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Evaluation of a planetary boundary layer subgrid-scale model that accounts for near-surface turbulence anisotropy

Philippe Drobinski,1 Jean-Luc Redelsperger,2 and Christophe Pietras3

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[1] In numerical modelling, far from regions of large gradients, the resolved scales contain most of the energy of turbulent motion whilst on subgrid scales (SGS), motions are less energetic. However, SGS contribution becomes larger than the resolved part near the surface. Recent studies have shown that near-surface turbulence anisotropy has a dramatic effect on the mixing length to be used in SGS models which are generally derived for free-stream isotropic turbulence or use the standard Prandtl mixing length $\kappa z$. Using the flux measurements collected at the SIRTA observatory located near Paris (France), this paper shows that a SGS model suitable for both the surface layer and free-stream turbulence must discard the Prandtl mixing length and account for near-surface turbulence anisotropy. It also shows that such an SGS scheme, which is suited for all numerical models of the atmosphere, improves the representation of planetary boundary layer processes in most stability conditions. Citation: Drobinski, P., J.-L. Redelsperger, and C. Pietras (2006), Evaluation of a planetary boundary layer subgrid-scale model that accounts for near-surface turbulence anisotropy, Geophys. Res. Lett., 33, L23806, doi:10.1029/2006GL027062.

1. Introduction

[2] Turbulent eddy motion in the planetary boundary layer (PBL) exhibits three spectral regions: (1) the energy-containing range (or large-scale turbulence), in which turbulent kinetic energy (TKE) is produced, (2) the inertial subrange (or small scale turbulence), in which TKE is neither produced nor dissipated, but is simply transported from large to small scales ($-1/3$ power law spectral signature [Kolmogorov, 1941]), and (3) the dissipation range, where TKE is converted to internal energy by the action of fluid viscosity. Recent studies reported well defined $-1$ power law at production scales for the velocity spectra in the atmospheric surface layer (ASL) (see Drobinski et al. [2006] for a review). This $-1$ power law is a signature of near-surface turbulence anisotropy caused by surface blocking and vertical shear of the wind in the ASL [Hunt and Carlotti, 2001; Drobinski et al., 2004]. Turbulence anisotropy modifies the transfer processes from the large-scales to the dissipation-scales and the spectral analysis shows how the energy deficit of blocked turbulence for a given dissipation dramatically affects the coefficients to be used in subgrid-scale (SGS) schemes.

[3] Redelsperger et al. [2001] developed a general approach, accounting for turbulence anisotropy, to allow standard SGS schemes to be suitable both for the ASL and free-stream turbulence. Using blocked turbulence theory, which assumes that a $-1$ power law exists at intermediate spectral subrange, Redelsperger et al. [2001] showed that the SGS length scales should be taken much larger than the Prandtl mixing length $L = \kappa z$ (where $\kappa = 0.4$ is the Von Karman constant and $z$ the height above ground), though often used in large-eddy simulations (LES) and large scale models. They proposed an approach which reconciles the standard SGS model and the Monin-Obukhov (MO) similarity laws. The same SGS model can thus be used in free-stream turbulence and in the ASL, allowing the retrieval of the MO similarity laws with a standard scheme using physically-based constants derived for free-stream turbulence. The key point is the use of a mixing length depicting the near-surface turbulence anisotropy ($L = 2.8z$ in neutral conditions, see Section 2). This new mixing length (herein-referred as RMC01 mixing length) is only function of vertical stability conditions. It is simple to implement in models, applicable to LES, limited area model (LAM) or global circulation model (GCM), and suitable for inhomogeneous surface conditions or complex topography.

[4] This study evaluates the ability of standard SGS model with “standard” and RMC01 mixing lengths to retrieve surface momentum and sensible heat fluxes. The data used to conduct this study are the sonic anemometer measurements at 10 and 30 m heights collected at the SIRTA observatory located at Palaiseau, 20 km south of Paris, France [Haefelin et al., 2005]. The mean wind speed and temperature at these two levels are used to compute surface fluxes from both MO similarity laws and SGS model (similarly to Poulos and Burns [2003]). The latter is evaluated using different mixing lengths. The retrieved surface momentum and sensible heat fluxes are compared to the fluxes measured by eddy correlation with the sonic anemometers. After the introduction in section 1, section 2 briefly describes the standard SGS scheme and defines the different mixing lengths. Section 3 discusses the performance of the SGS model for the different mixing lengths, stressing on the improved performance when turbulence anisotropy is accounted for. Section 4 concludes the study and point out some open research questions needing further investigation.

2. General Subgrid Model for Free-Stream and Near-Surface Turbulence

2.1. Monin-Obukhov Similarity Laws

[5] In the ASL, the MO similarity theory allows relating surface momentum and sensible heat fluxes to the wind
Table 1. Mixing Length Scales for the Neutral PBL

<table>
<thead>
<tr>
<th>Free-Stream Turbulence Mixing Length</th>
<th>Prandtl Mixing Length</th>
<th>RMC01 Mixing Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_K = C_k L$</td>
<td>$\ell_K = \alpha^{-1/2}Kz$</td>
<td>$\ell_K = C_k L_{K}$</td>
</tr>
<tr>
<td>$\ell_z = L/C_{\varepsilon}$</td>
<td>$\ell_z = \alpha^{-1/2}Kz$</td>
<td>$\ell_z = L/C_{\varepsilon}$</td>
</tr>
<tr>
<td>$L = \Delta$</td>
<td>$L = \kappa z$</td>
<td>$L = L_p/L_{\varepsilon} = A z$</td>
</tr>
<tr>
<td>$\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ (i.e. mesh size)</td>
<td>$K = 0.4$</td>
<td>$\Delta = \kappa C_k C_{\varepsilon}^{-3/4} \sim 2.8$</td>
</tr>
<tr>
<td>$C_k = (1/\pi)(2/3)\alpha_3^{3/2} = 0.066$</td>
<td>$C_k = \alpha^{-1/2} = 0.516–0.428$</td>
<td>$C_k = (1/\pi)(2/3)\alpha_3^{3/2} = 0.066$</td>
</tr>
<tr>
<td>$C_z = (2/3)\alpha_3^{1/2} = 0.7$</td>
<td>$C_z = \alpha^{-1/2} = 0.137–0.078$</td>
<td>$C_z = (2/3)\alpha_3^{1/2} = 0.7$</td>
</tr>
</tbody>
</table>

* $\alpha_3 = 1.6 \pm 0.02$ is the three-dimensional Kolmogorov constant; $\alpha$ ranges between 3.75 [Wyngaard and Coté, 1974] to 5.47 [Garratt, 1992].

speed and temperature gradients and stability functions $\phi_m$ and $\phi_h$:

$$u'^4 = \overline{u'^2 v'^2} + \overline{v'^2 w'^2} = (\kappa z)^4 \left( \frac{\partial u'}{\partial z} + \frac{\partial v'}{\partial z} \right)^2 \phi_m^{-4} \left( \frac{z}{L_{MO}} \right),$$

$$-\overline{w' \overline{\theta '}} = (\kappa z)^2 \frac{\partial \overline{\theta'}}{\partial z} \left( \frac{\partial u'}{\partial z} + \frac{\partial v'}{\partial z} \right)^2 \phi_h^{-1} \left( \frac{z}{L_{MO}} \right),$$

$$\cdot \phi_h^{-1} \left( \frac{z}{L_{MO}} \right),$$

where $u$, $v$, $w$ are the components of the wind along $x$, $y$ and $z$ directions, respectively ($z$ is the vertical direction, perpendicular to the ground), $\theta$ is the potential temperature. The prime stands for the fluctuations and the subscript $s$ for “surface” and $L_{MO}$ is the MO length scale defined by $L_{MO} = -u'_s [g/(\partial \theta'/\partial z)] w' \theta'$. The non-dimensional wind shear and temperature gradients are chosen as [Businger et al., 1971]:

$$\phi_m = \left( 1 - 15 \frac{z}{L_{MO}} \right)^{-1/4} \frac{z}{L_{MO}} \quad \text{for unstable case,}$$

$$\phi_h = 0.74 \left( 1 - 9 \frac{z}{L_{MO}} \right)^{-1/2} \frac{z}{L_{MO}} \quad \text{for stable case.}$$

2.2. Subgrid-Scale Model

[6] In common SGS eddy viscosity approach, momentum and sensible heat fluxes are modelled as:

$$\overline{u' w'} = K_m \frac{\partial u'}{\partial z} ; \quad \overline{v' w'} = K_m \frac{\partial v'}{\partial z} ; \quad \overline{w' \overline{\theta'}} = K_h \frac{\partial \overline{\theta'}}{\partial z}$$

$K_m$ and $K_h$ are the eddy diffusivities for the wind and the potential temperature, respectively. The eddy diffusivity $K_m$ is such that $K_m = \ell_K E^{1/2}$, where $E$ denotes the SGS TKE and $\ell_K$ has the dimension of a length and is related to an eddy diffusion length scale $L_k$ by $\ell_K = C_k L_k$ (where $C_k$ is a constant). The ratio $K_m/K_h$ is the turbulent Prandtl number which is a function of the Richardson number $R_i$ only [Cuxart et al., 2000]. The dissipation $\varepsilon$ can be expressed as $\varepsilon = E^{1/2}/\ell_\varepsilon$, where $\ell_\varepsilon$ has the dimension of a length related to the dissipation length scale $L_\varepsilon$ by $\ell_\varepsilon = L_\varepsilon/C_\varepsilon$ (where $C_\varepsilon$ is a constant).
the second-order moment statistics of these variables (variances and fluxes). The data set was collected over a seven-month period extending between April and October 2005, covering all types of westerly flow situations (cloud-free to precipitating situations) and stratification (from stable to convective). To compare the eddy-correlation fluxes to those derived from the formulations given in the previous section, the Richardson number is estimated from the vertical gradients of wind speed (\(\overline{\partial u/\partial z} \approx \Delta u/\Delta z\)) and temperature (\(\overline{\partial T/\partial z} \approx \Delta T/\Delta z\)) (for a discussion on the impact of calculating the gradient Richardson number across \(\Delta z\) see Poulos and Burns [2003]). This allows the computation of the MO length scale and stability functions \(\phi_m, \phi_h, \phi_L\) and \(\psi_L\) (equations (2) and (7)). The momentum and sensible heat fluxes are then conjointly deduced from equation (1) (MO similarity laws) and equations (3) and (4) (SGS model). In the latter case, fluxes are estimated using three different length scales: the standard length scales (\(L = \kappa z\) and \(L = \Delta\)) and the RMC01 length scale, using the free-stream values for \(C_a\) and \(C_k\). The surface fluxes deduced from the MO laws and the SGS model with the different length scales are then compared to the surface flux produced by eddy correlation using the measurements from the 10-m sonic anemometer. It must be noted that the use of a simple linear interpolation for the vertical gradients of wind speed and temperature between 10 and 30 m heights produces an overestimation of the estimated surface momentum and sensible heat fluxes of at most 9% in the neutrally stratified PBL.

Figure 1 shows an example of a full diurnal cycle of surface momentum and sensible heat fluxes estimated using the SGS model with the three different length scales and measured by the two sonic anemometers on 2 May 2005 at the SIRTA. The daytime period is typical of a sheared convective PBL flow with high values of momentum fluxes (0.5 m s\(^{-2}\) in average) associated with about 6 m s\(^{-1}\) wind speed and 5 \times 10^{-2} s\(^{-1}\) vertical shear of horizontal wind, and heat fluxes reaching 150 W m\(^{-2}\) on average. The nighttime period is slightly stably stratified with surface heat fluxes ranging between -15 W m\(^{-2}\) on the night beginning 1 May and -5 W m\(^{-2}\) on the night beginning 2 May. The momentum fluxes reach maximal values of 0.04 m\(^2\) s\(^{-2}\) on the night of 1 May and are nearly zero on the night beginning 2 May. The measurements show very similar values at 10 and 30 m, indicating that the measurements are representative of the ASL (constant-flux layer). The agreement with MO similarity theory is good (less than 0.04 m\(^2\) s\(^{-2}\) and -20 W m\(^{-2}\) mean absolute error for the momentum and sensible heat fluxes, respectively). Figure 1 shows very different estimates of the SGS momentum and sensible heat surface fluxes when different mixing length scales are used. The SGS with the RMC01 length scale gives the most accurate results when compared to the observations. In details, it slightly overestimates (in absolute value) the momentum and sensible heat fluxes during nighttime which has also been evidenced by Poulos and Burns [2003] for different Ri-based surface layer flux formulas. During daytime, the momentum fluxes are underestimated until 1400 UTC but it is not a systematic behaviour as it will be shown in section 3.2. As suggested by Poulos and Burns [2003], this can be explained by the fact that, at times, the local measurement by the sonic anemometer at a given
The SGS model using $L = \kappa z$ underestimates both the momentum and sensible heat fluxes by a factor 40 (sensible heat fluxes) to 70 (momentum fluxes) which reduces to a factor of about 2 at night. This is due to the fact that we keep the free-stream values for $C_e$ and $C_K$ (i.e. $C_e = 0.7$ and $C_K = 0.066$) even when $L = \kappa z$ is used (see discussion in section 3.2). This discards the Prandtl mixing length in SGS models that use the same constants in the ASL and in the free-stream PBL. The SGS model using RMC01 length scale gives accurate results with respect to the measurements and is close to the MO theory prediction (the difference between the surface momentum fluxes retrieved from MO theory and the SGS model which uses RMC01 length scale is hardly visible in Figure 1a). The SGS model which uses RMC01 length scales produces better momentum and sensible heat fluxes compared to the SGS model that uses $L = \Delta$, which underestimates (overestimates) those fluxes during daytime (night-time).

### 3.2. Statistical Analysis

Figures 2 and 3 show the scatter plots of the SGS momentum (Figure 2) and sensible heat (Figure 3) fluxes retrieved using MO similarity theory, RMC01 length scale, $L = \kappa z$, and $L = \Delta$ versus the measured corresponding fluxes using the whole data set over the seven month period. A distinction is made between stably ($z/L_{MO} > 0.01$ at 10 m height) and unstably ($z/L_{MO} < -0.01$ at 10 m height) stratified PBL. The data are averaged over a 20 min period. Figures 2 and 3 show a good agreement of the momentum and sensible heat surface fluxes retrieved from the MO similarity theory with the sonic anemometer measurements, both in stable (Figures 2a and 3a) and unstable situations (Figures 2b and 3b). The largest discrepancies are found for weak wind speed ($<2$ m s$^{-1}$) or in stably stratified PBL, most probably due to turbulence intermittency [Poulos and Burns, 2003]. Table 2 complements Figures 2 and 3 by summarizing statistical scores relevant to the representation of the ASL by the SGS model with the different mixing lengths.

The use of $L = \kappa z$ as SGS length scale leads to an underestimation of the momentum and sensible heat fluxes (Figures 2c–f and 3c–f and Table 2), consistent with the fact that it overestimates the values of the vertical gradients of wind speed and potential temperature when the free-

![Figure 2](image_url)

**Figure 2.** Scatter plot of momentum fluxes estimated using (a–b) MO similarity laws, (c–d) SGS model with RMC01 length scales, (e–f) $L = \kappa z$, and (g–h) $L = \Delta$ versus measured momentum flux at 10 m height for stable (Figures 2a, 2c, 2e, and 2g) and unstable (Figures 2b, 2d, 2f, and 2h) stratification.

![Figure 3](image_url)

**Figure 3.** Same as Figure 2 for sensible heat fluxes.
stream values of $C_s$ and $C_K$ are used [Redelsperger et al., 2001]. Table 2 shows that when the values of $C_s$ and $C_K$ are chosen so that MO similarity theory applies in the ASL with $L = \kappa z$ (i.e. $C_s = \alpha^{-3/2} = 0.078 – 0.137$ and $C_K = \alpha^{-1/2} = 0.428 – 0.516$, see Table 1), then the performance of the SGS model become very similar to those obtained with RMC01 length scale in the ASL (however, the SGS model become questionable above the ASL since the constants are no longer physically justified). Figure 2 and Table 2 show the improvements brought by the SGS scheme which uses RMC01 length scales over the usual methods. When RMC01 length scales are used, the SGS momentum fluxes match perfectly with those predicted by the MO similarity theory and measured with the sonic anemometers (Figures 2c and 2d). The RMC01 mixing length leads to an unbiased prediction of the surface momentum fluxes with a root mean square error of about 0.07 m$^2$ s$^{-2}$ (Table 2). The slopes of the best linear fit of the measured versus estimated momentum fluxes for unstable stratifications are 1.1 for the MO similarity theory and RMC01 length scale, whereas a value of 0.53 is found for $L = \Delta$ (Figures 2b, 2d, and 2h). The 10% overestimation of the SGS momentum fluxes using MO similarity theory and RMC01 length scale can, most probably, be attributed to the discretization of the vertical gradient of the wind speed. The use of $L = \Delta$ as SGS length scale leads to an underestimation of the fluxes by a factor of around 2. For stable conditions, the use of RMC01 and $L = \Delta$ as SGS length scales leads to a slight overestimation of the momentum fluxes [Poulos and Burns, 2003] (Table 2).

For unstably stratified PBL (Figures 3d and 3h), the use of RMC01 length scales instead of $L = \Delta$ does not improve the estimate of the sensible heat fluxes, it is even worse (Table 2). The SGS sensible heat fluxes are overestimated by about 40% when RMC01 length scale is used (Figure 3d and Table 2) whereas they are underestimated by about 30% when $L = \Delta$ (Figure 3h and Table 2). In contrast for stable stratification, the SGS model using RMC01 length scale gives results similar to the MO similarity theory even though there is a tendency for overestimation of the sensible heat fluxes in absolute value (Figures 3a and 3c and Table 2). It is even more striking when $L = \Delta$ is used (Figure 3e and Table 2). The night-time overestimation of the sensible heat fluxes (in absolute value) has also been shown by Poulos and Burns [2003] which suggest that the SGS model transfers cool air at too great a rate to the first model grid point over the ground.

### Table 2. For the Stability Category, the Bias and Root Mean Square Error (RMSE) Between the SGS Model Fluxes and the Measured Fluxes

<table>
<thead>
<tr>
<th>Mixing Length</th>
<th>Momentum Flux, m$^2$ s$^{-2}$</th>
<th>Sensible Heat Flux, W m$^{-2}$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = \Delta$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stable</td>
<td>+0.02</td>
<td>−15.41</td>
<td>0.07</td>
</tr>
<tr>
<td>Unstable</td>
<td>−0.08</td>
<td>−12.62</td>
<td>0.06</td>
</tr>
<tr>
<td>RMC01</td>
<td></td>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>Stable</td>
<td>+0.02</td>
<td>−2.37</td>
<td>0.07</td>
</tr>
<tr>
<td>Unstable</td>
<td>+0.01</td>
<td>+44.47</td>
<td>0.07</td>
</tr>
</tbody>
</table>

For $L = \kappa z$, the statistics are given for the free-stream turbulence values of $C_s$ and $C_K$ and for the values fitted for $L = \kappa z$ and MO similarity theory in the ASL. In average, the median values for the momentum fluxes are 0.10 (stable) and 0.17 m$^2$ s$^{-2}$ (unstable), and for the sensible heat fluxes, −19.06 (stable) and 80.28 W m$^{-2}$ (unstable), respectively.

Free-stream turbulence values for $C_s$ and $C_K$.

$C_K$ and $C_s$ values fitted for $L = \kappa z$ and MO similarity theory in the ASL.

### 4. Conclusion

[14] This paper is the first attempt to evaluate with actual and statistically representative atmospheric measurements the impact of the implementation of turbulence anisotropy on turbulent surface fluxes in SGS scheme. The paper focuses on the SGS model developed by Redelsperger et al. [2001] suitable both for the ASL and free-stream turbulence, which includes modifications physically justified by recent measurements of spectra close to the ground. This paper shows:

[15] 1. The interest to use the RMC01 mixing length to estimate ASL characteristics with a turbulent scheme having the same constants everywhere, in the free-stream PBL or in the ASL.

[16] 2. The danger to use the Prandtl mixing length in a turbulent scheme using a TKE equation (either diagnostic or prognostic) with the same constants as in the free-stream PBL. The Prandtl mixing length should be discarded as the mixing length of such SGS model [Redelsperger et al., 2001; Carlotti and Drobsinski, 2004].

[17] This paper shows that the use of RMC01 mixing length improves the retrieval of the turbulent surface fluxes (momentum and sensible heat fluxes) in most stability conditions. The momentum flux retrieval is clearly improved whatever the stratification despite the fact that the RMC01 length scale was first derived for neutral stratification and extended towards stably and unstably stratified PBL. For the surface sensible heat flux, no such improvement has been found for unstable conditions whereas for stable conditions, the SGS sensible heat fluxes using RMC01 length scale remain much closer to the MO similarity theory and the measurements than when the usual length scales are used. This paper also points out the difficulty to retrieve the surface fluxes from SGS models which generally rely on the computation of bulk Richardson number which may not be, at times, representative of the local fluxes at a given height [Poulos and Burns, 2003].

[18] This study must be completed by an evaluation of the SGS model implemented in a numerical model where the SGS model has a feedback on the resolved simulated...
such an evaluation has been recently made using a three-dimensional LES of a field experiment case in near-neutral stratification [Drobinski et al., 2006]. The comparison with the data proved the performance of such SGS to reproduce the near-surface flow structure and associated energetics. If encouraging, a much more extensive evaluation needs to be performed in non-neutral stratification as a natural follow-up of this study, in order to eventually be implemented in LES, LAM, and GCM.

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References


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