefly explained in the appendix of this document, are applicable and give good results if the matrix M is square and preconditioned (a preconditioned interval matrix is a regular matrix with all diagonal elements different from zero). However, in our case, this matrix is not square and the procedure must be adapted by using a sequential algorithm.

Step 1

Consider the first square matrix $[M^1] = [M(i,j)]$ where $i = 1 \dots s_n$, $j = 1 \dots s_n$ and $[H^1] = [H(i)]$ with $i = 1 \dots s_n$. Then, we calculate an estimation of $x(k,s)$ by solving the equation $[M^1]x(k,s) = [H^1]$. For that, we use successively the three methods mentioned previously (after preconditioning). The intervals $[\hat{x}^1_G(k,s)]$, $[\hat{x}^1_{GS}(k,s)]$ and $[\hat{x}^1_K(k,s)]$ represent respectively the estimations calculated by Gauss elimination, by Gauss-Seidel iteration and by the method of Krawczyk. To minimize the width of the estimation interval, the intersection between these three estimations is carried out:

$$[\hat{x}^1(k,s)] = [\hat{x}^1_G(k,s)] \cap [\hat{x}^1_{GS}(k,s)] \cap [\hat{x}^1_K(k,s)]$$

Step i**,** $i = 2, \dots, s_t - n + 1$

At step i a square matrix $[M^i]$ and the corresponding vector $[H^i]$ are built by replacing the row s_n in $[M^1]$ and $[H^1]$ by the row $s_n + i$ of $[M]$ and $[H]$. Thereafter, the ith interval estimation $[\hat{x}^i(k,s)]$ is computed from the system defined by $[M^i]$ and $[H^i]$ as indicated in the step 1.

The final estimation of $x(k,s)$ is obtained through the intersection of the different previous estimations:

$$\begin{aligned} [\hat{x}(k,s)] &= \bigcap_{i=1}^{s_t-n+1} [\hat{x}^i(k,s)] \\ [\hat{x}^i(k,s)] &= [\hat{x}^i_G(k,s)] \cap [\hat{x}^i_{GS}(k,s)] \cap [\hat{x}^i_k(k,s)] \end{aligned} \tag{8}$$

In the case of normal operation of the system and when the procedure of the state estimation on the first observation window is finished, we slip the observation window for one sampling period. The static form (7) is updated by using the data in the new observation window, then we repeat the preceding steps and so on.

In the case of abnormal operation of the system (presence of one or several faults), the intersection of the intervals obtained in (8) can be empty. So, to calculate an estimation of state variables coherent with the model of the system (1), we must detect and isolate these faults and then correct the corrupted data. The faults can affect the measurements and/or the system itself. In this presentation, we are only interested by the measurement faults.

4 DETECTION AND ISOLATION OF FAULTY MEASUREMENTS

The detection of an anomaly in a set of data is often carried out through the generation of a Boolean indicator. For that, at each step of calculation of the state estimation in the observation window, we test the coherence between the estimation resulting from this step and those resulting from the preceding steps.

At first, the dimension s of the observation window must be chosen in order to avoid faulty data in the first step of the estimation of $x(k,s)$. This initial choice guarantees the inclusion of the actual state $x(k,s)$ inside the interval estimation $[\hat{x}^1(k,s)]$ calculated by solving the equation $[M^1]x(k,s) = [H^1]$. Thus the width s of the observation window must verify the following constraint :

$$s_t + s.n \geq s_n + m \Rightarrow s \geq \frac{n}{m}\ ,\ s \in N^* \tag{9}$$

with $s_t = (s+1)m$ and $s_n = (s+1)n$

In fact, as the first set of observations will be used as a reference as it will be explained later, the optimal choice of s would be to take the smallest value which verify the condition (9). To implement the method, we suppose that there are no faults in the temporal interval $[k_0, k_0 + s - 1]$ (k_0 is the initial moment). This assumption makes it possible to have, in the first step of the estimation of $x(k,s)$, a matrix M^1 and a vector H^1 not affected by faults, therefore this estimation $[\hat{x}^1(k,s)]$ can be viewed as a reference to compare the forthcoming estimations in the following steps.

First, based on this idea of reference, the following interval $[\tau]$ is evaluated:

$$[\tau] = [\hat{x}^1(k,s)] \cap [\hat{x}^j(k,s)] \tag{10}$$

Let us note that the intersection between $[\hat{x}^j(k,s)]$ and $[\hat{x}^1(k,s)]$ is empty if at least one of the intersections between two components of the same index of these vectors is empty.

Second, a fault indicator τ is built based on the analysis of the previously computed interval:

1. If $[\tau] \neq \emptyset$, the estimations $[\hat{x}^j(k,s)]$ and $[\hat{x}^1(k,s)]$ are coherent. In this case the values of the state variables belong simultaneously to $[\hat{x}^j(k,s)]$ and $[\hat{x}^1(k,s)]$. Thus we declare that the data intervening in the equation $[M^j]x(k,s) = [H^j]$ are valid and the fault indicator τ is set to zero.
2. If : $[\tau] = \emptyset$, there is an inconsistency between the estimations $[\hat{x}^j(k,s)]$ and $[\hat{x}^1(k,s)]$. In this case, the values of the state variables belong to $[\hat{x}^1(k,s)]$ only, because $[\hat{x}^j(k,s)]$ is influenced by a fault. In that case, the fault indicator τ is set to one.

It is necessary now to eliminate the effect of the data (measurements) declared as invalid, since they will be used to test the coherence of the new measurement in the following observation windows. In fact, after isolating the faulty data, we a posteriori correct them "on line". A faulty measurement $y_i(k+s),\ i = 1 \dots m$ is a posteriori corrected by using the model of the system (1) and the estimation considered as a reference $[\hat{x}^1(k,s)]$. The corrected measurement $y_{i,cor}(k+s)$ must belong to the interval estimation $[\hat{y}_i(k+s)]$ calculated as follows:

$$y_{i,cor}(k+s) \in [\hat{y}_i(k+s)] = [\underline{C}_i,\ \overline{C}_i][\hat{x}^1(k+s)] \tag{11}$$

where $[\underline{C}_i,\ \overline{C}_i]$ is the i^{th} row of the matrix $[\underline{C},\ \overline{C}]$, $[\hat{y}_i(k+s)]$ is the interval estimation of the measurement $y_i(k+s)$ and $y_{i,cor}(k+s)$ is the corrected measurement which will be used instead of $y_i(k+s)$ in the following steps of the estimation of $x(k,s)$ and also in the forthcoming observation windows. Any value belonging to the interval defined in (11) is satisfactory, however the simplest choice consists to retain the center of the considered interval.

When the strategy described above has been applied to the set of data of a given observation window, we slip this window for one sampling step and update the static form (6) according to the data of this new observation window. As we replaced the eventual corrupted data in a given observation window by their corrected values, we are sure that the forthcoming observation windows may contain faults only on the data obtained at the new observation time. Thus it is possible to apply the proposed strategy using sliding windows of observations leading to the data validation on an overall time interval.

5 EXAMPLE

We consider a system described by the model (1) where :

$$[\underline{A},\, \overline{A}] = \begin{pmatrix} 0.45 & -0.29 & [0.19,\ 0.21) \\ [-0.084,\ -0.076] & 0.66 & -0.08 \\ 0.12 & [0.114,\ 0.126] & 0.37 \end{pmatrix},$$

$$[\underline{B},\, \overline{B}] = \begin{pmatrix} 1 \\ -0.8 \\ [0.949,\ 1.051] \end{pmatrix},$$

$$[\underline{C},\, \overline{C}] = \begin{pmatrix} -5.25 & [-1.576,\ -1.423] & 1.2 \\ [0.949,\ 1.051] & 0.2 & -5 \end{pmatrix}$$

The input $u(k)$ is represented on the figure 1. In the temporal intervals [20, 30], [50, 60], intervene the faults f_1 on y_1 and f_2 on y_2, the outputs affected by these faults are represented on the figure 2.

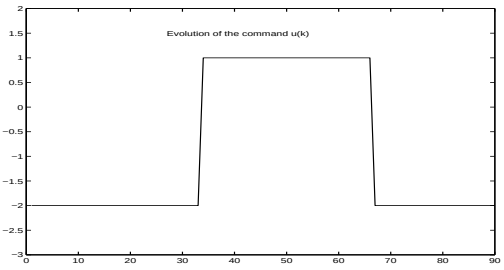

Figure 1. Input of the system

According to the equation (9), $(s \geq \frac{n}{m} = 1.5)$, we choose the dimension of the observation window $s = 2$. By piling up the state, the input and the measurement vectors on the horizon $[k, k+2]$, we obtain the static form (6):

$$[M] = \begin{pmatrix} [\underline{A},\, \overline{A}] & -I & 0 \\ 0 & [\underline{A},\, \overline{A}] & -I \\ [\underline{C},\, \overline{C}] & 0 & 0 \\ 0 & [\underline{C},\, \overline{C}] & 0 \\ 0 & 0 & [\underline{C},\, \overline{C}] \end{pmatrix},$$

$$[H] = \begin{pmatrix} -[\underline{B},\, \overline{B}]u(k) \\ -[\underline{B},\, \overline{B}]u(k+1) \\ y(k) \\ y(k+1) \\ y(k+2) \end{pmatrix} \text{ and } x(k,2) = \begin{pmatrix} x(k) \\ x(k+1) \\ x(k+2) \end{pmatrix}$$

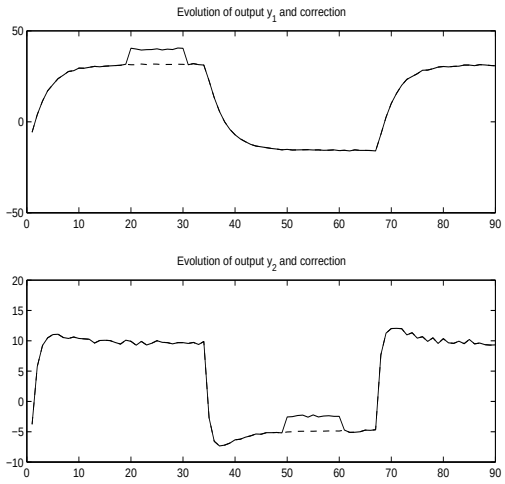

Figure 2. The measured and corrected outputs

By varying k from 1 to 88, the equations (8), (7) and (11) allow to detect the faults and isolate the measurements which are affected by these faults. The figure 3 shows the fault indicators related to y_1 and y_2. The faults are clearly detected and isolated.

The corrected measurements (according to the equation (11)) are represented on the figure 2 by discontinuous line.

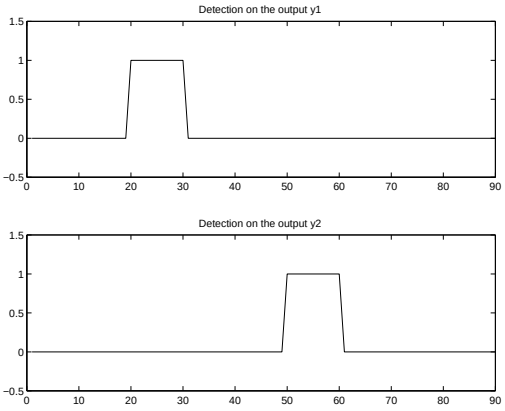

Figure 3. Fault detection indicators

An interval state estimation, coherent with the model of the system, is represented on the figure 4. It is clear that the real state (discontinuous line) is included inside the interval estimation.

6 CONCLUSION

The majority of physical systems present one or more uncertain parameters whose bounds are supposed to be known in general. In order to analyze the functioning of such systems and eventually to adjust the control parameters, it is important to correctly estimate the state from the collected measurements. However, due to uncertainties, the data validation resulting from such systems is delicate to realize. The uncertainties on the parameters of the system make

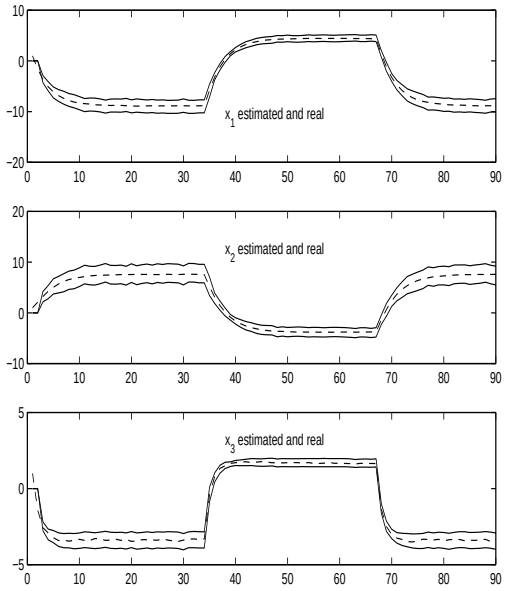

Figure 4. Actual state and its estimation interval

the data validation even more complicated in the presence of faults (acting on the measurement devices for example).

The suggested method presents a new approach for the detection and isolation of faulty measurements, moreover it allows to correct these measurements a posteriori. Thus we realize the data validation in the presence of faults on the measurements for systems with uncertain parameters, where the bounds of uncertainties are known. The method does not use a priori any knowledge on the statistical distribution of the faults affecting the measurements. The suggested method is easy to implement and uses only operations on intervals. It can be applied to all uncertain dynamic linear systems. In future works it will be important to extend the proposed method to certain class of non linear systems. Moreover, attention will be given to the characterization of the uncertainties and mainly to the estimation of their bounds.

APPENDIX

Gauss elimination

Consider an interval linear model $[A]x = [b]$ where $[A] \in \mathbb{R}^{n.n}$ is a regular matrix. The calculation of x by Gauss elimination is done by using a LU factorization, which transforms the matrix $[A]$ into two triangular matrices, lower $[L]$ and upper $[U]$ with $[A] \subseteq [L][U]$. Thus the original system becomes $[L][U]x \supseteq [b]$. As for certain systems, the solution is obtained by solving the following two subsystems:

$$[L]\, y \supseteq [b]$$
$$[U]\, x \supseteq [y]$$

The elements of L and U are obtained using the following iterative calculus:

$$[l_{i,j}] = \frac{\left([a_{i,j}] - \sum_{k=1}^{j-1} [l_{i,k}][u_{k,j}]\right)}{[u_{j,j}]}, \;\; 0 \notin [u_{j,j}], \quad j = 1...i-1$$

and

$$[u_{i,j}] = [a_{i,j}] - \sum_{k=1}^{i-1} [l_{i,k}][u_{k,j}] \quad j = 1...n$$

From the known matrices $[L]$ and $[U]$, the solution of the first triangular system :

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ [l_{2,1}] & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ [l_{n,1}] & [l_{n,2}] & \cdots & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \supseteq \begin{pmatrix} [b_1] \\ [b_2] \\ \vdots \\ [b_n] \end{pmatrix}$$

is obtained according to the relation :

$$[y_i] \supseteq [b_i] - \sum_{j=1}^{i-1} [l_{i,j}][y_j] \quad i = 1...n$$

The solution $[x]$ of the second triangular system:

$$\begin{pmatrix} [u_{1,1}] & 0 & \cdots & [u_{1,n}] \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & [u_{n-1,n-1}] & [u_{n-1,n}] \\ 0 & 0 & \cdots & [u_{n,n}] \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \supseteq \begin{pmatrix} [y_1] \\ [y_2] \\ \vdots \\ [y_n] \end{pmatrix}$$

is obtained in the same way using the formula:

$$[x_i] \supseteq \frac{[y_i] - \sum_{j=i+1}^{n} [u_{i,j}][x_j]}{[u_{i,i}]} \quad i = n...1$$

Gauss-Seidel iteration

The principle of this method consists to develop the interval linear model, described in the preceding method, row by row as follows :

$$[a_{i,1}]x_1 + [a_{i,2}]x_2 + ... + [a_{i,n}]x_n = [b_i] \quad i = 1...n$$

By solving the i^{th} equation in respect to v_i gives :

$$[x_i] = \frac{[b_i] - [v_i]}{[a_{i,i}]}$$

where :
$[v_i] = [a_{i,1}]x_1 + ... + [a_{i,i-1}]x_{i-1} + [a_{i,i+1}]x_{i+1}.... + [a_{i,n}]x_n$

If we suppose that x belongs to a paving $[X]$, in other words each x_j belongs to an interval $[X_j]$, then we can write the preceding expression in the form :

$$x_i \in [y_i] = \frac{[b_i] - [w_i]}{[a_{i,i}]}$$

where :

$$[w_i] = [a_{i,1}][X_1] + ... + [a_{i,i-1}][X_{i-1}] + [a_{i,i+1}][X_{i+1}]...+ \\ .... + [a_{i,n}][X_n]$$

To ensure that each component belongs to the research interval, we add the constraint:

$$x_i \in [x_i] = [y_i] \cap [X_i]$$

The resolution can be improved by using in the subsequent steps the partial results obtained after a particular step. Thus, solving the i^{th} equation leads to a solution interval $[x_i]$ which, in general, is smaller

than $[X_i]$. This intermediate results can be used in the expression of $[y_i]$:

$$[y_i] = \frac{1}{[a_{i,i}]} \left([b_i] - \sum_{j=1}^{i-1} [a_{i,j}]\,[x_j] - \sum_{j=i+1}^{n} [a_{i,j}]\,[X_j] \right)$$
$$[x_i] = [y_i] \cap [X_i]$$

We perform again this calculation until we cannot minimize any component of the vector x compared to the old value of this vector.

Krawczyk method

Krawczyk method is also an iterative method of resolution based on the following formula:

$$[x^{j+1}] = (A_c^{-1}[b] + (I - A_c^{-1}[A])[x^j]) \cap [x^j]$$

This iterative calculus is stopped when the estimation has converged, i.e. $[x^{j+1}] = [x^j]$. For more details on the three methods, the reader is invited to consult [9] and [5].

REFERENCES

[1] O. Adrot, *Diagnostic à base de modèles incertains utilisant l'analyse par intervalles : l'approche bornante*, Doctorat de l'Institut National Polytechnique de Lorraine, Spécialité automatique, Vandoeuvre-les-Nancy, France, 2000.

[2] M. Alhaj Dibo, D. Maquin, and J. Ragot, 'Validation de données par approche bornante', *Journées Doctorales d'Automatique - Valencienne - France*, 347–352, (2003).

[3] J. Armengol, J. Vehi, M. Angel Sainz, and P.J. Herrero, 'Fault detection in a pilot plant using interval models and multiple sliding time windows', *Safeprocess - Washington - USA*, 729–734, (2003).

[4] I.C. Chang, C.C. Yu, and C.T.J. Liou, 'Model-based approach for fault diagnosis. part 2. extension to interval systems', *Industrial Engineering Chemical Research*, **34**, 828–844, (1995).

[5] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, *Applied Interval Analysis*, Springer, London, 2001.

[6] A. Johansson and T. Norlander, 'Parametric uncertainty in sensor fault detection for a turbofan jet engine', *42nd IEEE - Conference on Decision and Control - Hawaii - USA*, 1950–1955, (2003).

[7] D. Maquin and J. Ragot, 'Validation de données issues de systèmes de mesure incertains', *Journal Européen des Systèmes Automatisés RS-série JESA numéro spécial Applications des outils de calculs ensemblistes*, **37, n° 9**, (2003).

[8] M. Milanese, J. Norton, H. Piet-Lahanier, and E. Walter, *Bounding approaches to system identification*, Plenum Press, New York and London, 1996.

[9] A. Neumaier, *Interval methods for systems of equations*, Cambridge University Press, Cambridge, 1990.

[10] R.J. Patton, P.M. Frank, and R.N. Clark, *Fault diagnosis in dynamic systems : theory and application*, Prentice Hall, New Jersey, 1989.

[11] S. Ploix, *Diagnostic des systèmes incertains : l'approche bornante*, Doctorat de l'Institut National Polytechnique de Lorraine, Spécialité automatique, Vandoeuvre-les-Nancy, France, 1998.

[12] J. Ragot, A. Boukhris, and D. Mandel, 'A propos de l'algèbre des intervalles. application à la validation de données', *Rencontres Francophones sur la Logique Floue - Lyon - France*, (1997).