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Elsa Lhotel, Olivier Coupiac, François Lefloch, M. Sanquer, Hervé Courtois. Divergence at low bias and down-mixing of the current noise in a diffusive superconductor-normal metal-superconductor junction. 2007. hal-00143895v1

HAL Id: hal-00143895

<https://hal.science/hal-00143895v1>

Preprint submitted on 27 Apr 2007 (v1), last revised 25 Oct 2007 (v2)

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Divergence at low bias and down-mixing of the current noise in a diffusive superconductor-normal metal-superconductor junction

E. Lhotel,¹ O. Coupiac,¹ F. Lefloch,^{1,*} H. Courtois,² and M. Sanquer¹

¹CEA-Grenoble, DRFCM / SPSMS / LaTEQS, 17 rue des Martyrs, F-38054 Grenoble cedex 09, France

²Institut Néel, CNRS and Université Joseph Fourier,
25 rue des Martyrs, F-38042 Grenoble cedex 09,
France. Also at Institut Universitaire de France

We present current noise measurements in a long diffusive superconductor-normal-superconductor junction, in the temperature range for which current-voltage characteristics are reversible. We focus on the low voltage regime, in which transport can be partially described in terms of coherent multiple Andreev reflections. We show that, when decreasing voltage, the current noise exhibits a strong divergence together with a broad peak. We ascribe this peak to the mixing between the Josephson current beating and the noise of the junction itself. At high temperature, we show that the junction noise corresponds to the thermal noise of a non linear resistor $4k_B T/R$ with $R = V/I(V)$ and no adjustable parameters.

PACS numbers: 74.45.+c, 72.70.+m, 74.40.+k

The full understanding of charge transport mechanisms in diffusive superconductor - normal metal - superconductor (S-N-S) junctions has not yet been achieved. In these junctions, it is well known that the main processes contributing to the current are the multiple Andreev reflections (MAR) [1]: incident quasiparticles with energy $\epsilon < \Delta$ go through successive Andreev reflections at the two S-N boundaries until they reach the gap energy Δ . With an applied bias V , the number of successive MAR is equal to $2\Delta/eV + 1$. Depending on the size of the junction, two situations can be distinguished to describe the non-equilibrium properties of the electronic transport.

The first situation applies to short junctions, $L < \xi < L_\Phi$ (ξ and L_Φ are the superconducting and single particle coherence lengths and L the length of the sample). In this case, successive Andreev reflections are coherent and lead to the formation of Andreev bound states at energy $\pm\Delta$. The calculated noise is strongly enhanced compared to the normal state and corresponds to the partition noise of effective charges $e^* \simeq 2\Delta/V$. These predictions apply to short diffusive S-N-S junctions [2] and to superconducting atomic point-contacts with a small number of conducting channels [3, 4].

The second situation corresponds to long junctions $\xi < L < L_\Phi$, in which the Thouless energy $E_{Th} (= \hbar D/L^2$ where D is the electron diffusion constant) is less than the gap Δ . In that case, two regimes can be distinguished depending on the applied bias. In the incoherent regime ($E_{Th} < eV < \Delta$), the electron and the hole behave, away from the interface, as independent quasiparticles. Consequently, the electronic transport occurs by incoherent MAR which induce excess noise as compared to a similar N-N-N system. Shot noise can then be described in the framework of the semiclassical theory [5, 6, 7], and has been measured experimentally [8, 9].

In the coherent regime obtained at low bias $eV < E_{Th}$,

coherent multiple Andreev reflections are possible within an energy window of E_{Th} . Because the Thouless energy is smaller than the superconducting gap, other processes, like incoherent MAR and inelastic interactions, must be considered. This makes the description of the electronic transport properties rather complex. Current-voltage (IV) characteristics have been calculated at zero temperature [10], but noise has not yet been derived. Recent measurements in such junctions [8, 9] showed that the noise is strongly enhanced at low voltage, which was partially described in the framework of coherent MAR. However, this analysis led to surprisingly large effective charges compared to the expected cut-off of coherent MAR due to inelastic processes.

The experimental work presented in this paper focuses on the coherent regime ($eV < E_{Th}$) in long diffusive S-N-S junctions ($E_{Th} < \Delta$). Our low temperature noise measurements show that, when decreasing bias, the current noise exhibits a strong divergence together with a broad peak. We identified this peak as due to the non-linear mixing between the Josephson current beating at the Josephson frequency $\omega_J/2\pi = 2eV/h$ and the noise of the junction itself. We could then extract, from the measured noise, the intrinsic noise of the junction. The central result of this paper is that the divergence corresponds exactly to the thermal current noise of a non linear resistor with $R = V/I$.

We measured the transport properties and the current noise of S-N-S junctions using a superconducting quantum interference device (SQUID) experimental setup [11] inserted in a dilution fridge ($T_{\min} = 30$ mK). The SQUID is connected to a resistance bridge (See left of Fig. 1) composed of the sample on one side and of a reference resistor R_{ref} on the other side. The current fluctuations propagating in this loop come from the reference resistor and from the sample. The value of the reference resistor $R_{\text{ref}} = 29$ m Ω is chosen so that most of

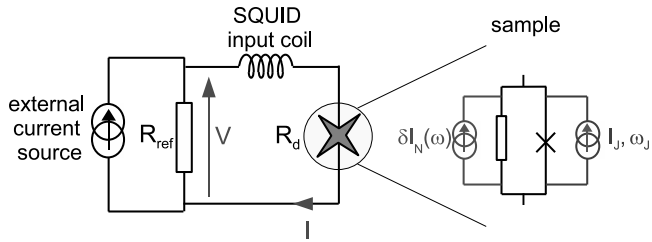


FIG. 1: Left: Scheme of the measurement circuit. The mean current in the sample and the current fluctuations are measured through the input coil of the SQUID. The reference resistor R_{ref} is at the same temperature than the sample of differential resistance R_d . Right: Scheme of the sample, considered as a mesoscopic conductor with a fluctuations source $\delta I_N(\omega)$ in parallel with a Josephson junction oscillating at ω_J .

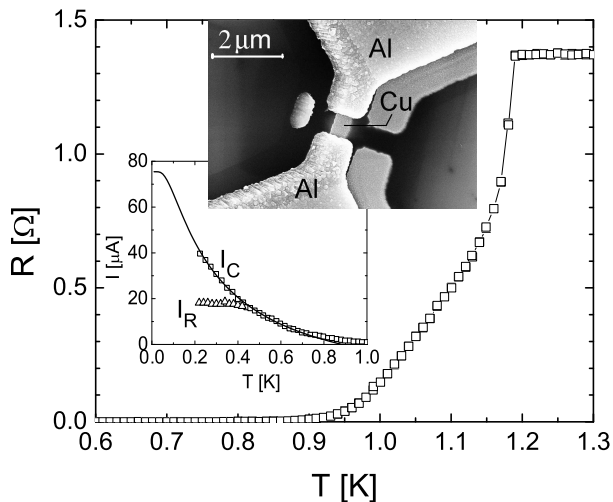


FIG. 2: Zero bias resistance $R_d = dV/dI$ vs. temperature T measured with $I_{ac} = 25$ nA. Top inset: scanning electron micrograph of the sample. Bottom inset: Critical current I_C (square symbols) and retrapping current I_R (triangles) vs. T .

the measured current noise comes from the sample. The intrinsic noise level is about $8 \mu\Phi_0/\sqrt{\text{Hz}}$, which corresponds to $1.6 \text{ pA}/\sqrt{\text{Hz}}$ in the SQUID input coil. Hereafter, the current noise measurements were performed in the frequency range between 4 kHz and 6.4 kHz.

The sample is a Cu bridge of 700 nm length and 425 nm width connected to two thick superconducting Al electrodes (see Fig. 2 inset). It has been fabricated by two-angles evaporation through a polymethyl-methacrylate (PMMA-PMMA/MAA) bilayer mask in a ultra-high vacuum chamber. The Cu and Al thicknesses are 50 and 500 nm respectively. From the Cu diffusion constant estimated to $D = 100 \text{ cm}^2 \cdot \text{s}^{-1}$ by resistance measurements, we get the Thouless energy of the sample E_{Th} equal to $13 \mu\text{eV}$.

The zero bias resistance is represented as a function of temperature in Fig. 2. It shows a small drop at 1.18 K due to the transition of the Al reservoirs. At lower

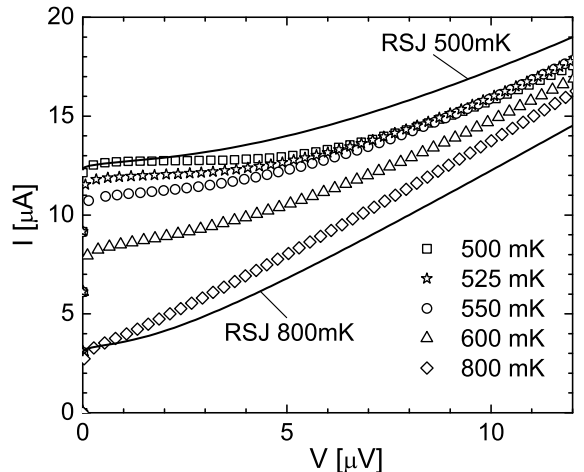


FIG. 3: IV curves at several temperatures between 500 and 800 mK. The lines are obtained from the RSJ model at 500 and 800 mK [13], with the parameters: $R_N = 0.85 \Omega$ and the critical currents set to 12.7 and $3.4 \mu\text{A}$ respectively.

temperatures, the resistance decreases slowly and goes to zero at about 850 mK. Below this temperature, a supercurrent is established. We measured the critical current I_C as a function of temperature down to 200 mK (see inset of Fig. 2). It follows the usual behavior observed in rather long diffusive S-N-S junctions and can be fitted using $eR_N I_C = aE_{Th} (1 - b \exp(-aE_{Th}/3.2k_B T))$. As for parameters, we took $R_N = 0.85 \Omega$ and $a \simeq 5$ and set $b = 1.3$ [12]. Such a value for a shows that the possible barrier resistance is negligible. Below 450 mK, the IV characteristic is hysteretic, with a retrapping current I_R significantly below the critical current. In the following, we will focus on the non-hysteretic regime, i.e. $T \geq 500$ mK.

IV characteristics measured in this regime are shown in Fig. 3 together with the resistively shunted junctions (RSJ) calculation at finite temperature [13] for $T = 500$ and 800 mK. As already mentioned in other studies on S-N-S junctions [8, 14], the curves cannot be described within the RSJ model. Because all the experimental curves are lying between the 500 mK and 800 mK theoretical curves, the discrepancy cannot be simply due to heating effect. From the overall shape of IV characteristics, we can state that some current in the junction is supported by a normal "channel" (see right of Fig. 1), since in pure Josephson junctions no current can flow at voltages less than the gap.

We first measured the shot noise over a large voltage scale (See Fig. 4 inset). The voltage dependence at $V > 50 \mu\text{V}$ can be nicely described within the semiclassical theory of incoherent MAR [5, 6] (with $\Delta = 163 \mu\text{eV}$) as already reported in a previous work on the same kind of junctions [9]. We also confirmed the existence of a noise minimum at the Thouless energy, which appears as a signature of the transition towards the coherent regime

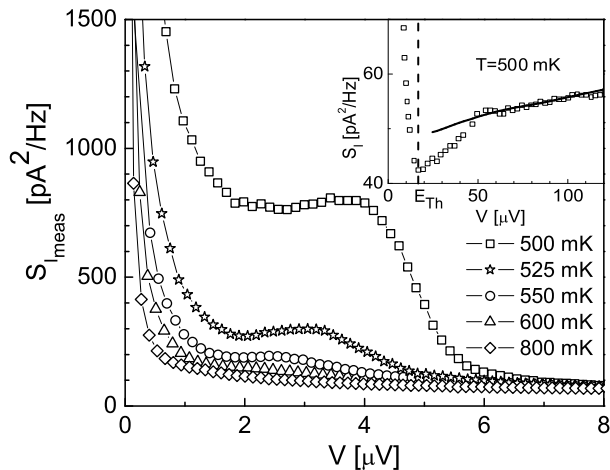


FIG. 4: Current noise $S_{I_{\text{meas}}}$ vs. bias voltage V at several temperatures between 500 and 800 mK. Inset: S_I at 500 mK up to 120 μV . The solid line is the noise prediction due to incoherent MAR [5, 6, 9].

at low voltage, on which we will focus now.

Fig. 4 shows the low voltage ($V < 8 \mu\text{V}$) part of the noise spectral density as a function of bias, at five temperatures from 500 to 800 mK. The first striking result is the overall divergence of the noise at low bias for every temperature. The second important feature is the broad anomaly with a maximum between 2 and 4 μV . When the temperature increases, the anomaly is less and less pronounced, moves to lower voltages and finally disappears around 800 mK.

We analyze this broad peak in terms of a mixing between the non-linear Josephson current beating I_J and the fluctuating current δI_N generated by the junction. In such a situation, the voltage fluctuations $S(\omega)$ at low frequency are determined not only by the current fluctuations at the frequency ω , but also by those close to the Josephson frequency ω_J and its harmonics, inducing the so-called mixed down noise. Following Likharev and Semenov [15], the resulting noise spectral density is:

$$S_V(\omega) = \sum_k |Z_k|^2 S_I(k\omega_J - \omega) \quad (1)$$

where S_I is the current noise spectral density of the fluctuations source and Z_k are the Fourier coefficients of the junction impedance ($V(\omega) = \sum_k Z_k I(\omega - k\omega_J)$). Note that, due to the large SQUID impedance at the Josephson frequency ω_J , there is no coupling between the S-N-S junction and the reference resistor at ω_J . Thus, the reference resistor can not contribute to the mixed-down noise.

At low frequency ($\omega \ll \omega_J$), only the first harmonics ($k = -1, 0, 1$) of the Josephson beating are relevant [16]. Therefore, Eq. (1) becomes, at zero frequency, $S_V(0) = R_d^2 S_I(0) + 2|Z_1|^2 S_I(\omega_J)$. For an arbitrary current-phase relation, Kogan and Nagaev [17] have shown that the coefficient $|Z_1|^2$ is equal to $-V/4 \times \partial R_d / \partial I$. This results

in:

$$S_V(0) = R_d^2 [S_I(0) - \eta S_I(\omega_J)] \quad (2)$$

$$\text{with } \eta = \frac{V}{2R_d^2} \frac{\partial R_d}{\partial I}$$

In our experimental set-up, the SQUID measures the resulting low frequency current noise that propagates through the SQUID input coil. It is given by $S_{I_{\text{meas}}}(0) = S_V(0)/R_d^2 = S_I(0) - \eta S_I(\omega_J)$. Because the thermal energy $k_B T$ is larger than the Josephson frequency $\omega_J/2\pi = 2eV/h$, we can assume that the spectral noise $S_I(\omega_J)$ of the junction does not differ from its zero-frequency value $S_I(0)$. Then,

$$S_{I_{\text{meas}}}(0) = S_I(0)(1 - \eta) \quad (3)$$

We obtained the coupling factor η from the measured quantities V and R_d by numerically differentiating R_d with respect to the calculated I . The quantity $1 - \eta$ is plotted in the inset of Fig. 5. We can then extract, from the measured noise, the intrinsic junction noise $S_I(0) = S_{I_{\text{meas}}}(0)/(1 - \eta)$ (See Fig. 5). For all the curves the broad peak has disappeared or, at least, has strongly been reduced [18]. This clearly justifies the mixed down noise analysis using the generalized coupling factor η . To our knowledge, this prediction of Kogan and Nagaev [17] had never been verified experimentally.

From Fig. 5, we see that $S_I(0)$ exhibits a strong divergence at low voltage. Therefore, it cannot be described by the thermal noise of the normal resistance $4k_B T/R_N$ as it was considered in Ref. 19 and 20. We cannot either understand the data by introducing an effective temperature. Indeed, we would get an effective temperature as high as 100 K which appears unrealistic since the aluminum critical temperature is 1.2 K.

We propose that, in our temperature and voltage regime, the noise of the S-N-S junction can be written as:

$$S_I(0) = \frac{4k_B T}{V/I} \quad (4)$$

The comparison with experimental data of $S_I(0)$ is shown in Fig. 5 with *no adjustable parameter*. Between 800 and 550 mK, the agreement is remarkable.

The use of expression (4) can be understood in two ways. First, it is the low voltage limit ($eV \ll k_B T$) of the quasiparticle noise of a junction with low transmitting channels: $S_I(0) = 2eI \coth(eV/2k_B T)$ [21, 22]. If effective charges $e^* = Ne$ are responsible for the transport, this low voltage limit should be defined as $e^*V = NeV \ll k_B T$. This inequality stays valid as long as $N < 30$ at 1 μV and 550 mK. In this description, only weakly transmitting channels contribute to the current and to the noise.

Eq. (4) might also be related to the fluctuations-dissipation theorem in a non-linear system, in which the

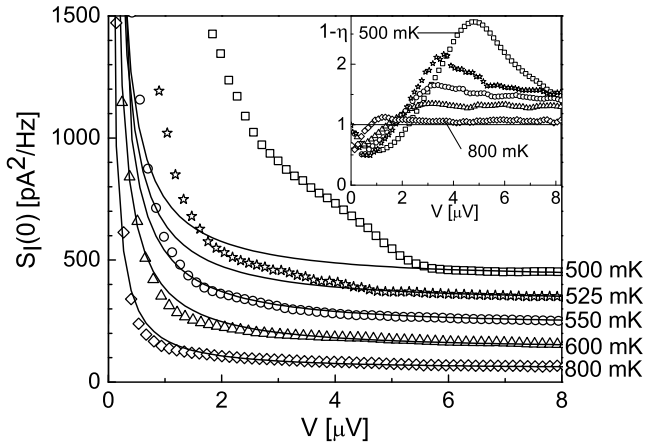


FIG. 5: Current noise $S_I(0) = S_{I_{\text{meas}}}/(1-\eta)$ vs. bias voltage V at several temperatures between 500 and 800 mK (Data are successively shifted by 100 pA²/Hz for clarity). Lines are the calculated noise $4k_B T/(V/I)$. Inset: $1-\eta$ vs. V at the same temperatures, where η is the coupling factor (See Eq. (2)).

resistance R_N would be replaced by $V/I(V)$. Here, dissipation arises from electron-electron interactions that necessarily occur at low voltage due to the large number of Andreev reflections needed for the quasiparticles to escape into the superconducting electrodes. In both cases, we conclude that, at very low voltage and "high" temperature, the current noise in long S-N-S junctions is governed by the transport of incoherent quasiparticles between the two superconducting reservoirs, which induces a noise divergence when the voltage goes to zero.

When decreasing further the temperature, the experimental data present a large excess noise compared to the noise obtained from Eq. (4). Therefore, additional noise sources must be involved. It is worth noting that this deviation from a pure quasiparticles noise occurs below 525 mK, which corresponds to $k_B T \simeq 3.5 E_{Th}$, very close to the width of the mini-gap ($3.1 E_{Th}$) [23]. This suggests that coherent processes, that become relevant in that energy scale, should play a role here.

In conclusion, we have presented current noise measurements in the non-hysteretic temperature range of a long diffusive S-N-S junction at very low bias ($eV < E_{Th} < \Delta$). In this regime, the current-voltage response is strongly non linear. The observed noise spectral density is enhanced by several orders of magnitude compared to the thermal noise of a normal junction and exhibits a broad peak. We clearly identify this peak to be a consequence of the non linear coupling between the current noise of the junction and the Josephson current beating. We could then extract the noise of the junction from the data. It corresponds extremely well to the thermal-like noise $4k_B T/R$ of a non linear resistor $R = V/I(V)$, which results in a divergence at low voltage and appears to be analogous to the quasiparticles noise of a junction with

low transmitting channels.

We acknowledge M. Houzet and F. Hekking for fruitful and continuous discussions. Samples have been fabricated at Nanofab-CNRS Grenoble. This work was funded by EU STREP project 'SFInx' and the Institut de Physique de la Matière Condensée IPMC/Grenoble.

* Corresponding author: francois.lefloch@cea.fr

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