



Fuzzy nominal scales

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2 Measurements approaches

Scale formalism

Fuzzy description

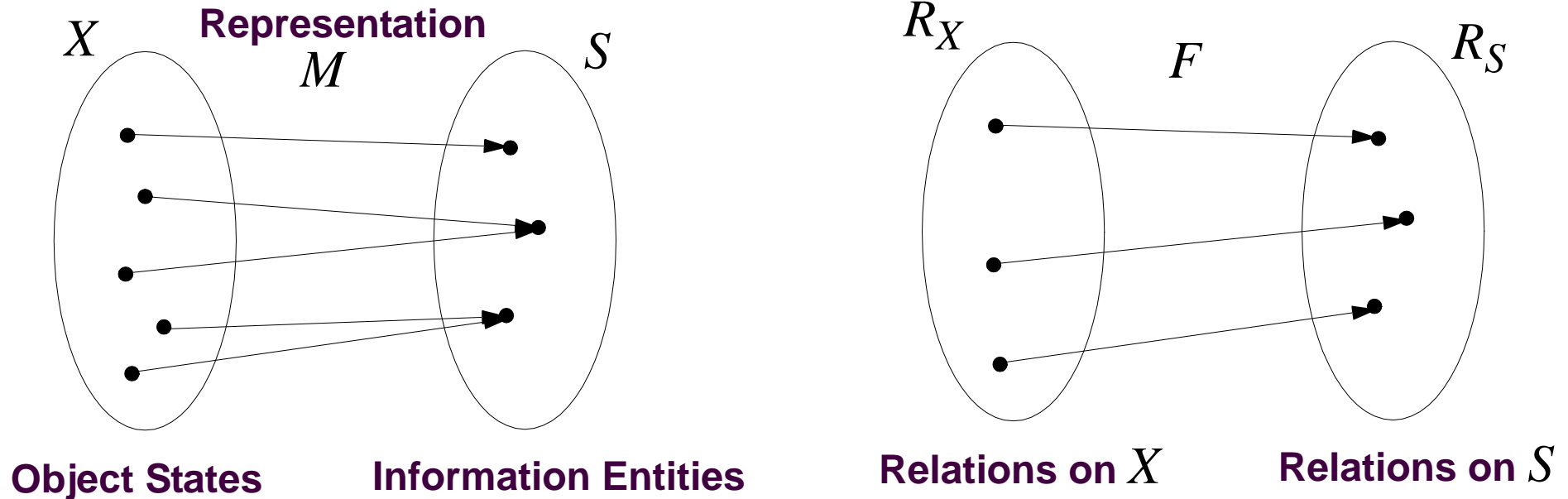
Is a fuzzy description be a scale?

If yes, what kind of scale? A nominal one?

Scales

Definition of a scale:

$$C = \langle X, S, M, R_X, R_S, F \rangle$$



Nominal Scales: $F = \{(\sim, =)\}$

$$\forall(x_i \in X), \forall(x_j \in X), x_i \sim x_j \Leftrightarrow M(x_i) = M(x_j)$$

Used to perform calculus on information entities

Induce limitation on the set of allowed relations:

$F(R_X)$ is the set of significant relations on S

Duality precision/complexity in nominal scales

large set of information entities => precision

small set of information entities => easy to use

Fuzzy description

idea: perform a precise representation into a small set
solution: Fuzzy description

Introduce a Fuzzy relation R between X and a set of words W

Fuzzy description

$$d \quad X \rightarrow S(W)$$

$$\forall x \in X \quad d(x) = \{w \in W | xRw\}$$

Fuzzy meaning

$$m \quad W \rightarrow S(X)$$

$$\forall w \in W \quad m(w) = \{x \in X | xRw\}$$

Fuzzy description

Fuzzy relation characterized by the membership function: $\mu_R(x, w)$

R	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
w_1	0	0	0.9	0	0.5	0.7	0	0.2
w_2	0	0.1	0	0	0	0.3	0.6	0
w_3	1	0.1	0	0.3	0	0	0.4	0.3
w_4	0	0.8	0.1	0.7	0.5	0	0	0.5

$d(x_5)$

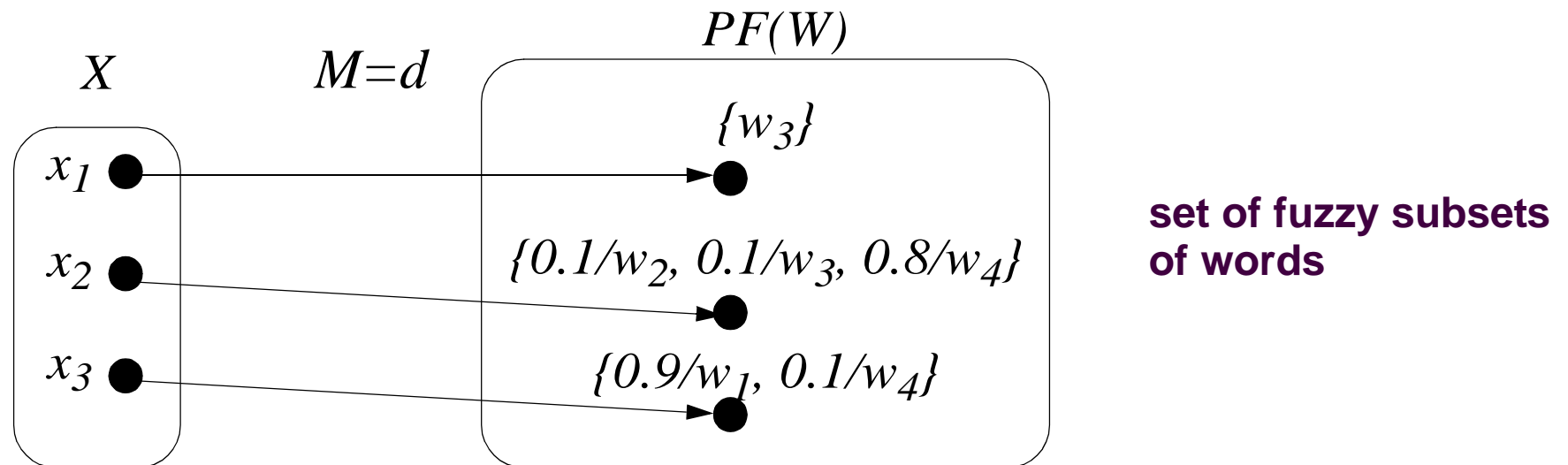
$m(w_3)$

$$\forall (x \in X), \sum_{w \in W} \mu_{m(w)}(x) = 1$$

Fuzzy partition constraint:

Fuzzy description is a scale?

Considering fuzzy subsets of words as information entities



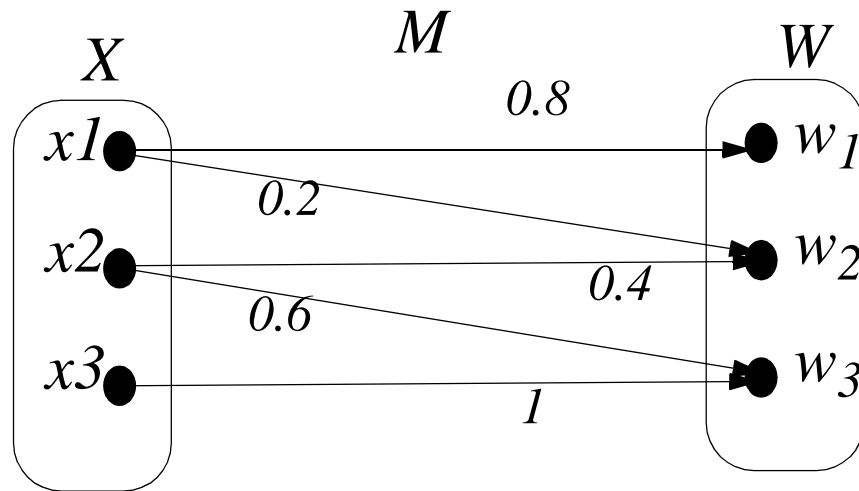
$$C = \langle X, FS(W), d, \{\sim\}, \{\equiv\}, \{(\sim, \equiv)\} \rangle$$

this scale is precise

Lot of information entities => not easy to use

Fuzzy description is a scale?

Considering the fuzzy relation as the representation



$$C = \langle X, W, R, \{\sim\}, \{=\}, \{(\sim, =)\} \rangle$$

$$\forall (w, z) \in W^2 \quad w=z \Leftrightarrow \{w\} \equiv \{z\}$$

this scale stay precise

small set of information entities

«Fuzzy nominal scale»

What kind of scale?

Fuzzy nominal scale defines at least a nominal scale

Fuzzy nominal scale induces a proximity relation on W

w_1 is near w_2 if at least one $d(x)$ of $d(X)$ verifies: $d(x)$ includes both w_1 and w_2

$$\mu_{\text{near}}(w, z) = \sup_{A \in d(X)} (\min(\mu_A(w), \mu_A(z)))$$

Fuzzy nominal scale is more than a simple nominal scale

Proximity scale?

«Recommendations»

**Case 1, a proximity relation on object states is present
fuzzy nominal scales can take this relation into account.**

**Case 2, no proximity relation on object states is present
using a fuzzy description introduces an artificial
proximity relation on information entities.**

**=> operations that use this artificial relation can
perform wrong conclusions**

Conclusion

Fuzzy description is at least as a nominal scale

Fuzzy description is between nominal and ordinal scales

Using fuzzy description can be a bad idea

next studies:

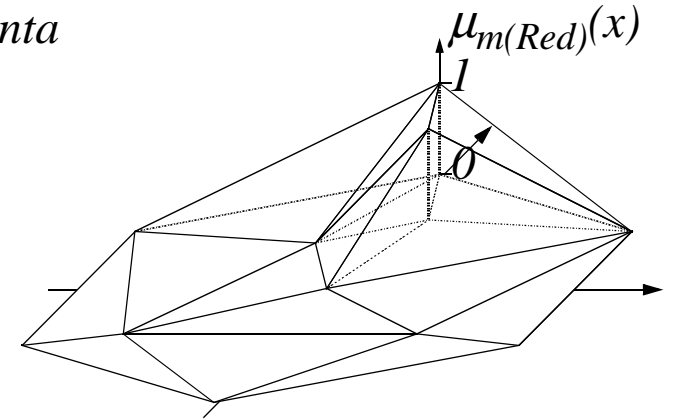
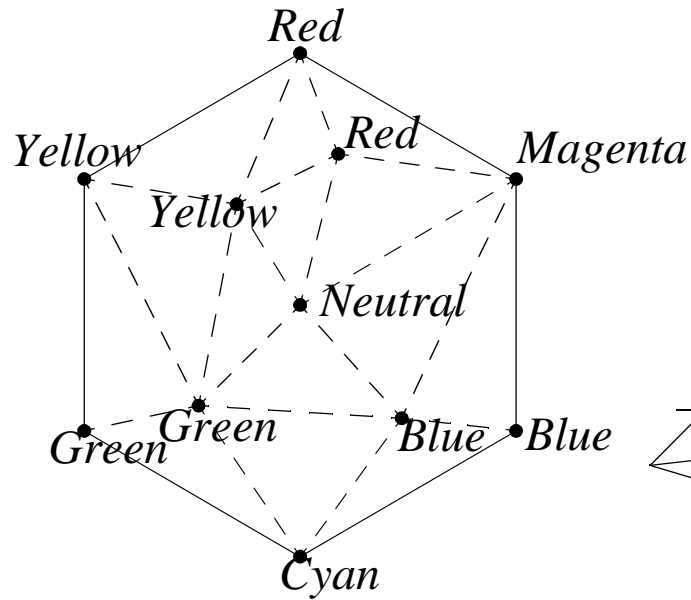
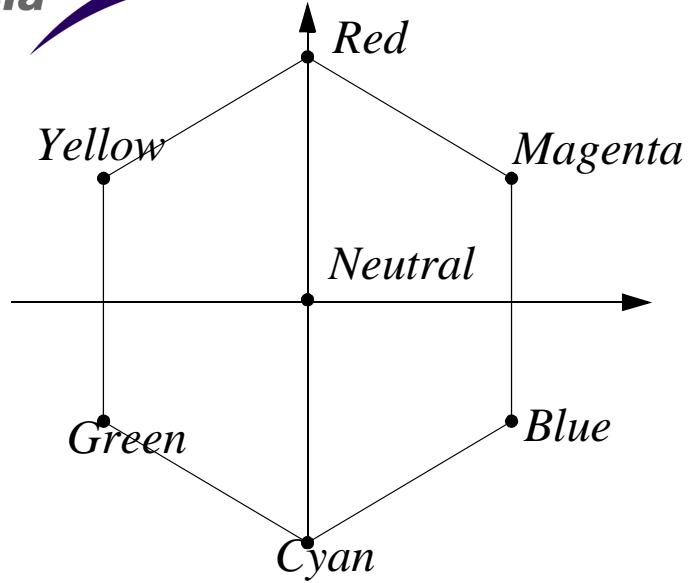
fuzzy nominal scales based on different kind of fuzzy relations

ordinal scales, interval scales, linear scales, ...

operators that use proximity relation (fuzzy rules, topology)

application to formalisms based on nominal scales

Example



	Red	Mag.	Yell.	Neut.	Green	Cyan	Blue
Red	1	0.5	0.5	0.5	0	0	0
Mag.	0.5	1	0	0.5	0	0	0.5
Yell.	0.5	0	1	0.5	0.5	0	0
Neut.	0.5	0.5	0.5	1	0.5	0	0.5
Green	0	0	0.5	0.5	1	0.5	0.5
Cyan	0	0	0	0	0.5	1	0.5
Blue	0	0.5	0	0.5	0.5	0.5	1