HAL
open science

Simulated annealing coupled with the tabu search method for continuum optimization in electromagnetics

Joao Antonio Vasconcelos, Laurent Krähenbühl, Alain Nicolas

To cite this version:

Joao Antonio Vasconcelos, Laurent Krähenbühl, Alain Nicolas. Simulated annealing coupled with the tabu search method for continuum optimization in electromagnetics. IEEE Transactions on Magnetics, 1996, 32 (3), pp.1206-1209. 10.1109/20.497460 . hal-00140683

HAL Id: hal-00140683

https://hal.science/hal-00140683

Submitted on 23 Apr 2007

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Simulated Annealing coupled with the Tabu Search Method for Continuum Optimization in Electromagnetics

J. A. Vasconcelos
CPDEE - UFMG
Av. Antônio Carlos 6627 - 31270-901 - B. Hte. - MG - Brazil

L. Krähenbühl A. Nicolas
Centre de Génie Electrique de Lyon - ECL
BP163 - 69131 Ecully Cedex - France

Abstract - **In this paper the simulated annealing algorithm coupled with the tabu search method for continuum optimization problems is presented. Basically, the new algorithm is obtained by assembling certain characteristics of the tabu search method into the standard simulated annealing. The numerical performance of the new algorithm is discussed based on the optimization of some classical test functions. Finally, an insulator geometry is optimized using this algorithm coupled with the boundary element method in order to test it in electromagnetics.**

INTRODUCTION

The simulated annealing algorithm (SA) was first proposed in the area of combinatorial optimization, that is, when the cost function is defined in a discrete domain [1]. In this case, the design variables are discrete and the random moves correspond to permutations in the list of possible moves. It happens as in the traveling salesman problem, where the random moves correspond to permutations in the list of cities to be visited. Therefore, many important problems are defined as functions of continuous variables and the application of the SA requires an efficient strategy to choose the random step h. This strategy must be problem independent and self-regulatory to allow SA algorithm to be implemented efficiently and reliably.

Some authors, e.g., Vanderbilt and Louie [2] and Corana et al. [3] have proposed strategies to deal with this problem. The former chose the step as

$$h = Qu \tag{1}$$

where the matrix Q controls the step distribution and u is a vector of random numbers $(u_1, u_2, \ldots, u_n)$. Each u_i is chosen independently. This matrix can be generated as an isotropic distribution with an RMS average step length. The strategy suggested by Corana et al to adjust the step vector is based on the ratio of accepted to rejected moves. This idea consists in changing the step to keep the ratio close to one. A lower rate means that many moves are rejected and the search was made in a too large region, thus misusing computational effort. On the other hand, a higher rate means that many moves are accepted and the search was made in a region too small also wasting computational time. In other words, this strategy searches to obtain a state of quasi-equilibrium for each temperature in analogy with the physical process of annealing. They proposed two equations to update the step vector h as given bellow:

$$h_i' = h_i\,[1 + c_i(p_i - 0.6)/0.4] \qquad if\ p_i > .6$$

$$h_i' = h_i\,[1 + c_i(0.4 - p_i)/0.4]^{-1} \qquad if\ p_i < .4 \tag{2}$$

where p_i is the ratio of the number of accepted moves to the number of proposed ones and c_i is a control parameter for the i-th direction. Using the equations above, a self-regulatory strategy is obtained. In the initial process, Corana et al. used the value of $h_i = 0.5(b_i - a_i)$, where a_i and b_i are the inferior and superior bounds for the i-th step.

In the solution of some analytical problems using this last strategy, it is remarked that the results obtained presented some dependency from the starting temperature. The principal cause is due to the difficulty to adjust the step length h.

This paper examines how the characteristics of the tabu search algorithm assembled into the standard simulated annealing improves it. The principal idea is to deal with a vector of different step lengths in place of only one step.

TABU SEARCH ALGORITHM

The Tabu Search method (TS), initially proposed for combinatorial optimization by Glover, was extended to continuous variables by Hu [4]. Glover's idea, as summarized by Hu, is as follows:

- For the given point x and the given step h, let $N(x,h)$ be the *neighbour* of x, defined as $N(x,h) = \{x : |x-y| \le h\}$.
- For the step h and the direction r, let y be the move from x, i.e., $y = x + rh$, where r is a random number that belongs to (-1,1).
- Given the set $H=\{h_1, h_2, \ldots, h_{nd}\}$, the list of inactive steps T, which is initially empty, and an initial feasible point x, the search of the next points are made over a set of active neighbours $N(x,h_k)$, where k belongs to H - T. Only one feasible random move is generated for each neighbour. If a decrement in the objective function is obtained for y,

Manuscript received July 10, 1995.
J.A. Vasconcelos (joao@cpdee.ufmg.br); L. Krähenbül (krahenb@trotek.ec-l.fr); A. Nicolas (alain@trotek.ec-l.fr)
This work was supported in part by the Conselho Nacional de Pesquisa Científica e Tecnológica (CNPq) and the Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG) - Brazil.

0018-9464/96$05.00 © 1996 IEEE

then save y as the current solution x and add its corresponding step h to T. When T is full, T is updated empty and the total process is reiterated, otherwise the above procedure is repeated for the next active step.

In global optimization problems, the moves in neighbours of different sizes prevent the search from trapping at a local optimum.

THE METHOD

The flow chart of the modified simulated annealing algorithm (MSA) is given bellow. Basically, it is the SA algorithm proposed by Corana et al (as applied by Simkin & Trowbridge [5]) with some characteristics of the TS algorithm incorporated [6].

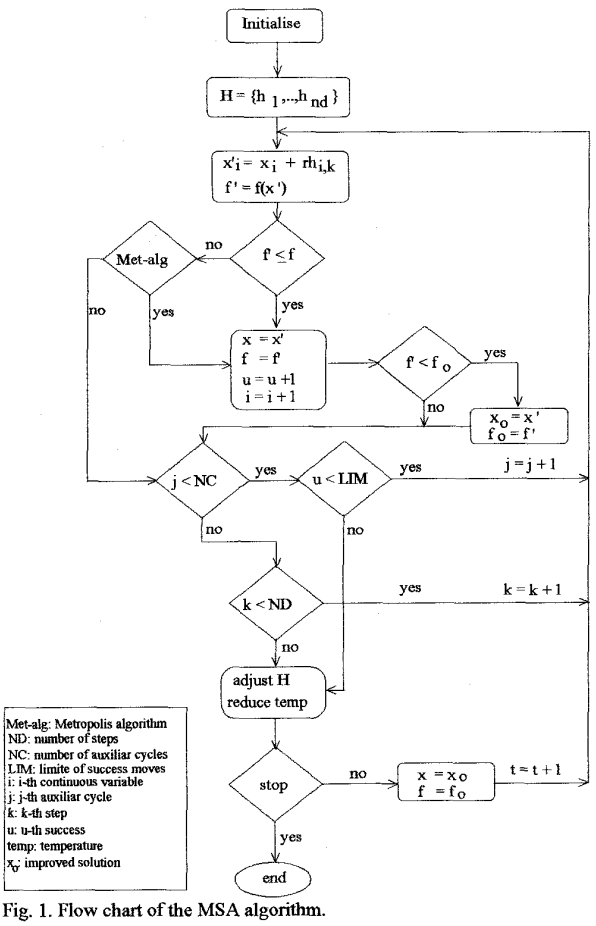

Fig. 1. Flow chart of the MSA algorithm.

The main difference between the MSA proposed here and the SA is that in the former the vector of steps h is replaced by ND vectors of steps, which are stored in matrix H. Also, NC auxiliary cycles are executed for each active vector h_k, $k \in \{1,...,ND\}$. After $NC*ND$ cycles or if u > LIM, matrix H is adjusted, using the same criteria as in [3] and the temperature in the SA is reduced by a factor smaller but close to 1.

The MSA starts with a given initial point and a succession of them is generated by changing the value of one variable at time with the rule $x_i' = x_i + rh_{i,k}$. In this equation i denotes the i-th variable, k the k-th step and r is a number whose values could be randomly $+1$ or -1. The last suggestion is proposed in [7].

The procedure used to calculate matrix H [4], is:

- First, consider that the objective function is defined as a function of n continuous variables x_i, $i \in \{1,...,n\}$, and each $x_i \in \{a_i, b_i\}$.
- Now, the ND vectors of H can be calculated as follows:

$$h_{i,1} = (b_i - a_i)/c$$
$$h_{i,2} = h_{i,1}/c$$
$$\vdots$$
$$h_{i,nd} = h_{i,nd-1}/c$$

where c is a factor greater than 1, for example $c=2$.

The condition when the number of success moves u is less than LIM in the MSA is used as a supplementary test. It enables the algorithm to update matrix H and the temperature TEMP, when the number of success moves reaches the limit imposed. This condition save computational time because of the total number of auxiliary cycles that is not completely iterated.

The deviation between two or more consecutive values of the objective function, in addition to the test if the value of the temperature is less than a small number, say TOLTEMP, and a limit in the number of function evaluations can be used as a stop criteria.

COUPLING THE MSA WITH A CODE OF ELECTROMAGNETIC FIELD CALCULATION

A code based on the finite element method or boundary element method can be coupled with the MSA algorithm via the objective function. The constrained non-linear optimization problem in electromagnetics generally involves problems of the form:

$$\begin{aligned} &min \quad f_1(x) \\ &with \quad g_l(x) \le 0 \quad l = 1,\dots,m \end{aligned} \tag{3}$$

where $f_1(x)$ is the objective function, $g_l(x)$ is the l-th constraint function and $x=(x_1,x_2,...x_n)$ is the vector of n degrees of freedom (*x can be either a vector of geometric parameters and/or electrical field or other electrical quantity that can be implicitly function of the geometric parameters*).

Using a transformation as in the penalty methods, the original problem stated above can be rewritten as an unconstrained minimization problem, as shown in (4).

$$min \quad f(x) = f_1(x) + p\sum_{l=1}^{m}[g_l(x)]_+ \tag{4}$$

Above, p is the penalty parameter and the term $[g_l(x)]_+$ ensures that only the violated constraint function is

considered. Now, the transformed problem can be solved by the MSA algorithm.

RESULTS

Analytical Results

A computer code which was developed based on the MSA algorithm was tested using the same two test functions reported in [8]. They are the "exponential" function f_1 and the Rastrigin's function f_2.

The function f_1 is a continuous, bimodal function of two variables $x = (x_1,x_2)^T$.

$$f_1(x) = 20 - e^{(-\Sigma(x_i-1.5)^2+1)} + e^{(-\Sigma(x_i-2.5)^2+1.05)} - e^{(-\Sigma(x_i-3.5)^2+1.1)} \qquad (5)$$

$$0 \le x_i \le 10 \qquad i = 1,2$$

This function has two local minima. The global minimum can be found at $x^* = (3.59585;3.59585)^T$ to give $f_1(x^*) = 17.30889$.

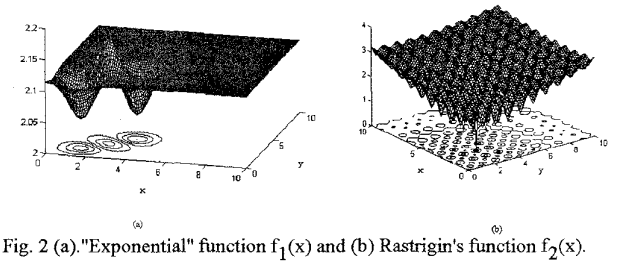

Fig. 2 (a)."Exponential" function $f_1(x)$ and (b) Rastrigin's function $f_2(x)$.

The second function f_2 is a continuous, multimodal test function of n continuous variables. For $x_i \in \{0,10\}$ there are 10^n local minima and the global minimum can be found at $x^* = (2.5;...;2.5)^T$ giving $f_2(x^*)=0$.

$$f_2(x) = 10n + \sum_{i=1}^{n} [(x_i - 2.5)^2 - 10\cos 2\pi(x_i - 2.5)] \qquad (6)$$

$$n = 10 \quad x = (x_1,..,x_{10})^T \quad 0 \le x_i \le 10$$

Several computation using different start points and ND values were executed. The start points A, B, C and D for the first function and A', B', C' and D' for the second one were used:

A={1.0,9.0}^{T}
B={0.0,1.0}^{T}
C={4.0,1.0}^{T}
D={7.0,9.0}^{T}
A'={0.5,0.2,0.3,0.4,5.0,9.0,8.2,2.0,4.0,3.2}^{T}
B'={1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0,10.0}^{T}
C'={1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0}^{T}
D'={0.0,10.0,0.0,10.0,0.0,10.0,0.0,10.0,0.0,10.0}^{T}.

The termination criteria used was only the test on the temperature value. The main parameters used were: ND={1,2,..,20}, NC=10*n/ND, LIM=MIN(n,ND), $TEMP^0$=0.1 ("Exponential" function), $TEMP^0$=1.0 (Rastrigin's function), $TEMP^{t+1}$ = 0.95*$TEMP^t$, TOLTEMP=0.001 and C = 2.

The results are shown in TABLE I and II, where $q_1^* = f_1(x_o) - f_1(x^*)$ and $q_2^* = f_2(x_o)$. The number of function evaluations is denoted by NBC. Note that the total number of possible cycles at each temperature *NC*ND* were made constant and equal to *10*n*, where *n* is the number of design variables. These computation were performed using simple precision.

TABLE I
"EXPONENTIAL" FUNCTION F_1.

Point	ND	1	5	10	15	20
A	NBC	909	1102	855	751	842
	q_1*	0.18334	0.00037	0.00003	0.00014	0.00006
B	NBC	811	1097	832(*)	724	724
	q_1*	0.03072	0.00076	0.28051	0.00033	0.00033
C	NBC	712	1091	921	763	834
	q_1*	0.07934	0.00059	0.00017	0.00003	0.00011
D	NBC	758	1056	880	763	777
	q_1*	0.49174	0.00026	0.00016	0.00019	0.00011

(*) Local minimum.

TABLE II
RASTRIGIN'S FUNCTION F_2.

Point	ND	1	5	10	15	20
A'	NBC	4825(*)	7738	7534	6311	5504
	q_2*	18.07629	0.32427	0.00920	0.00468	0.00539
B'	NBC	3345(*)	7145	7453	5919	5450
	q_2*	10.52338	0.54167	0.00784	0.00360	0.00636
C'	NBC	4679(*)	8145	7864	6242	5692
	q_2*	5.40686	0.20988	0.01038	0.00281	0.00237
D'	NBC	2166(*)	7382	7219	5617	5282
	q_2*	10.52417	0.00000	0.01090	0.00634	0.00000

(*) No convergence.

The performance comparison in terms of accuracy and the number of function evaluations, for several values of ND, using three points generated randomly, is presented in Fig. 3. It was used the same parameter values as before except for *TOLTEMP*, which it was made equal to 1E-5.

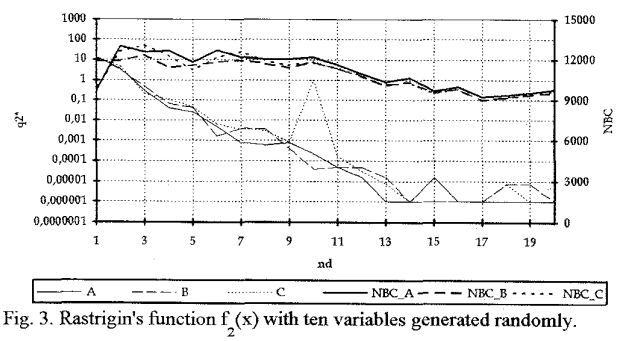

Fig. 3. Rastrigin's function $f_2(x)$ with ten variables generated randomly.

The MSA works quite well for all *ND* values. It has been noted that the best results are given by using a number of steps greater than ten (*ND > 10*). For *ND=1*, the algorithm works as the standard SA. It is interesting to point out that for *ND=1*, the number of function evaluations necessary to reach the optimal result is large, and the results are dependent upon the starting temperature in the SA. Changing the test *u < LIM* in the MSA to u < 5*LIM, after

to u < 10*LIM and u < 100*LIM, for the point A', the standard SA reaches a point near the global minimum only for the last case. In this test, the number of field evaluations was 8224, 10380 and 17218 respectively. The values of the objective function in this three cases were 4.28085, 8.38059 and *2.09673* in the same order.

Electrostatic Insulator

The goal in this problem is to find a maximum normal electrical field on the lower electrode equal to E_0. The optimization problem was formulated in the following way:

$$\begin{aligned} &min \quad f(x) = (E_{max}^2 - E_0^2)^2 \\ &with \quad x_{min_i} \le x_i \le x_{max_i} \qquad i = 1,\ldots,5 \end{aligned} \tag{7}$$

where x is the vector of design variables (Fig. 4). E_{max} and E_0 (E_0 = *130 V/m*) are respectively the maximum calculated and the desired normal electric fields on the movable boundary.

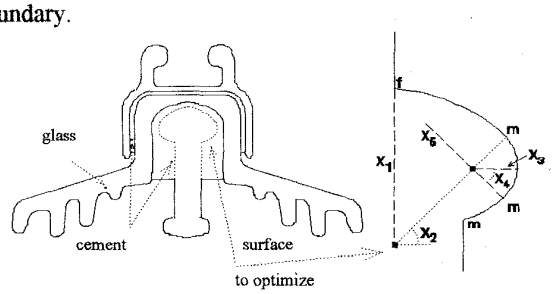

Fig. 4. Insulator for distribution lines (left) and a zoom of the boundary to be optimized showing the design parameters.

Two optimization are presented for different values of *ND*, in order to show the power and the applicability of the new algorithm. The parameter values used in the MSA to solve this problem were: $ND = \{10, 20\}$, $NC = 16*n/ND$, $LIM = 8*n$, $TEMP^{t+1} = 0.95*TEMP^t$, $TEMP^0 = 0.1$ and $C = 2$.

The termination criterion used in this case was simply the test on the number of function evaluations, after each cycle of temperature. This choice was made to prevent a great number of field analysis. Using this test, the number of cycles of temperature performed were only one for the first case and two for the second one.

The results are shown in TABLE III, where the optimized parameters for *ND* = 10 and ND = 20 are denoted by $Xopt_{10}$ and $Xopt_{20}$, respectively. The maximum normal electric field calculated before the optimization was 188.61 V/m.

The optimized parameters for the two simulation cases are approximately equals. Note that the parameter x_3 reaches its maximum value in both the cases. Also, these results show that MSA can be used to optimize real problems.

Fig. 5 shows the normal electric fields for the initial and the optimized geometries for the different optimizations as functions of the ordinate z along the movable boundary.

The result obtained represents a considerable gain on the maximum normal electric field, which can be considered a satisfactory result.

TABLE III
INITIAL AND FINAL DESIGN VARIABLE VALUE FOR ND ={10,20}.

Parameters	Xinit	Xmin	Xmax	$Xopt_{10}$	$Xopt_{20}$
radius x_1 (m)	.019	.010	.025	.01736	.01719
angle x_2	48.0	40	60	54.88	54.73
radius x_3 (m)	.0025	.002	.010	.010	.010
angle x_4	-10.0	-20	0	-18.11	-18.69
radius x_5 (m)	.019	.010	.025	.014	.01402
Emax (V/m)	188.61			132.99	132.97
NBC				80	138

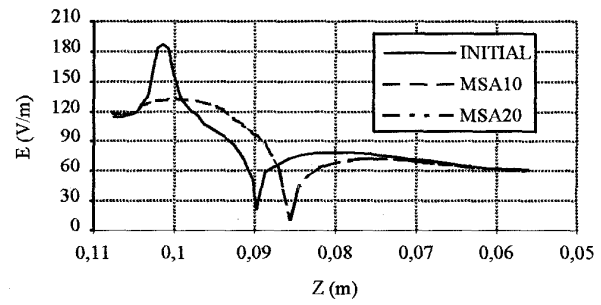

Fig. 5. Normal electric field on the movable boundary for the initial and the optimized geometries (*ND={10,20}*).

CONCLUSION

A new simulated annealing algorithm coupled with the tabu search method for design optimization was presented in this paper. The analytical results have shown that MSA can be applied to solve complex global optimization problems. Among the advantages of MSA, it can be mentioned that no calculation of gradient is necessary and also that its implementation is very simple. When solving a real problem, if the main interest is to find an improved solution, the number of function evaluations can be used to prevent numerous electromagnetic field computation. Moreover, a small number of auxiliary cycles in addition to a small TEMP value will also work in the same way.

REFERENCE

[1] S. Kirkpatrick, C.D. Gelatt, and M.P. Vecchi, "Optimization by simulated annealing", Science, 220, 671-680, 1983.
[2] D. Vanderbilt and S.G. Louie, "A Monte Carlo simulated annealing approach to optimization over continuous variables", Journal of Computational Physics, 56, 259-271, 1984.
[3] A. Corana, M. Marchesi, C. Martini and S. Ridella, "Minimizing multimodal functions of continuous variables with the 'Simulate Annealing' algorithm", ACM Trans. on Mathematical Software, 13, 262-280, 1987.
[4] N. Hu, "TABU search method with random moves for globally optimal design", Int. J: Num. Meth. Engineering, 35, 1055-1070, 1992.
[5] J. Simkin and C.W. Trowbridge, "Optimizing electromagnetic devices combining direct search methods with simulated annealing", IEEE Trans. on Magnetics, 28, 1545-1548, 1992.
[6] J.A. Vasconcelos, "Optimisation de forme des structures électromagnetics", Doctoral Thesis, ECL 94-32, France.
[7] G. Drago, A. Manella, M. Nervi, M. Repetto, G. Secondo, "A combined strategy for optimization in non-linear magnetic problems using simulated annealing and search techniques", IEEE Trans. on Magnetics, 28, 1541-1544, 1992.
[8] J.A. Vasconcelos, L. Krähenbühl, L. Nicolas, A. Nicolas, "Design optimisation using the BEM coupled with Genetic Algorithm", Proc. of the 2nd IEE Inter. Conference on Comp. in Electromagnetics, Nottinghan (UK), 1994.