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OBSERVABILITY AND VARIABLES CLASSIFICATION IN BILINEAR PROCESSES

Journal A - 1990

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ABSTRACT

An algorithmic approach is presented which allows the separation of observable variables of a process described by bilinear equations. Consequently, this separation gives the equations which contain all the observable variables and allows the reconciliation of the observable data. Several examples demonstrate the developed algorithms and indicate the usefulness of the proposed approach in adjusting the process data through the use of balance equations.

INTRODUCTION

For a process operating under steady state conditions, the variables are constrained by conservation relations ; if we consider as variables the total flowrate and the concentration in chemical species, the conservation laws of material give linear and bilinear equations. In the simple case where all the variables are measured, it is a common practice to test the consistency of data by evaluating the residues of balance equations. Generally, data are conflicting in the sense that these residues are too large ; it is then necessary to reconcile these data by using hypotheses on the statistic distribution of the measurements.

Unfortunately this case of complete observations almost never occurs in reality for reasons of cost, convenience or technical feasibility. Typically, we have a partial set of measurements and we would like to know whether each variable is observable or unobservable. The observability concept takes into account the fact that some variables

are directly measured and that some others can be deduced from the conservation relations.

The concept of redundancy and observability is very easy to understand. Let us consider the small network of *Figure 1* where the points indicate the presence of measurements.

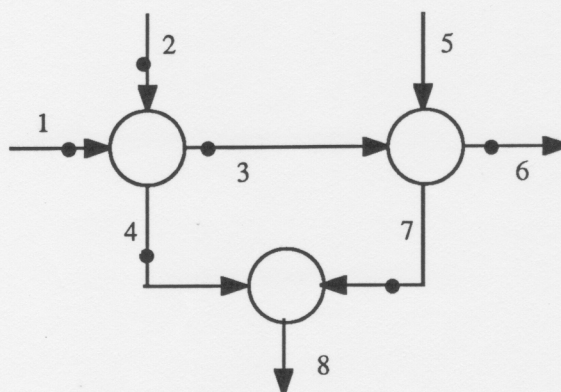


Fig. 1 : A simple network

It is obvious that we can delete the measurement of stream 1 without making the associated variable unobservable ; it is due to the fact that stream 1 can be deduced from 2, 3 and 4 ; so the measurement 1 is said to be redundant. On the contrary stream 6 is not a redundant variable. For complex flowsheets described by graphs, the decomposition based on the concept of observability has been proposed by several authors [2], [7].

The importance of observability concept has been discussed in previous papers [7], [4], [5] and [8]. As an important consequence, classification allows the size reduction of the problem of data reconciliation. In this paper, our approach for treating bilinear systems could be based on using and combining observability for linear system successively for the total flow and for the components.

I - CLASSIFICATION OF VARIABLES : THE LINEAR CASE

The structure of a plant is conveniently represented by mean of its incidence matrix. The balance equations can be represented by :

$$\sum_{j=1}^m a_{kj} X_j = 0 \quad (1)$$

In these equations X_j represents the total flowrate of the stream j ; a_{kj} is the element of the incidence matrix which denotes the topology of units and streams with $a_{kj} = 1$, if stream j is an input to unit k and $a_{kj} = -1$, if stream j is an output from unit k . In matrix notation, equation (1) can be written as follows :

$$A X = 0 \quad (2)$$

In the flowsheet of figure 1, we have observed that some streams are measured, some others are unmeasured but deducible, and the others are not measured and not deducible. A very simple scheme can be used to obtain this classification and specially the redundant part of the flowsheet ; this scheme, called further the linear strategy, can be formalized by a series of matrix transformations. The demonstration of these transformations can be founded in [8] and we only report here the final result. Regular matrix transformation can express the initial incidence matrix A under the equivalent form :

measured variables	unmeasured variables			
A1	I	O	O	deduction equations
A2	O	I	B	
A3	O	O	O	redundancy equations
	observable variables		unobservable variables	

For instance, the flowsheet of *Figure 2* (the point indicates the position of the measurements) is characterized by the following incidence matrix :

nodes	streams								
	1	2	3	4	5	6	7	8	9
I	1	-1	-1
II	.	.	1	-1	-1
III	1	-1	-1	.	.
IV	1	-1	-1

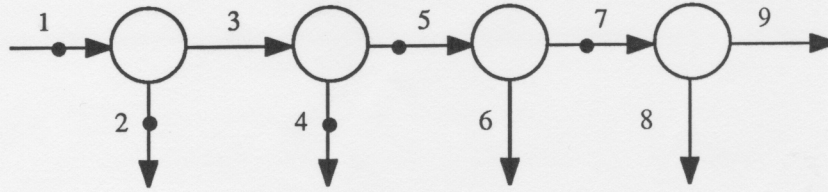


Fig. 2 : Partially measured flowsheet

The application of the procedure LINEAR gives the equivalent partitioned matrix :

equations	measured streams					unmeasured streams			
	1	2	4	5	7	3	6	8	9
1	-1	1	.	.	.	1	.	.	.
3	.	.	.	-1	1	.	1	.	.
4	-1	.	.	1	1
2	1	-1	-1	-1

By direct examination of this matrix, we deduce the classification :

- streams 1, 2, 4 and 5 are measured and redundant
- stream 7 is measured but not redundant
- streams 3 and 6 are unmeasured but deducible
- streams 8 and 9 are unmeasured and undeducible.

II - CLASSIFICATION OF VARIABLES : THE BILINEAR CASE

Let us consider now a component balance equation around unit k of the following form :

$$\sum_{j=1}^m a_{kj} X_j Y_j = 0 \quad (3)$$

where X_j is the flow rate of stream j and Y_j is the molar or mass fraction of the considered component in the j th stream. Note that equation (1) is always available in order to describe the total flow conservation. In matrix notation, equation (3) can be written as follows :

$$A X * Y = 0 \quad (4)$$

where the sign * is used to indicate a product of two vectors element by element.

In order to introduce the classification of variables, we consider a simple node with n streams (n greater than 2), n_x streams measured for the X variables and n_y streams measured for the Y variables. Different cases of interest can appear :

(a) $n_x = n$ and $n_y = n$. In this case, all the variables X and Y are measured ; the considered node is observable in X and Y.

(b) $n_x = n$ and $n_y = n - 1$. Because the X variables are observable (all are measured), the bilinear balance equation permits the deduction of the only one unmeasured Y variable ; the node is completely observable in X and Y.

(c) $n_x = n$ and $n_y < n - 1$. Because there are at least two unmeasured Y variables they cannot be deduced with only one equation. The node is said to be observable in X but unobservable in Y.

(d) $n_x = n - 1$ and $n_y = n$. The only one unmeasured X variable is deducible from the linear balance equation ; the node is completely observable in X and Y.

(e) $n_x = n - 1$ and $n_y = n - 1$. Because the X variables are observable (the only one unmeasured is deducible from the linear balance equation), the bilinear balance equation permits the deduction of the only one unmeasured Y variable.

(f) $n_x = n - 1$ and $n_y < n - 1$. The only one unmeasured X variable is deducible ; the node is completely observable in X. In the bilinear balance equation there are at least 2 unmeasured variables in Y which are not deducible.

(g) $n_x = n - 2$ and $n_y = n$. Because all the Y variables are measured, the two unknown X variables may be deduced (at the condition that the streams which are unmeasured in X have distinct numeric values for their Y measures) by using the linear and the bilinear balance equations.

The interactions, for the observability concept, between the variables X et Y are only present and used in cases (b), (e), (g). The given results are available for a node but can be directly generalized for an agregation of nodes and for a whole flowsheet.

Let us now summarize, in the so called procedure BILINEAR1, the different aspects discussed earlier in a systematized algorithmic approach, in order to classify the variables for a given system. Note that on a numerical point of view, the procedure only uses very simple calculus : extraction of a regular part of a matrix, permutation of rows and columns of a matrix, counting measured streams on a node or on a group of nodes.

Procedure BILINEAR1

SOBSX is the list of the observable streams in X and SOBSY is the list of the observable streams in Y. At the beginning, these two lists are formed with the measured streams. The procedure is divided into two major steps. The first one is concerned with the linear observability : we detect the nodes which have streams completely observable in X and the nodes which have streams completely observable in Y. The second step is devoted to the deduction of unmeasured streams by using the informations given mutually by the X and Y measurements.

Initialization : the lists SOBSX and SOBSY are filled with the numbers of the measured streams in X, Y.

Step 1 :

1a - apply procedure LINEAR to classify the X variables

if some X variables are deducible, complete the list SOBSX

obtain the list NOBSX of the nodes with streams completely observable in X

1b - apply procedure LINEAR to classify the Y variables

obtain the list NOBSY of the nodes with streams completely observable in Y

Step 2 :

2a - find the nodes in NOBSX with one unobserved Y stream

as the corresponding value can be deduced, complete the list SOBSY

2b - find the nodes in NOBSY with two unobserved X streams

as the corresponding values can be deduced, complete the list SOBSX

take care of the case of nodes with only two streams : the knowledge of Y values is not sufficient to determine the X values

if SOBSX or SOBSY have changed, go to step 1, else stop the procedure

Consider by example the flowsheet of *Figure 3*.

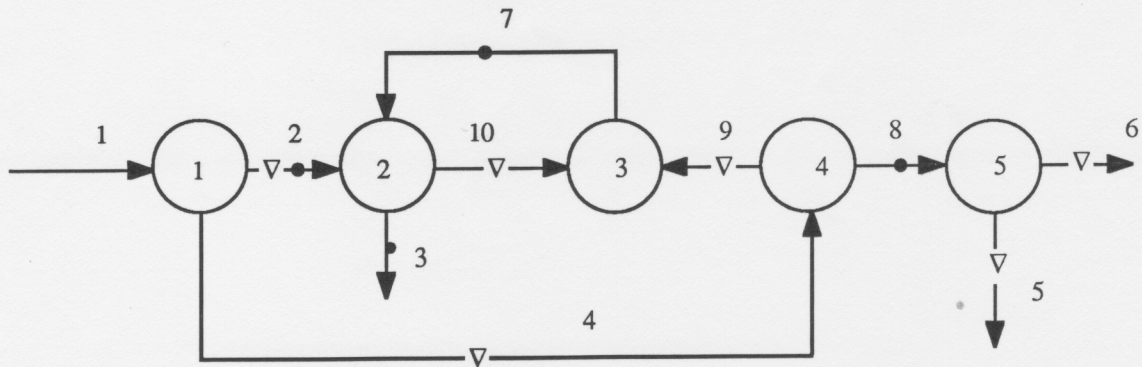


Fig. 3 : A simple flowsheet with measurements of flow (point) and concentration (triangle)

Due to the simplicity of the flowsheet, the classification procedure can be applied by only visual inspection of the measurement positions. The successive results obtained by applying the procedure are the following.

Initialization :

SOBSX = (2, 3, 7, 8)

SOBSY = (2, 4, 5, 6, 9, 10)

NOBSX = ()

NOBSY = ()

Fisrt cycle

Step 1 :

1a - Applying procedure LINEAR for X shows that streams 1, 4, 9 and 10 are observable. Therefore SOBSX and NOBSX are updated :

SOBSX = (2, 3, 7, 8, 1, 4, 9, 10)

NOBSX = (I, II, III, IV)

1b - Applying procedure LINEAR for Y shows that the agregated node IV+V has all its streams observed. Then :

NOBSY = (IV+V)

Step 2 :

2a - All the nodes of NOBSX have only one unmeasured Y stream except node II. Therefore SOBSY and NOBSY are updated as :

SOBSY = (2, 4, 5, 6, 9, 10, 8, 7, 1)

NOBSY = (I, III, IV, V)

2b - Because the node V is completely observed in Y, the two unmeasured streams (5 and 6) in X can be deduced. The corresponding lists are updated as :

SOBSX = (2, 3, 7, 8, 1, 4, 9, 10, 5, 6)

NOBSX = (I, II, III, IV, V)

Second cycle

Step 1 : nothing

Step 2 :

2a - The node II has only one unmeasured stream (stream 3) ; it can then be deduced and the lists are updated as :

SOBSY = (2, 4, 5, 6, 9, 10, 8, 7, 1, 3)

NOBSY = (I, II, III, IV, V)

As a conclusion, the flowsheet is totally observable in X and Y.

III - THE GENERALIZED BILINEAR CASE

Let us consider now a component balance equation around unit k of the following form :

$$\sum_{j=1}^m a_{kj} X_j Y_{ij} = 0 \quad i = 1, \dots, n_c \text{ (number of components)} \quad (5)$$

where X_j is the flow rate of stream j and Y_{cj} is the molar or mass fraction of the considered component c in the jth stream.

In matrix notation, equation (5) can be written as follows :

$$A X * Y_i = 0 \quad i = 1, \dots, n_c \quad (6)$$

Procedure BILINEAR2 is given for two components but can be easily extended for a number n_c of components. As BILINEAR1, the procedure BILINEAR2 is divided into two major steps. The first one is concerned with the linear observability : we detect the nodes which have streams completely observed in X and the nodes which have streams completely observed in Y_1 and Y_2 . The second step is devoted to the deduction of

unmeasured streams by using the mutual information given by the X , Y_1 and Y_2 variables.

Procedure **BILINEAR2**

Initialization : the lists SOBSX, SOBSY1 and SOBSY2 are filled with the numbers of the measured streams in X , Y_1 and Y_2 ; the lists NOBSX, NOBSY1 and NOBSY2 are formed with the numbers of the corresponding nodes.

Step 1 :

1a - apply procedure LINEAR to classify the X variables

if some X variables are deducible, complete the list SOBSX

complete the list NOBSX of the nodes with streams completely observed in X

1b - apply procedure LINEAR to classify the Y_1 , Y_2 variables

complete the list NOBSY1 of the nodes with streams completely observed in Y_1

complete the list NOBSY2 of the nodes with streams completely observed in Y_2

Step 2 :

2a - find the nodes in NOBSX with one unobserved Y_1 stream ; as this value can be deduced, complete the lists SOBSY1 and NOBSY1

find the nodes in NOBSX with one unobserved Y_2 stream ; as this value can be deduced, complete the lists SOBSY2 and NOBSY2

2b - find the nodes in NOBSY1 with two unobserved X streams ; as these values can be deduced, complete the lists SOBSX and NOBSX

find the nodes in NOBSY2 with two unobserved X streams ; as these values can be deduced, complete the lists SOBSX and NOBSX

2c - find the nodes in NOBSY1 and NOBSY2 with three unobserved X streams and complete the lists SOBSX and NOBSX (take care of the case of nodes with only two and three streams ; similar remark as the one given in step 2 of BILINEAR1 algorithm)

if SOBSX, SOBSY1 or SOBSY2 have changed, go to step 1.

The flowsheet of *Figure 4* has been drawn in order to illustrate the different possible situations.

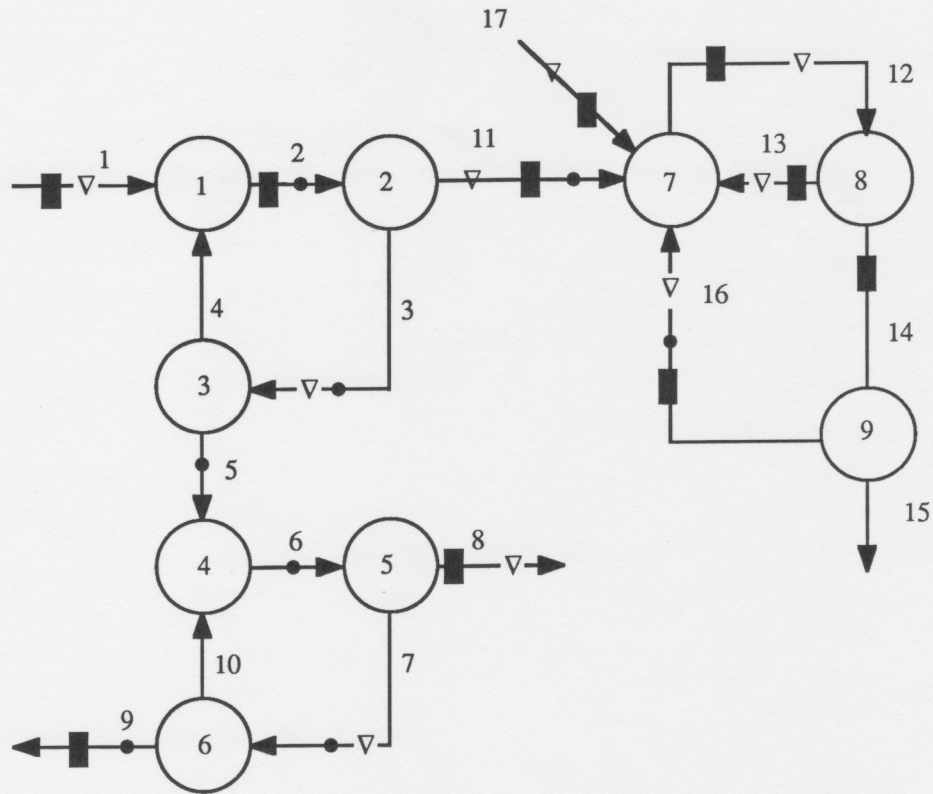


Fig. 4 : Points indicate flow X, squares the first component Y1 and triangles the second component Y2

Initialization

streams "X" observable	2, 3, 5, 6, 7, 9, 11, 16	[direct measurement]
streams "Y1" observable	1, 2, 8, 9, 11, 12, 13, 14, 16, 17	[direct measurement]
streams "Y2" observable	1, 3, 7, 8, 11, 12, 13, 16, 17	[direct measurement]

First cycle

Step 1

streams "X" observable	1, 4, 8, 10	[obtained with observability in X]
------------------------	-------------	--------------------------------------

Step 2

stream "Y1" observable	3, 4, 5	[deduced from a complete node in X]
stream "Y2" observable	2, 4, 5, 6, 9, 10	[deduced from a complete node in X]

Second cycle

Step 1

nothing

Step 2

streams "X" observable	12, 13, 17	[deduced from a complete node in Y]
------------------------	------------	---------------------------------------

Third cycle

Step 1

streams "X" observable 14, 15 [obtained with observability in X]

Step 2

stream "Y1" observable 15 [deduced from a complete node in X]

streams "Y2" observable 14, 15 [deduced from complete nodes in X]

A fourth cycle gives no more information on the observability point of view. Finally, the observable variables are :

$$X = [2, 3, 5, 6, 7, 9, 11, 16, 1, 4, 8, 10, 12, 13, 17, 14, 15]$$

$$Y1 = [1, 2, 8, 9, 11, 12, 13, 14, 16, 17, 3, 4, 5, 15]$$

$$Y2 = [1, 3, 7, 8, 11, 12, 13, 16, 17, 2, 4, 5, 6, 9, 10, 14, 15]$$

It is also possible to obtain the observation equations which would be useful to reconcile all the observable measurements. The preceding classification algorithm simultaneously gives the corresponding equations.

For the linear equations :

$$X_1 - X_2 + X_4 = 0$$

$$X_2 - X_3 - X_{11} = 0$$

$$X_3 - X_4 - X_5 = 0$$

$$X_5 - X_6 + X_{10} = 0$$

$$X_6 - X_7 - X_8 = 0$$

$$X_7 - X_9 - X_{10} = 0$$

$$X_{11} - X_{12} + X_{13} + X_{16} + X_{17} = 0$$

$$X_{12} - X_{13} - X_{14} = 0$$

$$X_{14} - X_{15} - X_{16} = 0$$

For the bilinear equations due to component 1 :

$$X_1 Y_{1,1} - X_2 Y_{1,2} + X_4 Y_{1,4} = 0$$

$$X_2 Y_{1,2} - X_3 Y_{1,3} - X_{11} Y_{1,11} = 0$$

$$X_5 Y_{1,5} - X_8 Y_{1,8} - X_9 Y_{1,9} = 0$$

$$X_3 Y_{1,3} - X_4 Y_{1,4} - X_5 Y_{1,5} = 0$$

$$X_{11} Y_{1,11} - X_{12} Y_{1,12} + X_{13} Y_{1,13} + X_{16} Y_{1,16} + X_{17} Y_{1,17} = 0$$

$$X_{12} Y_{1,12} - X_{13} Y_{1,13} - X_{14} Y_{1,14} = 0$$

$$X_{14} Y_{1,14} - X_{15} Y_{1,15} - X_{16} Y_{1,16} = 0$$

For the bilinear equations due to component 2 :

$$\begin{aligned} X_1 Y_{2,1} - X_2 Y_{2,2} + X_4 Y_{2,4} &= 0 \\ X_2 Y_{2,2} - X_3 Y_{2,3} - X_{11} Y_{2,11} &= 0 \\ X_3 Y_{2,3} - X_4 Y_{2,4} - X_5 Y_{2,5} &= 0 \\ X_5 Y_{2,5} - X_6 Y_{2,6} + X_{10} Y_{2,10} &= 0 \\ X_6 Y_{2,6} - X_7 Y_{2,7} - X_8 Y_{2,8} &= 0 \\ X_7 Y_{2,7} - X_9 Y_{2,9} - X_{10} Y_{2,10} &= 0 \\ X_{11} Y_{2,11} - X_{12} Y_{2,12} + X_{13} Y_{2,13} + X_{16} Y_{2,16} + X_{17} Y_{2,17} &= 0 \\ X_{12} Y_{2,12} - X_{13} Y_{2,13} - X_{14} Y_{2,14} &= 0 \\ X_{14} Y_{2,14} - X_{15} Y_{2,15} - X_{16} Y_{2,16} &= 0 \end{aligned}$$

We note that these equations, which they are used all together to reconcile the data, contain measured variables (some of them are redundant) and unmeasured but deducible variables.

IV - DATA VALIDATION

From the whole process, we are now able to isolate the part which is observable (deducible and redundant). We can reconcile the measured data of this subprocess by taking into account the corresponding balance equations. As we are concerned with bilinear equations, the estimation problem has, generally, no analytical solution. However we can propose an iterative procedure.

Generally speaking, the balance equations for the redundant variables, can always be written as :

$$A \hat{X} = 0 \tag{7}$$

$$(B_i \otimes \hat{Y}_i) S_i \hat{X} = 0 \tag{8}$$

with A of dimension $n_a.v_a$, B_i of dimension $n_{bi}.v_{bi}$, and S_i , a selection matrix. We recognize that A is the incidence matrix for the linear equations, B_i is the incidence matrix for the bilinear equations ($i=1, \dots, nc$) and S_i selects the components of X used with Y in order to form the bilinear equations. The operator \otimes is applied between the

matrix B_i and the vector \hat{Y}_i in order to form a matrix in which the line k is obtained by multiplying each term of the line k of B_i by each term of \hat{Y}_i .

The least squares estimation problem of true values is reduced to find the minimum with regard to \hat{X} et \hat{Y}_i of the criterion (with $i = 1, \dots, nc$ the number of components for the partial balance equations) :

$$\Phi = \frac{1}{2} \| H\hat{X} - X \|_{V_X^{-1}}^2 + \sum_{i=1}^{nc} \frac{1}{2} \| H_i \hat{Y}_i - Y_i \|_{V_{Y_i}^{-1}}^2 \quad (9)$$

subject to :

$$A \hat{X} = 0 \quad (10)$$

$$(B_i \otimes \hat{Y}_i) S_i \hat{X} = 0 \quad i = 1, \dots, nc \quad (11)$$

where H and H_i are the matrices of measurement function for X and Y_i ,

X and Y_i are the measurement vectors of respective dimensions m and v_{bi} ,

\hat{X} and \hat{Y}_i , the estimation vectors of dimensions v_a and v_{bi} ,

V_X^{-1} and $V_{Y_i}^{-1}$ are the inverse variance matrices of the measurements X and Y_i .

In order to simplify the notation let us also define the diagonal matrix $\Lambda_{S_i X}$ formed, on its diagonal, with the elements of $S_i \hat{X}$. It is then clear that equation (11) can also be expressed as :

$$B_i \Lambda_{S_i X} \hat{Y}_i = 0 \quad i = 1, \dots, nc \quad (12)$$

Similarly, we define the diagonal matrix Λ_{Y_i} formed with \hat{Y}_i vector ; the preceding equation takes also the form :

$$B_i \Lambda_{Y_i} S_i \hat{X} = 0 \quad i = 1, \dots, nc \quad (13)$$

The Lagrangian associated to problem (9, 10, 11) is :

$$L = \Phi + \lambda^T A \hat{X} + \sum_{i=1}^{nc} \mu_i^T (B_i \otimes \hat{Y}_i) S_i \hat{X} \quad (14)$$

The stationary conditions of first order require that :

$$\frac{\partial L}{\partial \hat{X}} = H^T V_X^{-1} (H \hat{X} - X) + A^T \lambda + \sum_{i=1}^{nc} (B_i \Lambda_{Y_i} S_i)^T \mu_i = 0 \quad (15a)$$

$$\frac{\partial L}{\partial \hat{Y}_i} = H_i^T V_{Y_i}^{-1} (H_i \hat{Y}_i - Y_i) + (B_i \Lambda_{S_i X})^T \mu_i = 0 \quad i=1, \dots, nc \quad (15b)$$

$$\frac{\partial L}{\partial \lambda} = A \hat{X} = 0 \quad (15c)$$

$$\frac{\partial L}{\partial \mu_i} = (B_i \otimes \hat{Y}_i) S_i \hat{X} = 0 \quad i=1, \dots, nc \quad (15d)$$

Generally, the system of equations (15) has not any analytical solution ; approximate solutions must be found (for example by linearization of equations), or iterative procedures based on hierarchical calculus, must be examined.

We propose a simple solution which is based on a two levels relaxation principle. In order to simplify the notations, let :

$$N_{X_i} = B_i \Lambda_{S_i X} \quad (16)$$

$$N_{Y_i} = B_i \Lambda_{Y_i} S_i \quad (17)$$

The following calculus allows to determine the estimation \hat{X} ; premultiply equation (15c) by A^T :

$$A^T A X = 0 \quad (18)$$

then (15d) by $N_{Y_i}^T$:

$$N_{Y_i}^T N_{Y_i} \hat{X} = 0 \quad i=1, \dots, nc \quad (19)$$

By adding equations (15a), (18) and (19), we obtain :

$$\hat{X} = G (H^T V_X^{-1} X - A^T \lambda - \sum_{i=1}^{nc} N_{Y_i}^T \mu_i) \quad (20)$$

$$\text{with } G = (H^T V_X^{-1} H + A^T A + \sum_{i=1}^{nc} N_{Y_i}^T N_{Y_i})^{-1} \quad (21)$$

Notice that G matrix is always regular because of the global observability of the system.

Thus, by combining (20) and (15c), the estimation \hat{X} can be written as :

$$\hat{X} = (I - G A^T (A G A^T)^{-1} A) G \bar{X} \quad (22)$$

$$\text{with } \bar{X} = H^T V_X^{-1} X - \sum_{i=1}^{nc} N_{Y_i}^T \mu_i \quad (23)$$

Therefore, from equations (15b) and (15d), we obtain :

$$\hat{Y}_i = (I - G_i N_{X_i}^T (N_{X_i} G_i N_{X_i}^T)^{-1} N_{X_i}) G_i H_i^T V_{Y_i}^{-1} Y_i \quad (24)$$

$$\mu_i = (N_{X_i} G_i N_{X_i}^T)^{-1} N_{X_i} G_i H_i^T V_{Y_i}^{-1} Y_i \quad (25)$$

$$G_i = (H_i^T V_{Y_i}^{-1} H_i + N_{X_i}^T N_{X_i})^{-1} \quad (26)$$

From (15b) we have :

$$B^T \mu_i = \Lambda_{S_i X}^{-1} H_i^T V_{Y_i}^{-1} (H_i \hat{Y}_i - Y_i) \quad (27)$$

Then \bar{X} vector given by equation 23, can be written as a function of X , Y_i et \hat{Y}_i :

$$\bar{X} = H^T V_X^{-1} X - \sum_{i=1}^{nc} S_i^T \Lambda_{Y_i} \Lambda_{S_i X}^{-1} H_i^T V_{Y_i}^{-1} (H_i \hat{Y}_i - Y_i) \quad (28)$$

Then, using the definition (16), the estimation \hat{Y}_i becomes :

$$\hat{Y}_i = \Lambda_{S_i X}^{-1} [\Gamma_i - \Gamma_i B_i^T (B_i \Gamma_i B_i^T)^{-1} B_i \Gamma_i] \Lambda_{S_i X}^{-1} H_i^T V_{Y_i}^{-1} Y_i$$

$$\text{with } \Gamma_i = \Lambda_{S_i X} G_i \Lambda_{S_i X} \quad (29)$$

Finally, the solution is obtained through a direct iteration algorithm. The first level estimates \hat{X} (equations 22 and 28) after calculating G as a function of the estimation \hat{Y}_i , initialized to Y_i . The second level estimates \hat{Y}_i (equation 29) after calculating Γ_i as a function of \hat{X} which is produced by the upper level.

Notice that the calculus at each level are relatively reduced, due to the particularly forms of equations (22) and (29). Moreover, it is possible to use a recurrence formula for matrix inversion which degenerates into computing the reciprocal of scalars [8]. The calculus is stopped when all the derivatives of the Lagrangian are kept below a given threshold.

For the flowsheet of Figure 2, the *Table 1* indicates the numerical results ; the first column contains the number of the streams, the second one is related to the measurements and the last to the estimations. The reconciliation has been performed with a unit variance on all the measured data.

Stream	Measurements			Estimations		
	X	Y1	Y2	X	Y1	Y2
1	-	1.525	1.234	20.844	1.493	1.234
2	25.400	1.599	-	25.451	1.599	1.133
3	9.616	-	2.481	9.565	3.117	2.481
4	-	-	-	4.607	2.080	0.676
5	5.182	-	-	4.958	4.080	4.158
6	6.801	-	-	7.026	-	3.425
7	5.432	-	2.000	5.207	-	2.000
8	-	2.470	7.508	1.818	2.472	7.507
9	2.911	5.007	-	3.140	5.012	2.218
10	-	-	-	2.067	-	1.668
11	15.937	0.656	0.319	15.885	0.685	0.321
12	-	0.802	0.944	30.786	0.811	0.942
13	-	1.004	1.578	3.066	1.003	1.578
14	-	0.805	-	27.720	0.789	0.871
15	-	-	-	22.634	0.609	0.833
16	5.086	1.592	1.040	5.086	1.593	1.040
17	-	0.427	2.039	6.748	0.429	2.039

Table 1 : Measurements and estimations

CONCLUSION

An algorithm has been developed for determining the observability of variables in a process network operating under steady state condition. Starting with only one mass flow network, the complexity has been increased by investigation of multicomponent balances. By making intensive use of linear observability we are able to develop and implement a classification algorithm which is computationally quite efficient.

REFERENCES

- [1] CROWE, C.M., Reconciliation of process flow rates by matrix projection, *AIChE journal*, vol. 32, p. 616-623, 1986.
- [2] DAROUACH, M., Observabilité et validation des données de systèmes de grandes dimensions. Application à l'équilibrage des bilans de mesure, thèse de doctorat-ès-sciences, Université de Nancy I, France, 1986.
- [3] ROMAGNOLI, J.A., STEPHANOPOULOS, G., On the rectification of measurement errors for complex chemical plants, *Chemical Engineering Science*, vol. 35, p. 1067-1081, 1980.
- [4] STANLEY, G.M., MAH, R.S.H., Observability and redundancy in process data estimation, *Chemical Engineering Science*, vol. 36, p. 259-272, 1981.
- [5] KRETISOVALIS, A., MAH, R.S.H., Observability and redundancy classification in generalized process networks - I, theorems, *Computer Chemical Engineering*, vol. 12, N° 7, p. 671- 687, 1988.
- [6] KRETISOVALIS, A., MAH, R.S.H., Observability and redundancy classification in generalized process networks - II, algorithms, *Computer Chemical Engineering*, vol. 12, N° 7, p. 689-703, 1988.
- [7] VACLAVEK, V., LOUCKA, M., Selection of measurements necessary to achieve multicomponent mass balance in chemical plant, *Chemical Engineering Science*, vol. 31, N° 2, p. 1199-1205, 1976.
- [8] MAQUIN, D., Observabilité, diagnostic et validation de données des procédés industriels, thèse de doctorat de l'université de Nancy I, France, 1987.