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# Navier-Stokes equations with periodic boundary conditions and pressure loss 

Chérif Amrouche ${ }^{\text {a }}$, Macaire Batchi ${ }^{\text {a,b }}$, Jean Batina ${ }^{\text {b }}$.<br>${ }^{\text {a }}$ Laboratoire de Mathématiques Appliquées, CNRS UMR 5142<br>${ }^{\mathrm{b}}$ Laboratoire de Thermique Energétique et Procédés<br>Université de Pau et des Pays de l'Adour<br>Avenue de l'Université 64000 Pau, France


#### Abstract

We present in this note the existence and uniqueness results for the Stokes and Navier-Stokes equations which model the laminar flow of an incompressible fluid inside a two-dimensional channel of periodic sections. The data of the pressure loss coefficient enables us to establish a relation on the pressure and to thus formulate an equivalent problem.

Keywords : Navier-Stokes equations, incompressible fluid, bidimensional channel, periodic boundary conditions, pressure loss.


## 1 Introduction

The problem which one proposes to study here is that modelling a laminar flow inside a two-dimensional plane channel with periodic section. Let $\Omega$ be an open bounded connected lipschtzian of $\mathbb{R}^{2}$ (see figure hereafter), where $\left.\Gamma_{0}=\{0\} \times\right]-1,1\left[\right.$ and $\left.\Gamma_{1}=\{1\} \times\right]-1,1[$.

One defines the space

$$
V=\left\{\boldsymbol{v} \in \mathbf{H}^{1}(\Omega) ; \operatorname{div} \boldsymbol{v}=0, \boldsymbol{v}=\mathbf{0} \text { on } \Gamma_{2}, \boldsymbol{v}_{\left.\right|_{\Gamma_{0}}}=\boldsymbol{v}_{\left.\right|_{\Gamma_{1}}}\right\}
$$

and for $\lambda \in \mathbb{R}$ given, one considers the problem

$$
(\mathcal{S})\left\{\begin{array}{c}
\text { Find } \boldsymbol{u} \in V, \text { such that } \\
\forall \boldsymbol{v} \in V, \int_{\Omega} \nabla \boldsymbol{u} . \nabla \boldsymbol{v} d \boldsymbol{x}=\lambda \int_{-1}^{+1} v_{1}(1, y) d y
\end{array}\right.
$$



Figure 1. Geometry of channel

## 2 Resolution of the problem (S)

Initially one proposes to study the problem $(\mathcal{P})$. One has it

Theorem 2.1 Problem $(\mathcal{S})$ has an unique solution $\boldsymbol{u} \in V$. Moreover, there is a constant $C(\Omega)>0$ such that:

$$
\begin{equation*}
\|\boldsymbol{u}\|_{\mathbf{H}^{1}(\Omega)} \leq \lambda C(\Omega) . \tag{1}
\end{equation*}
$$

Proof: Let us note initially that space $V$ provided the norm $H^{1}(\Omega)^{2}$ being a closed subspace of $H^{1}(\Omega)^{2}$ is thus an Hilbert space. Let us set

$$
a(\boldsymbol{u}, \boldsymbol{v})=\int_{\Omega} \nabla \boldsymbol{u} . \nabla \boldsymbol{v} d \boldsymbol{x}, \quad l(\boldsymbol{v})=\lambda \int_{-1}^{+1} v_{1}(1, y) d y
$$

It is clear, thanks to the Poincaré inequality, that the bilinear continuous form is $V$-coercive. It is easy to also see that $l \in V^{\prime}$. One deduces from Lax-Milgram Theorem the existence and uniqueness of $\boldsymbol{u}$ solution of $(\mathcal{S})$. Moreover,

$$
\int_{\Omega}|\nabla \boldsymbol{u}|^{2} d \boldsymbol{x} \leq \lambda \sqrt{2}\left(\int_{-1}^{+1}\left|u_{1}(1, y)\right|^{2} d y\right)^{1 / 2}
$$

i.e.

$$
\begin{aligned}
\|\nabla \boldsymbol{u}\|_{L^{2}(\Omega)}^{2} \leq & \lambda \sqrt{2}\|\boldsymbol{u}\|_{L^{2}(\Gamma)} \leq \lambda \sqrt{2}\|\boldsymbol{u}\|_{H^{1 / 2}(\Gamma)} \\
\leq & \lambda C_{1}(\Omega)\|\boldsymbol{u}\|_{H^{1}(\Omega)}
\end{aligned}
$$

Thus there is the estimate (1).

We now will give an interpretation of the problem $(\mathcal{S})$. One introduces the space

$$
\mathcal{V}=\left\{\boldsymbol{v} \in \mathcal{D}(\Omega)^{2} ; \quad \operatorname{div} \boldsymbol{v}=0\right\} .
$$

Let $\boldsymbol{u}$ be the solution of $(\mathcal{S})$. Then, for all $\boldsymbol{v} \in \mathcal{V}$, one has :

$$
\langle-\Delta \boldsymbol{u}, \boldsymbol{v}\rangle_{\mathcal{D}^{\prime}(\Omega) \times \mathcal{D}(\Omega)}=0
$$

So that thanks to De Rham Theorem, there exists $p \in \mathcal{D}^{\prime}(\Omega)$ such that

$$
\begin{equation*}
-\Delta \boldsymbol{u}+\nabla p=0 \text { in } \Omega . \tag{2}
\end{equation*}
$$

Moreover, since $\nabla p \in H^{-1}(\Omega)^{2}$, it is known that there exists $q \in L^{2}(\Omega)$ such that (see [1])

$$
\begin{equation*}
\nabla q=\nabla p \quad \text { in } \Omega \tag{3}
\end{equation*}
$$

The open $\Omega$ being connected, there exists $C \in \mathbb{R}$ such that $p=q+C$, what means that $p \in L^{2}(\Omega)$. Let us recall that (see [1])

$$
\inf _{K \in \mathbb{R}}\|p+K\|_{L^{2}(\Omega)} \leq C\|\nabla p\|_{H^{-1}(\Omega)^{2}}
$$

One deduces from the estimate (1) and from (2) that

$$
\inf _{K \in \mathbb{R}}\|p+K\|_{L^{2}(\Omega)} \leq C\|\Delta \boldsymbol{u}\|_{H^{-1}(\Omega)^{2}} \leq C\|\boldsymbol{u}\|_{H^{1}(\Omega)^{2}} \leq \lambda \mathrm{C}(\Omega)
$$

Since $\boldsymbol{u} \in H^{1}(\Omega)^{2}$ and $\mathbf{0}=-\Delta \boldsymbol{u}+\nabla p \in L^{2}(\Omega)^{2}$, it is shown that $-\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}}+p \boldsymbol{n}$ $\in H^{-1 / 2}(\Gamma)^{2}$ and one has the Green formula: for all $\boldsymbol{v} \in V$

$$
\begin{equation*}
\int_{\Omega}(-\triangle \boldsymbol{u}+\nabla p) \cdot \boldsymbol{v} d \boldsymbol{x}=\int_{\Omega} \nabla \boldsymbol{u} \cdot \nabla \boldsymbol{v} d \boldsymbol{x}+\left\langle-\frac{\partial \boldsymbol{u}}{\partial \mathbf{n}}+p \boldsymbol{n}, \boldsymbol{v}\right\rangle, \tag{4}
\end{equation*}
$$

where the bracket represents the duality product $H^{-1 / 2}(\Gamma) \times H^{1 / 2}(\Gamma)$. Moreover, as $p \in L^{2}(\Omega)$ and $\triangle p=0$ in $\Omega$, one has $p \in H^{-1 / 2}(\Gamma)$. Consequently, one has therefore $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}} \in H^{-1 / 2}(\Gamma)^{2}$. The function $\boldsymbol{u}$ being solution of $(\mathcal{S})$, for all $\boldsymbol{v} \in V$ one has:

$$
\begin{equation*}
\left\langle\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}}-p \boldsymbol{n}, \boldsymbol{v}\right\rangle=\lambda \int_{-1}^{+1} v_{1}(1, y) d y \tag{5}
\end{equation*}
$$

i.e.

$$
\begin{equation*}
\left\langle\frac{\partial \boldsymbol{u}}{\partial x}-p \mathbf{e}_{1}, \boldsymbol{v}\right\rangle_{\Gamma_{1}}+\left\langle-\frac{\partial \boldsymbol{u}}{\partial x}+p \mathbf{e}_{1}, \boldsymbol{v}\right\rangle_{\Gamma_{0}}=\left\langle\lambda \mathbf{e}_{1}, \boldsymbol{v}\right\rangle_{\Gamma_{1}} . \tag{6}
\end{equation*}
$$

where $\mathbf{e}_{1}=(1,0)$.
i) Let $\mu \in H_{00}^{1 / 2}\left(\Gamma_{1}\right)$ and let us set

$$
\mu_{2}=\left\{\begin{array}{l}
\mu \text { on } \Gamma_{0} \cup \Gamma_{1} \\
0 \text { on } \Gamma_{2}
\end{array} \quad \text { and } \quad \boldsymbol{\mu}=\binom{0}{\mu_{2}}\right.
$$

where (see [2])

$$
H_{00}^{1 / 2}\left(\Gamma_{1}\right)=\left\{\varphi \in \mathbf{L}^{2}\left(\Gamma_{1}\right) ; \exists \boldsymbol{v} \in \mathbf{H}^{1}(\Omega), \text { with }\left.\boldsymbol{v}\right|_{\Gamma_{2}}=\mathbf{0},\left.\boldsymbol{v}\right|_{\Gamma_{0} \cup \Gamma_{1}}=\varphi\right\} .
$$

It is checked easily that

$$
\boldsymbol{\mu} \in H^{1 / 2}(\Gamma)^{2} \quad \text { and } \quad \int_{\Gamma} \boldsymbol{\mu} . \boldsymbol{n} d \sigma=0
$$

So that there exists $\boldsymbol{v} \in H^{1}(\Omega)^{2}$ satisfying (see [3])

$$
\operatorname{div} \boldsymbol{v}=0 \quad \text { in } \quad \Omega \quad \text { and } \quad \boldsymbol{v}=\boldsymbol{\mu} \quad \text { on } \quad \Gamma .
$$

In particular $\boldsymbol{v} \in V$ and according to (6), one has

$$
\left\langle\frac{\partial u_{2}}{\partial x}, \mu\right\rangle_{\Gamma_{1}}=\left\langle\frac{\partial u_{2}}{\partial x}, \mu\right\rangle_{\Gamma_{0}},
$$

which means that

$$
\begin{equation*}
\left.\frac{\partial u_{2}}{\partial x}\right|_{\Gamma_{1}}=\left.\frac{\partial u_{2}}{\partial x}\right|_{\Gamma_{0}} . \tag{7}
\end{equation*}
$$

One deduces now from (6) that for all $\boldsymbol{v} \in V$,

$$
\begin{equation*}
\left\langle\frac{\partial u_{1}}{\partial x}-p, v_{1}\right\rangle_{\Gamma_{1}}+\left\langle-\frac{\partial u_{1}}{\partial x}+p, v_{1}\right\rangle_{\Gamma_{0}}=\left\langle\lambda, v_{1}\right\rangle_{\Gamma_{1}} . \tag{8}
\end{equation*}
$$

But, $\operatorname{div} \boldsymbol{u}=0$ and $\left.u_{2}\right|_{\Gamma_{1}}=\left.u_{2}\right|_{\Gamma_{0}}$, one thus has

$$
\begin{equation*}
\left.\frac{\partial u_{2}}{\partial y}\right|_{\Gamma_{1}}=\left.\frac{\partial u_{2}}{\partial y}\right|_{\Gamma_{0}} \quad \text { and }\left.\quad \frac{\partial u_{1}}{\partial x}\right|_{\Gamma_{1}}=\left.\frac{\partial u_{1}}{\partial x}\right|_{\Gamma_{0}} . \tag{9}
\end{equation*}
$$

Consequently, thanks to (8) one has:

$$
\begin{equation*}
\left\langle-p, v_{1}\right\rangle_{\Gamma_{1}}+\left\langle p, v_{1}\right\rangle_{\Gamma_{0}}=\left\langle\lambda, v_{1}\right\rangle_{\Gamma_{1}} \tag{10}
\end{equation*}
$$

ii) Let $\nu \in H_{00}^{1 / 2}\left(\Gamma_{1}\right)$ and let us set

$$
\nu_{1}=\left\{\begin{array}{l}
\nu \text { on } \Gamma_{0} \cup \Gamma_{1} \\
0 \text { on } \Gamma_{2}
\end{array} \quad \text { and } \quad \boldsymbol{\nu}=\binom{\nu_{1}}{0} .\right.
$$

One easily checks that

$$
\boldsymbol{\nu} \in H^{1 / 2}(\Gamma)^{2} \quad \text { and } \quad \int_{\Gamma} \boldsymbol{\nu} \boldsymbol{n} d \sigma=0
$$

So that there exists $\boldsymbol{v} \in H^{1}(\Omega)^{2}$ satisfying

$$
\operatorname{div} \boldsymbol{v}=0 \quad \text { in } \quad \Omega \quad \text { and } \quad \boldsymbol{v}=\boldsymbol{\nu} \quad \text { on } \quad \Gamma .
$$

In particular $\boldsymbol{v} \in V$ and according to (14), one has

$$
\langle-p, \nu\rangle_{\Gamma_{1}}+\langle p, \nu\rangle_{\Gamma_{0}}=\langle\lambda, \nu\rangle_{\Gamma_{1}}
$$

i.e.

$$
\begin{equation*}
p_{\mid \Gamma_{1}}=p_{\mid \Gamma_{0}}-\lambda \tag{11}
\end{equation*}
$$

where the equality takes place with the $H^{1 / 2}$ sense. In short, if $\boldsymbol{u} \in H^{1}(\Omega)^{2}$ is solution of $(\mathcal{S})$, then there exists $p \in L^{2}(\Omega)$, unique with an additive constant such that:

$$
\begin{align*}
& -\Delta \boldsymbol{u}+\nabla p=\mathbf{0} \quad \text { in } \quad \Omega,  \tag{12}\\
& \operatorname{div} \boldsymbol{u}=0 \quad \text { in } \quad \Omega,  \tag{13}\\
& \boldsymbol{u}=\mathbf{0} \quad \text { on }\left.\quad \Gamma_{2,} \quad \boldsymbol{u}\right|_{\Gamma_{1}}=\left.\boldsymbol{u}\right|_{\Gamma_{0}},  \tag{14}\\
& \left.\frac{\partial \boldsymbol{u}}{\partial x}\right|_{\Gamma_{1}}=\left.\frac{\partial \boldsymbol{u}}{\partial x}\right|_{\Gamma_{0}},  \tag{15}\\
& p_{\mid \Gamma_{1}}=p_{\mid \Gamma_{0}}-\lambda . \tag{16}
\end{align*}
$$

It is clear that if $(\boldsymbol{u}, p) \in H^{1}(\Omega)^{2} \times L^{2}(\Omega)$ checks (12)-(16), then $\boldsymbol{u}$ is solution
of $(\mathcal{S})$. Thus it

Theorem 2.2 The problem (12)-(16) has an unique solution $(\boldsymbol{u}, p) \in H^{1}(\Omega)^{2} \times$ $L^{2}(\Omega) \quad$ up to an additive constant for $p$. Moreover, $\boldsymbol{u}$ verifies $(\mathcal{S})$ and

$$
\|\boldsymbol{u}\|_{\mathbf{H}^{1}(\Omega)}+\|p\|_{L^{2}(\Omega) / \mathbb{R}} \leq \lambda C(\Omega)
$$

Remark 1 The pressure verifies the relation (16), which means that p satisfies the relation of Patankar et al.[5].

## 3 Navier-Stokes Equations

One takes again the assumptions of the Stokes problem given above. For $\lambda \in \mathbb{R}$ given, the one considers the following problem

$$
(\mathcal{N S})\left\{\begin{array}{c}
\text { Find } \boldsymbol{u} \in V, \text { such that } \\
\forall \boldsymbol{v} \in V, \int_{\Omega} \nabla \boldsymbol{u} \cdot \nabla \boldsymbol{v} d \boldsymbol{x}+b(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{v})=\lambda \int_{-1}^{+1} v_{1}(1, y) d y
\end{array}\right.
$$

with

$$
b(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})=\int_{\Omega}(\boldsymbol{u} . \nabla) \boldsymbol{v} \cdot \boldsymbol{w} d \boldsymbol{x}
$$

With an aim of establishing the existence of the solutions of the problem $(\mathcal{N S})$, one uses the Brouwer fixed point theorem (see [4], [6]). One will show it

Theorem 3.1 The problem (NS) has at least a solution $\boldsymbol{u} \in V$. Moreover, $\boldsymbol{u}$ checks the estimate (1).

Proof: To show the existence of $\boldsymbol{u}$, one constructs the approximate solutions of the problem $(\mathcal{N S})$ by the Galerkin method and then thanks to the arguments of compactness, one makes a passage to the limit.
i) For each fixed integer $m \geq 1$, one defines an approximate solution $\boldsymbol{u}_{m}$ of (NS) by

$$
\begin{gather*}
\boldsymbol{u}_{m}=\sum_{i=1}^{m} g_{i m} \boldsymbol{w}_{i}, \quad \text { with } \quad g_{i m} \in \mathbb{R}  \tag{17}\\
\left(\left(\boldsymbol{u}_{m}, \boldsymbol{w}_{i}\right)\right)+b\left(\boldsymbol{u}_{m}, \boldsymbol{u}_{m}, \boldsymbol{w}_{i}\right)=\left\langle\lambda \boldsymbol{n}, \boldsymbol{w}_{i}\right\rangle_{\Gamma_{1}}, i=1, \ldots, m
\end{gather*}
$$

where $V_{m}=\left\langle\boldsymbol{w}_{1}, \ldots, \boldsymbol{w}_{m}\right\rangle$ vector spaces spanned by the vectors $\boldsymbol{w}_{1}, \ldots, \boldsymbol{w}_{m}$ and $\left\{\boldsymbol{w}_{i}\right\}$ is an Hilbertian basis of $V$ which is separable. Let us note that (17) is equivalent to:

$$
\begin{equation*}
\forall \boldsymbol{v} \in V_{m}, \quad\left(\left(\boldsymbol{u}_{m}, \boldsymbol{v}\right)\right)+b\left(\boldsymbol{u}_{m}, \boldsymbol{u}_{m}, \boldsymbol{v}\right)=\lambda \int_{-1}^{+1} v_{1}(1, y) d y \tag{18}
\end{equation*}
$$

With an aim to establish the existence of the solutions of the problem $\boldsymbol{u}_{m}$, the operator as follows is considered

$$
\begin{aligned}
\mathrm{P}_{m}: V_{m} & \rightarrow V_{m} \\
\boldsymbol{u} & \longmapsto \mathrm{P}_{m}(\boldsymbol{u})
\end{aligned}
$$

defined by

$$
\forall \boldsymbol{u}, \boldsymbol{v} \in V_{m}, \quad\left(\left(\mathrm{P}_{m}(\boldsymbol{u}), \boldsymbol{v}\right)\right)=((\boldsymbol{u}, \boldsymbol{v}))+b(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{v})-\lambda \int_{-1}^{+1} v_{1}(1, y) d y
$$

Let us note initially that $\mathrm{P}_{m}$ is continuous and

$$
\forall \boldsymbol{u} \in V, \quad b(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{u})=0
$$

Indeed, thanks to the Green formula, one has

$$
b(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{u})=-\frac{1}{2} \int_{\Omega}|\boldsymbol{u}|^{2} \operatorname{div} \boldsymbol{u} d \boldsymbol{x}+\frac{1}{2} \int_{\Gamma}(\boldsymbol{u} . \boldsymbol{n})|\boldsymbol{u}|^{2} d \sigma=0 .
$$

But, $\operatorname{div} \boldsymbol{u}=0$ in $\Omega$ and

$$
\int_{\Gamma}(\boldsymbol{u} . \boldsymbol{n})|\boldsymbol{u}|^{2} d \sigma=\int_{\Gamma_{0}}(\boldsymbol{u} . \boldsymbol{n})|\boldsymbol{u}|^{2} d \sigma+\int_{\Gamma_{1}}(\boldsymbol{u} . \boldsymbol{n})|\boldsymbol{u}|^{2} d \sigma .
$$

since the external normal to $\Gamma_{0}$ is opposed to that of $\Gamma_{1}$ and $\boldsymbol{u} \in V$.
Thanks to Brouwer Theorem, there exists $\boldsymbol{u}_{m}$ satisfying (18) and

$$
\begin{equation*}
\left\|\boldsymbol{u}_{m}\right\|_{\mathbf{H}^{1}(\Omega)} \leq \lambda C(\Omega) \tag{19}
\end{equation*}
$$

ii) We can extract a subsequence $\boldsymbol{u}_{\nu}$ such that

$$
\boldsymbol{u}_{\nu} \rightharpoonup \boldsymbol{u} \text { weakly in } V,
$$

and thanks to the compact imbedding of $V$ in $L^{2}(\Omega)^{2}$, we obtain

$$
\forall \boldsymbol{v} \in V,((\boldsymbol{u}, \boldsymbol{v}))+b(\boldsymbol{u}, \boldsymbol{u}, \boldsymbol{v})=\lambda \int_{-1}^{+1} v_{1}(1, y) d y
$$

As for the Stokes problem, one shows the existence of $p \in L^{2}(\Omega)$, unique except for an additive constant, such that

$$
\begin{cases}-\Delta \boldsymbol{u}+(\boldsymbol{u} . \nabla) \boldsymbol{u}+\nabla p=\mathbf{0} & \text { in } \quad \Omega, \\ \operatorname{div} \boldsymbol{u}=0 & \text { in } \quad \Omega, \\ \boldsymbol{u}=\mathbf{0} & \text { on } \quad \Gamma_{2}, \\ \left.\boldsymbol{u}\right|_{\Gamma_{1}}=\left.\boldsymbol{u}\right|_{\Gamma_{0}} . & \end{cases}
$$

It is checked finally that

$$
\begin{aligned}
\left.\frac{\partial \boldsymbol{u}}{\partial x}\right|_{\Gamma_{1}} & =\left.\frac{\partial \boldsymbol{u}}{\partial x}\right|_{\Gamma_{0}} \\
p_{\mid \Gamma_{1}} & =p_{\mid \Gamma_{0}}-\lambda
\end{aligned}
$$

Remark 2 i) Theorem 3.1 of problem ( $\mathcal{N S}$ ) takes place in three dimension.
ii) One can show that the solution $(\boldsymbol{u}, p)$ belongs to $H^{2}(\Omega)^{2} \times H^{1}(\Omega)$.

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