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CONSISTENCY ENFORCING IN SCHEDULING: A GENERAL FORMULATION BASED ON ENERGETIC REASONING

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1 Introduction

Since the last decade, hard combinatorial problems such as scheduling have been the target of many approaches combining Operations Research and Artificial Intelligence techniques, focussed on constraint satisfaction as a general paradigm for representing and solving efficiently such problems (Van Hentenryck 89). Amongst these approaches, the so-called *Constraint-Based Analysis* has proposed to characterize feasible solutions (Erschler *et al.* 80), hence providing a decision-aid based alternative to optimization approaches through a panel of consistency enforcing techniques dedicated to resource and time constrained scheduling problems, viewed as special instances of Constraint Satisfaction Problems.

In order to prevent the combinatorics of solving conflicts between tasks in competition for limited resources, the approach based on the *energetic reasoning* has enabled the integration of both resource and time constraints (Erschler & Lopez 90, Erschler *et al.* 91). The underlying mechanisms are akin to other techniques already developed in the scheduling field, such as *immediate selections* (Carlier & Pinson 94), or *edge-finding* (Applegate & Cook 91).

Next section introduces the problem statement, gives a background information, and presents the problematics of the constraint-based reasoning. Section 3 introduces the energy concept and lists the associated inference rules. Section 4 examines the particular case of job-shop scheduling. Section 5 concludes on the possible extensions of the energy-based reasoning.

2 Constraint-based reasoning in scheduling

This paper deals with the following basic scheduling problem:

- a set of n tasks is to be realized, a task cannot be splitted;
- each task i is characterized by its duration p_i and has to be achieved within a time window $[r_i, d_i]$, which results either from initial individual time constraints, or from an initial makespan constraint, propagated to each task through potential constraints (*e.g.*, precedence ones);
- each task i needs a constant known amount a_i of a given resource available in a constant amount A .

The set $\{s_i / i = 1, \dots, n\}$ denotes *constrained variables* which represent the starting times consistent with both time windows and resource utilization constraints.

Constraint propagation which is under interest here, aims to remove inconsistent values of starting times so as to reduce the combinatorial search of the solutions. Several works have already demonstrated how solving algorithms, most of time built around Branch & Bound

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procedures, can take benefit of the integration of constraint propagation mechanisms. These works concern mostly *disjunctive* scheduling problems (Applegate & Cook 91, Carlier & Pinson 94, Caseau & Laburthe 95, Baptiste & Le Pape 95).

The present contribution addresses both *disjunctive* and *cumulative* scheduling problems, as in (Erschler & Lopez 90, Nuijten 94), but tries to compile partial results obtained in these previous works and recent ones, in a more general presentation from which each technique appears as a particular instantiation.

3 Energy-based reasoning

The concept of *energy*, which is the product of a duration by a consumption intensity, is an elegant way of considering simultaneously time and resource constraints in an unique reasoning. The problem of scheduling n tasks of duration p_i using a given resource available in a constant amount A over a time horizon p_Δ , is isomorphic to the placement problem of n rectangles of surface $p_i \cdot a_i$ on a rectangle of surface $A \cdot p_\Delta$.

3.1 Definitions and global consistency

The *maximal available energy* that a resource can provide on a given time interval $\Delta = [s_\Delta, f_\Delta]$, is defined by:

$$W^\Delta = A \cdot (f_\Delta - s_\Delta) \quad (1)$$

Let us consider one task, starting at time s_i (finishing at $f_i = s_i + p_i$), and a given time interval Δ . The *energy required by i on Δ* , termed w_i^Δ , is:

$$w_i^\Delta = \begin{cases} a_i \cdot [\min(f_i, f_\Delta) - \max(s_i, s_\Delta)] & \text{if } [s_i, f_i] \cap \Delta \neq \emptyset \\ 0 & \text{else} \end{cases}$$

yielding:

$$w_i^\Delta = a_i \cdot \max[0, \min(f_i, f_\Delta) - \max(s_i, s_\Delta)] \quad (2)$$

As s_i is a variable, w_i^Δ is also a variable, for which one can derive minimal and maximal bounds, taking account of the time window constraint $[r_i, d_i]$.

The *minimal energy required by i on Δ* , termed \underline{w}_i^Δ , is obtained for positions of i that overlap Δ as less as possible:

$$\underline{w}_i^\Delta = a_i \cdot \max[0, \min(p_i, f_\Delta - s_\Delta, r_i + p_i - s_\Delta, f_\Delta - d_i + p_i)] \quad (3)$$

Symmetrically, the *maximal energy required by i on Δ* , termed \overline{w}_i^Δ , is obtained for positions of i that overlap Δ as much as possible:

$$\overline{w}_i^\Delta = a_i \cdot \max[0, \min(p_i, f_\Delta - s_\Delta, d_i - s_\Delta, f_\Delta - r_i)] \quad (4)$$

Considering simultaneously the energy provided by the resource on a given interval and energies required by tasks, one can derive a *necessary global consistency condition* to be respected by any schedule on any time interval Δ :

$$\forall \Delta, \quad \sum_{i=1}^n w_i^\Delta \leq W^\Delta \quad (5)$$

3.2 Local consistency conditions

Although Formula (5) can serve to check whether a particular schedule is admissible or not, it does not lead to immediate restrictions on the *domain* of the variables. As a result, we now propose local consistency rules that take benefit from extremal bounds handled in Formulae (3) & (4) to derive time windows adjustments for some particular tasks.

3.2.1 □Domain adjustments of starting times

When considering any particular task i , Formula (5) can be rewritten as:

$$\forall i, \forall \Delta, w_i^\Delta \leq W^\Delta - \sum_{\substack{j=1 \\ j \neq i}}^n w_j^\Delta \quad (6)$$

Considering the maximal value of the right-hand side of Formula (6), one can derive a maximal value for its left-hand side:

$$\forall i, \forall \Delta, \bar{w}_i^\Delta \leq W^\Delta - \sum_{\substack{j=1 \\ j \neq i}}^n w_j^\Delta \quad (7)$$

The term $A_i^\Delta = W^\Delta - \sum_{\substack{j=1 \\ j \neq i}}^n w_j^\Delta$ is called the *maximal available energy for i on Δ* left by other

tasks. Any starting time of task i that leads to an energy consumption greater than the maximal available energy for i must be removed. Let be $forb_i^\Delta$ (forbidden) the set of these values (the expression of $forb_i^\Delta$ is detailed in (Lopez & Esquirol 96)). The following inference rule propagates maximal resource consumptions onto given intervals on the set of possible starting times for a task:

$$\text{if } \bar{w}_i^\Delta > A_i^\Delta \text{ then } \{s_i\} \leftarrow \{s_i\} - forb_i^\Delta \quad (8)$$

Note that the new set of values $\{s_i\}$ obtained through the application of Formula (8) may be either an interval, or the union of several disjointed intervals.

3.2.2 Consistency condition upon pairs of tasks

By applying the same type of reasoning upon a pair of tasks (i, j) , one can derive the following necessary condition:

$$\forall i, j, \forall \Delta, \bar{w}_i^\Delta + \bar{w}_j^\Delta \leq W^\Delta - \sum_{\substack{l=1 \\ l \neq i, j}}^n w_l^\Delta \quad (9)$$

Formula (9) expresses the impossibility for i and j to have an excessive simultaneous consumption on Δ . In other words, i and j must be a minimal *absolute distance* apart when both consuming on Δ . To make this condition more efficient, it is worth choosing special instances of Δ assuming that maximal required energies of i and j are obtained for their earliest and latest start times, *i.e.*, either $\Delta = [r_i, d_j]$, or $\Delta = [r_j, d_i]$.

4 Job-shop scheduling

We now consider the special case of job-shop scheduling in which $A = a_i = 1, \forall i$. For any task \bar{i} , several time intervals Δ may be chosen when trying to trigger the adjustment rule given in (8), for example:

$$[s_{\Delta}, f_{\Delta}] = [\min_{J_i} r_j, \max_{J_i \cup \{i\}} d_j] \text{ with } J_i = \{j = 1, \dots, n / j \neq i, r_i < r_j < d_i\}$$

which concludes on the sequencing of \boxed{i} before all tasks of J_i . A symmetrical rule would conclude on the sequencing of \boxed{j} after all tasks of J_i . Rule (8) thus encompasses edge-finding techniques, but is not limited to the sequencing of given tasks and the associated adjustments, as it can also deduce intermediate forbidden starting times.

For particular intervals such as $[r_i, d_j]$ (resp. $[r_j, d_i]$), triggering the rule associated to Formula (9) gives immediate selections on \boxed{i} and \boxed{j} . Note that it is interesting to check this rule only if \boxed{i} and \boxed{j} (resp. \boxed{j} and \boxed{i}) are not already sequenced by a non-energetic and simpler rule, checking whether $d_j - r_i \geq p_i + p_j$ (resp. $d_i - r_j \geq p_i + p_j$).

5 Conclusions and open issues

Energy-based reasoning addresses mainly both disjunctive and cumulative non-preemptive scheduling problems and proposes several inference rules that enforce the problem consistency, not only by adjustments of limit times of the tasks, but also by revealing inconsistent starting dates and sometimes global inconsistencies. The benefits concern both the problem of the feasibility characterization, but also the solving strategies, since the search space is reduced by constraint propagation.

This work must be developed in several directions. The complexity of the energetic reasoning depends essentially of the number of intervals Δ generated. Since the completeness is not realistic, it is helpful to restrict the intervals generated to those which produce the strongest adjustments.

Preemptive scheduling problems may be *a priori* tackled with energetic consistency rules. In this case, it seems possible to derive preemption from zero-maximal energy consumption on given intervals.

Finally, problems with variable durations seem also be addressable since a minimal consumption can be linked to a minimal duration, and conversely.

A first implementation of these techniques has been realized through the use of a Constraint Logic Programming tool. Computational results, still under developments, show the rather good performances obtained for lower bounds of the makespan in some classical benchmarks (Fisher & Thompson, Lawrence,...), and certain modified instances of these benchmarks in order to evaluate the techniques on multiple capacitated job-shop scheduling problems.

References

- D. Applegate and W. Cook (1991), "A computational study of job-shop scheduling", Research report CMU-CS-90-145.
- P. Baptiste and C. Le Pape (1995), "A theoretical and experimental comparison of constraint propagation techniques for disjunctive scheduling", *14th IJCAI*, Montréal (Québec).
- J. Carlier and E. Pinson (1994), "Adjustment of heads and tails for the job-shop problem", *EJOR* 78, 146-161.
- Y. Caseau and F. Laburthe (1995), "Improving branch and bound for job-shop scheduling with constraint propagation", *Combinatorics and Computer Science 95*, Brest (France).
- J. Erschler, F. Roubellat, and J.P. Vernhes (1980), "Characterizing the set of feasible sequences for n jobs to be carried out on a single machine", *EJOR* 4(3), 189-194.
- J. Erschler and P. Lopez (1990), "Energy-based approach for task scheduling under time and resource constraints", *Second International Workshop on Project Management and Scheduling*, pp.115-121, Compiègne (France).

- J. Erschler, P. Lopez, and C. Thuriot (1991), "Raisonnement temporel sous contraintes de ressources et problèmes d'ordonnancement", *Revue d'Intelligence Artificielle*, 5(3), 7-36.
- P. Lopez and P. Esquirol (1996), "An energy-based constraint approach for scheduling problems", Forthcoming paper.
- W.P.M. Nuijten and E.H.L. Aarts (1994), "A computational study of constraint satisfaction for multiple capacitated job-shop scheduling", *Fourth International Workshop on Project Management and Scheduling*, pp.166-173, Leuven (Belgium).
- P. Van Hentenryck (1989), *Constraint satisfaction in logic programming*, Logic programming series, MIT Press, Cambridge.