Band Structure of Crystals with Periodically Loaded Metallic Wires

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Introduction

The propagation of electromagnetic waves in periodic structures has received recently an important interest [1]. Potential applications have been suggested in microwave and antenna domains, such as suppressing surface waves [2], designing directive antennas [3], or creating controllable microwave components [4].
The propagation of waves in periodic structures is described by means of a band theory. Different methods have been proposed for computing the band structure of periodic structures, *e.g.*, the average field method [6], the order-N method [7], and the hybrid plane-wave-integral-equation method [8]. A particular interest has been given to the dispersion characteristics of periodic structure formed by infinitely long metallic wires [1-10]. The band structure of periodic materials with loaded wires has not been studied enough. These materials are interesting for designing reconfigurable microwave components. The band structure of the discontinuous wire medium for different wire diameters and lengths has been studied in [11] in order to design controllable crystals. However, the effects of the active element have not been taken into account. In [12], an analysis of the dispersion of crystals with loaded wires has been presented. However, in open literature, there is no parametrical study for showing the effect of the value of the active elements.
In this contribution, numerical results are presented for the pass-bands and stop-bands of these 3-D periodic structures, at normal incidence. To compute the propagation constant, a transmission line model is used, where a 2-D periodic structure (grid) of discontinuous wires is modelled by a T-circuit. The T-circuit parameters are written in terms of the S-parameters of the grid, computed rigourously using the FDTD method.

Computation of the propagation constant

An infinite 3-D periodic structure of perfect metallic wires shown in Fig. 1 is considered. Its parameters are the periods P_x, P_y and P_z, the wire diameter a and the width w. The propagation of the transverse electric field in x-direction is considered. To compute the propagation constant β_x, the transmission line model is used, where a 2-D periodic structure in y-direction (see Fig. 1) is modelled by a T-circuit [11]. The T-circuit parameters are written in terms of the S-parameters of the grid, computed rigourously using the FDTD method, where Floquet boundaries conditions and a thin mesh ($\triangle = Period/80$) are used. Only the fundamental mode is considered, then the limitations $P_y \leq P_x$, $P_x \leq \lambda$ and $P_z \leq \lambda$ are used.
In a first approximation, an electronic switch can be simulated by an equivalent circuit including R-L-C elements. The inductive term L, which essentially represent the connection wires to the device, can be considered included in the metallic wire, then active device can be represented only by an R-C circuit [4].
For a parallel or a series combination of a capacitor, C, a resistor, R, and an inductance, L, we integrated a model in our FDTD code, based on the scheme introduced by Piket-May *et al* [13]. The R-C circuits are periodically distributed along the wires,

which form the 2-D photonic lattice.

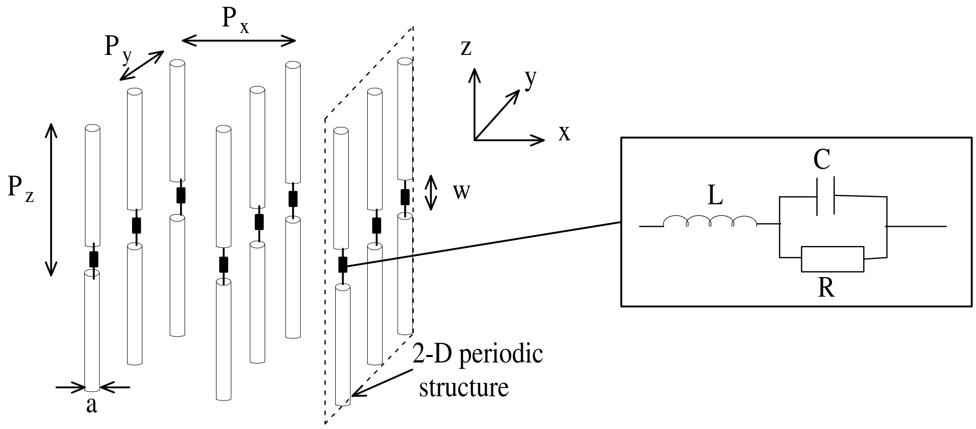

Figure 1: Infinite 3-D periodic structure of loaded metallic wires in air and equivalent RLC circuits for numerical simulations.

Results and discussion

The dual behavior in the pass-band and stop-band of the on-state and off-state structures is nearly obtained in the two first bands [11]. The limits of the two first bands of these structures are now studied for different wire diameters. We consider $P_x = P_y = P_z = P$. The R-C elements are chosen in agreement with characterization results obtained on high-speed commercial devices [4]. Based on practical considerations, we consider, for the on-state, $R = 10\Omega$; for off-state, we consider $R = 30k\Omega$. Three capacitance values are chosen: $C = 150\ fF$, $30\ fF$ and $13\ fF$.

Fig. 2 presents the limits of the two first bands for the on-state case, and for the continuous-wire structure, versus the fill factor a/P. From this figure, it can be seen that the active element has less influence when the wire diameter is small. This is due to the fact that, for large diameter wires, the contrast between the thickness of the wire and the thickness of the active element is more important.

Fig. 3 presents the limits of the two first bands for the off-state case, for the continuous-wire structure, and for the discontinuous-wire structure, versus the fill factor a/P. According to this figure, compared to the discontinuous wire case, the active element have effect on small diameter wires and has no influence on large diameter wires. In addition, it can be also observed that for small diameter wires, the increase of the capacitance has the same effect that the increase of the width between wires for the discontinuous-wire case [11].

Conclusion

In this paper, the band structure for normal propagation of crystal formed by periodically loaded metallic wires has been analyzed for different wire diameters and for different values of the load, which are assimilated as diodes. The diodes have been simulated by an equivalent R-C circuit, which has been chosen in agreement with characterization results obtained with high-speed commercial devices. The influences of the values of the R-C elements on on-state and off-state have been analyzed and the results have been compared to the previous results presented for continuous and

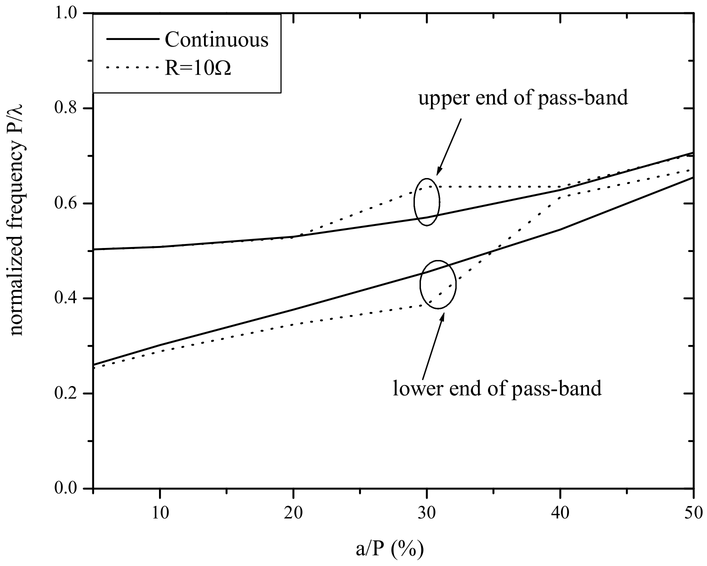

Figure 2: Two first bands limits for structures with continuous wires an for wires periodically loaded with $R = 10\Omega$ wires versus fill factor a/P.

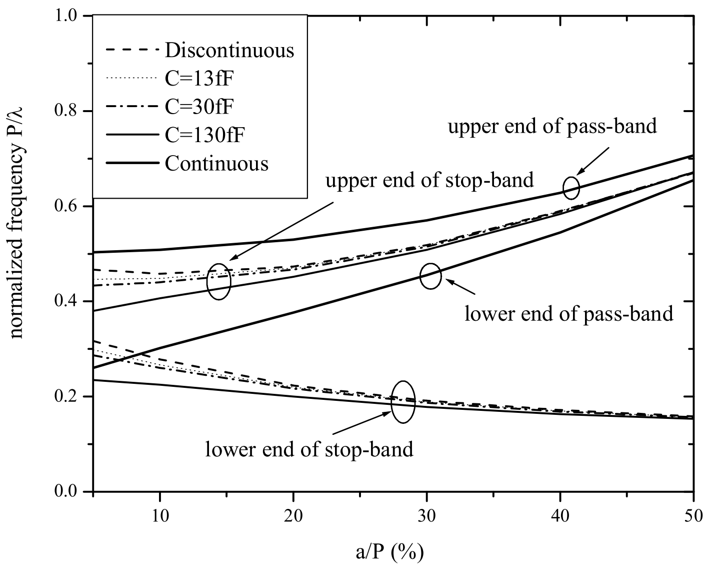

Figure 3: Two first bands limits for structures with continuous, discontinuous, and loaded wires versus fill factor a/P, with $R = 30k\Omega$, for different values of C.

discontinuous-wire structures. A potential application of this work is the design of controllable antennas.

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