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A Time-Domain Integral Formulation for the Scattering by Thin Wires

François Bost, Laurent Nicolas, and Gérard Rojat

***Abstract*—A time-domain model to provide the transient response of complex 3D wire structure is presented. It is based on the antennas theory. The Electric Field Integral Equation is solved by the Method of Moments and an iterative time-stepping procedure. The current is described by a first order expanding scheme. The delta function is chosen as testing function for point-matching.**

***Index Terms*—Integral equations, numerical analysis, time domain analysis, wire scatterers.**

I. INTRODUCTION

DIFFERENT formulations have been developed to perform the electromagnetic scattering by complex structures: finite difference methods first, boundary element (BEM) and finite element methods later. The study of thin wires structures using the BEM leads to very interesting simplification. The development of an integral formulation in time-domain for thin wires seems then very attractive. However only one computer code using such a time-domain formulation has been previously reported [1]. Our purpose is to apply this technique to model transient phenomena in complex structures such as electric circuits or Printed Circuits Boards.

This paper presents a rigorous three-dimensional time-domain integral formulation for thin wires based upon the antennas theory. The Electric Field Integral Equation (EFIE) is solved by using the Method of Moments and an iterative time-stepping procedure. The current is described by a first order expanding scheme and the delta function is chosen as testing function for point-matching. The formulation provides directly the induced current in the wire.

The basic hypotheses and the integral formulation are presented in the first section. The solving method is then described. The accuracy and the stability of the numerical scheme is then discussed. Examples are finally given in the last section.

II. INTEGRAL FORMULATION

By using the delayed vector and scalar potential $\boldsymbol{A}$ and φ, the classical EFIE is written with the assumption that the scatterers are perfect conductors [2]:

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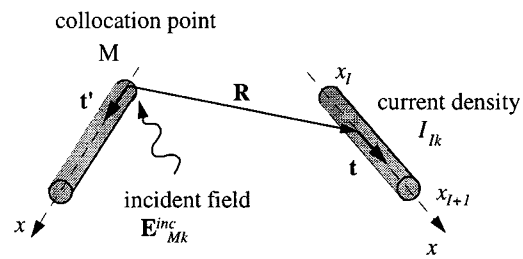

Fig. 1. Contribution of a straight segment at the collocation point M.

$$\boldsymbol{n} \times \boldsymbol{E}^{inc}(\boldsymbol{r}', t) = \frac{\boldsymbol{n}}{4\pi} \times \int_S \left[-\frac{\boldsymbol{R}}{c\varepsilon_0 R^2} \frac{\partial}{\partial \tau} \sigma_s(\boldsymbol{r}, \tau) - \frac{\boldsymbol{R}}{\varepsilon_0 R^3} \sigma_s(\boldsymbol{r}, \tau) + \frac{\mu_0}{R} \frac{\partial}{\partial \tau} \boldsymbol{J}_S(\boldsymbol{r}, \tau) \right] ds \tag{1}$$

where

$\boldsymbol{E}^{inc}$ is the incident field,

$\boldsymbol{n}$ is the unit vector normal to the surface S of the scatterer,

R is the distance between observation and source points, $\tau = t - R/c$ is the retarded time,

σ_s and $\boldsymbol{J}_s$ are the induced charges and current densities on S.

When conductors are thin wires, the current density $\boldsymbol{J}_s$ can be restricted to its tangential term along the x-direction of the wire axis:

$$\boldsymbol{J}_s(\boldsymbol{r}, \tau) = I(x, \tau)\boldsymbol{t}(\boldsymbol{r}) \tag{2}$$

where $\boldsymbol{t}$ is the tangent vector along the wire axis and $I(x, \tau)$ is the total current at the x curvilinear coordinate.

The continuity equation allows to replace the charges density with the current density:

$$\nabla \boldsymbol{J}_s(\boldsymbol{r}, \tau) + \frac{\partial \sigma_s(\boldsymbol{r}, \tau)}{\partial t} = 0 \tag{3}$$

With the thin-wire approximation, (3) leads to:

$$\frac{\partial I(x, \tau)}{\partial x} = -\frac{\partial \sigma_s(x, \tau)}{\partial t} \tag{4}$$

By multiplying by the tangent vector $\boldsymbol{t}'$ to the wire at observation point (Fig. 1) and using vector identities, the EFIE for thin wires may then be written as:

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$$\begin{aligned}\boldsymbol{t}' \cdot \boldsymbol{E}^{inc}(x', t) &= \frac{\mu_0}{4\pi} \int_L \left[\frac{\boldsymbol{t}' \cdot \boldsymbol{t}}{R} \frac{\partial}{\partial \tau} I(x, \tau) + c \frac{\boldsymbol{t}' \cdot \boldsymbol{R}}{R^2} \frac{\partial}{\partial x} I(x, \tau) \right. \\ &\quad \left. + c^2 \frac{\boldsymbol{t}' \cdot \boldsymbol{R}}{R^3} \int_{-\infty}^{\tau} \frac{\partial}{\partial x} I(x, t) dt \right] dx \qquad (5)\end{aligned}$$

This EFIE is of the first kind, and its kernel has a second order singularity. Since the singular integral evaluated for the self-patch in the right hand side of (5) leads to a nondiagonal matrix, the time scheme cannot be explicit. The entire integral is then separated into two parts, and the unknown current at the observation time can then be extracted from the self-patch value by solving an implicit scheme [3]:

$$\begin{aligned}&\left\{ \int_{\Delta x'} \begin{pmatrix} \text{self-patch} \\ \text{terms} \end{pmatrix} I(x', t)\, dx \right\} \\ &\quad = \boldsymbol{t}' \times \boldsymbol{E}^{\boldsymbol{inc}}(x', t) \\ &\qquad + \left\{ \int_{L-\Delta x'} \begin{pmatrix} \text{terms depending} \\ \text{on retarded time} \end{pmatrix} I(x, \tau)\, dx \right\} \qquad (6)\end{aligned}$$

Note that the magnetic field integral equation (MFIE) could have been written in the same way. The MFIE is generally preferred to the EFIE to solve a surface problem since it is simplier to use. However it becomes unstable, and hence unsuitable, when it is assumed that there is no variation of the induced current along the circumference of the wire. Indeed, in the case of a straight line, multiplying by $\boldsymbol{t}'$ leads to cancel all the terms except the source terms and the MFIE is no longer available.

III. Solving Method

The EFIE is solved by the Method of Moments (MoM) [4]. The unknown current is interpolated by a set of basis functions. The testing function is the delta function, so that the MoM becomes the so-called point-matching method or collocation method.

A. Interpolation Scheme for the Induced Current

The thin wires structure is divided into N segments, and a set of basis functions expresses the unknown current in each of these segments:

$$I(x, t) = \sum_{i=1}^{N} \sum_{j=1}^{Nt} I_{i,j}(x, t) \cdot V(x_i) \cdot U(t_j) \qquad (7)$$

with

$$\begin{aligned}V(x_i) &= \begin{cases} 1 & \text{if } x \in [x_{i-1}, x_i] \\ 0 & \text{anywhere else} \end{cases}; \\ U(t_j) &= \begin{cases} 1 & \text{if } x \in \lfloor t_{j-1}, t_j \rfloor \\ 0 & \text{anywhere else} \end{cases} \qquad (8)\end{aligned}$$

A linear scheme is used for time and space, on the contrary of the quadratic scheme used in [3], [5]:

for $\quad x \in [x_i, x_{i+1}] \quad$ and $\quad t \in [t_{j-1}, t_j]$

$$\begin{aligned}I_{ij}(x, t) &= \frac{(x - x_{i+1})}{(x_i - x_{i+1})} \left[\frac{(t - t_j)}{(t_{j-1} - t_j)} I_{i,j-1} + \frac{(t - t_{j-1})}{(t_j - t_{j-1})} I_{i,j} \right] \\ &\quad + \frac{(x - x_i)}{(x_{i+1} - x_i)} \left[\frac{(t - t_j)}{(t_{j-1} - t_j)} I_{i+1,j-1} \right. \\ &\qquad \left. + \frac{(t - t_{j-1})}{(t_j - t_{j-1})} I_{i+1,j} \right] \qquad (9)\end{aligned}$$

where I_{ij} represents the magnitude of the current in space-time at point x_i and time t_j. This first order interpolation scheme for the current leads to a constant distribution of charge on each space segment of the wire. Consequently a discontinuity appears at the junction between two adjacent segments. When modeling straight antennas, the computation of the induced current is actually not affected by this discontinuity. However, at singular points like angular points or multiple junctions, this can lead to a charge accumulation, and special numerical treatment has to be introduced in the model [9].

B. Kernel Singularity

The kernel shows a $1/R^2$ type singularity. This second-order singularity is circumvented by assuming that the current is located on the wire axis, because of the small radius compared to the wavelength and compared to the length of the wire. Since the observation point is located on the surface of the scatterer, the distance R is always greater or equal to the radius of the wire.

The regular integral terms representing the contribution of the far segments are computed by Gauss integration. In order to guarantee accurate results, the self-patch integration is calculated analytically.

C. Point Matching

The delta function is used as testing function for the Method of Moments. By using the point-matching method, (5) becomes:

$$\begin{aligned}\boldsymbol{t}' \cdot \boldsymbol{E}^{inc}(x'_m, t_k) &= \frac{\mu_0}{4\pi} \sum_i \int_{L_i} \left[\frac{\boldsymbol{t}' \cdot \boldsymbol{t}}{R_{mi}} \frac{\partial}{\partial \tau} I(x, \tau_k) + c \frac{\boldsymbol{t}' \cdot \boldsymbol{R}}{R_{mi}^2} \frac{\partial}{\partial x} I(x, \tau_k) \right. \\ &\quad \left. + c^2 \frac{\boldsymbol{t}' \cdot \boldsymbol{R}}{R_{mi}^3} \int_{-\infty}^{\tau_k} \frac{\partial}{\partial x} I(x, t)\, dt \right] dx \qquad (10)\end{aligned}$$

By writing the EFIE at each node $i = 1, N$ taken as matching point M, a $N \times N$ matrix system is obtained. From Fig. 1 it is seen that the contribution of a straight segment at the matching point M is affected with a delay R/c, which intends the propagation term.

Because the computation at the time t_k requires the currents at earlier times, the solving procedure uses an iterative scheme. The system of N equations is solved at each time step by a matrix inversion. The final matrix system may then be written symbolically as:

$$\boldsymbol{E}_{Mk}^{inc} = Z_{MI} \cdot \boldsymbol{I}_{Ik} + \boldsymbol{E}_{Mj}^{sca} \quad \text{for } j = 1, k-1 \qquad (11)$$

where

$\boldsymbol{E}_{Mk}^{inc}$ is the incident field vector tangent at points M and time t_k,

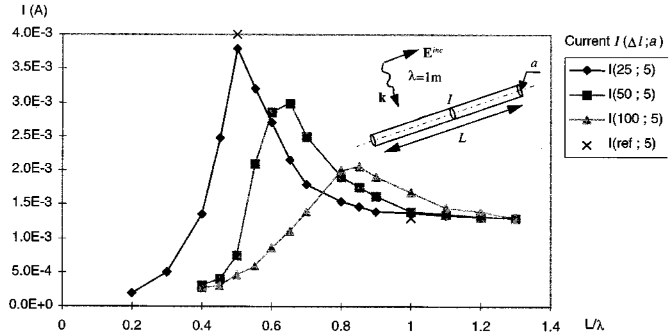

Fig. 2. Current induced by a 1 V/m-300 MHz incident electric field at the middle of a wire antenna for a radius $a = 5$ mm for several values of L and Δl. Reference values (denoted ref) are presented in [6].

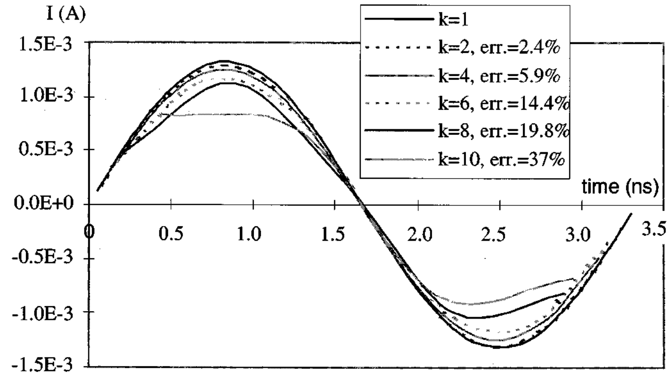

Fig. 3. Current induced at the middle of a straight antenna ($L = 1$ m, $a = 5$ mm) by a 300 MHz incident plane wave for several values of the time step ($\Delta t = k\Delta l/c$). Err. shows the difference with the reference value obtained for $k = 1$.

Z_{MI} is the geometrical matrix of the elementary contributions between nodes M and I (it corresponds actually to an incomplete impedance matrix),

$\boldsymbol{I}_{Ik}$ is the vector of currents at nodes I and time t_k, and

$\boldsymbol{E}_{Mj}^{sca}$ is the scattered field vector tangent at points M depending on the earlier currents at time t_j.

IV. Validation

The accuracy and the stability of the scheme depend on the characteristic parameters of the wire: its total length L, its radius a, the discretization Δl, the time step Δt, the wave-length λ. The thin wire approximation implies of course that the radius of the wire has to be small compared to the wavelength, in order to neglect the variation of the current along the circumference of the wire. On the other hand, two relations are predominant: $\Delta l/a$, and $\Delta l/\Delta t$.

A. Study of the Ratio $\Delta l/a$

The length of the line segment is related to the incident wavelength by $\Delta l = \lambda/N$, where N varies generally from 6 to 20 for the thin wires. Theoretically, the more the length of the segments decreases, the more the numerical model may be accurate. However the radius becomes significant in comparison with the segments length. Consequently the thin wires approximation is no longer valid, and the integral formulation becomes unstable.

There is then a compromise to find to obtain both accuracy and stability. Fig. 2 presents the current induced at the middle of antenna of various lengths when illuminated by a 300 MHz wave for several values of the space discretization Δl. At this frequency, a resonance is observed when the length of the antenna is equal to $\lambda/2 = 0.5$ m. Best results are obtained with a ratio $\Delta l/a = 5$. These values of the magnitude of the current for both antenna lengths $L = 1$ m and $L = 0.5$ m are in agreement with the results presented in [6]. For values of $\Delta l/a$ lower than 5, the problem becomes ill-conditioned and then unstable. For values greater than 10 (Fig. 2), one can note a displacement of the resonance peak, and the values of the current become underestimated.

B. Study of the Ratio $\Delta l/\Delta t$

Depending on the relative value of $c\Delta t$ compared to Δl, the system matrix may be more or less dense: for $c\Delta t < \Delta l$, it becomes for example tridiagonal. When the time step is increased, more and more nonadjacent segments are tied up at the same time, leading to a time scheme unstable.

Fig. 3 shows the induced current in the middle of a straight antenna illuminated by a 300 MHz plane wave for different values of the time step Δt. It can be observed that the current remains sinusoidal when the time step is lower than $4 \times \Delta l/c$. Furthermore the error made on the magnitude of the current remains also acceptable for such values of the time step.

C. Conclusion: Optimal Values of the Discretization Parameters

From the previous study, it appears that the optimal values for a L length wire with a λ wavelength are:

Radius of the wire: $a \ll L$ and $a \ll \lambda$

Space discretization: $\Delta l = 5a$, with $\Delta l \leq \lambda/10$

Time discretization: $\Delta t = \Delta l/c$ to $4\Delta l/c$ with $\Delta t \leq \lambda/10c$

D. Comparison with a Finite Element Code

The field scattered by a thin wire is computed by our model and by 2D and 3D finite element (FE) method [7]. Fig. 5 shows the comparison along a line perpendicular to the direction of propagation and centered on the wire, as defined at the Fig. 4. The results given by our model are in accordance with those obtained using 3D FE. The difference observed for the near field (<0.1 m) is due to the way that the thin wire formulation computes the scattered field outside the wire: it is obtained by inserting a small dipole at the measurement point. There is a coupling effect between the wire and the dipole, which is not negligible when both are close.

V. Example of Transient Responses of Wire Antennas

A. Wire Antenna Illuminated by a Gaussian Pulse

This example shows the response of a 1 meter long antenna exposed to a gaussian pulse of the form $E^{inc} = \exp[-a_0^2(t - t_{\max})^2]$, with $a_0^2 = 24.95$ c^2 and $t_{\max} = 0.52/c$. The radius of the wire is 6.7 mm. As shown in Fig. 6, the calculated current agrees with the results obtained with a second order interpolated current [8]. Slight differences

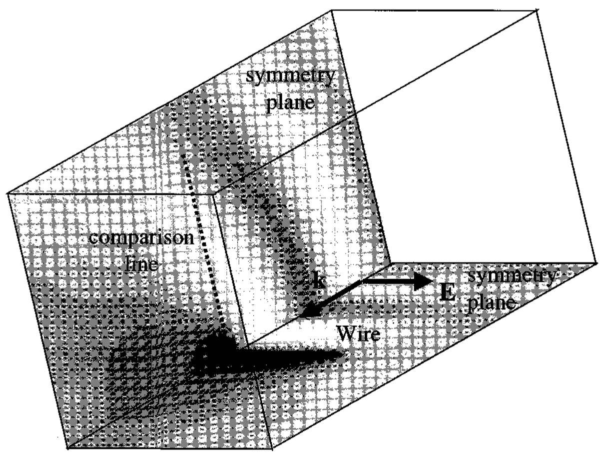

Fig. 4. Scattering by a thin wire antenna ($L = 1$ m, $a = 5$ mm) for a 1V/m-300 MHz incident plane wave. Computation by 3D finite element method [7]. Visualization of the total field in the symmetry planes.

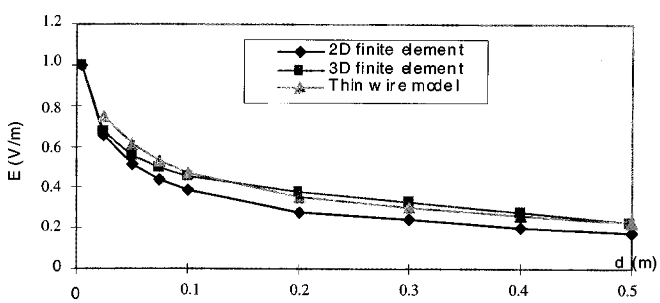

Fig. 5. Comparison of the scattered field between 2D FEM, 3D FEM and our model along a line perpendicular to the wire (Fig. 4).

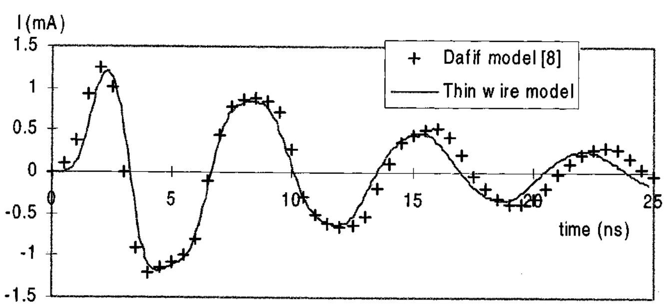

Fig. 6. Current induced in a thin wire antenna ($L = 1$ m, $a = 6.7$ mm) $E^{inc} = \exp[-a_0^2(t - t_{\max})^2]$, with $a_0^2 = 24.95c^2$ and $t_{\max} = 0.52/c$.

may be observed due to approximations made on numerical parameters defining the pulse. Discretization parameters and computation times are summarized in Table I.

B. Wire Antenna with a Voltage Step Supply

This integral formulation allows also actually the computation of conducted problems [9]. As example, a voltage generator supplying a 1 V voltage step is connected at the center of a wire antenna. The length of the antenna is 1m, and its radius is 6.74 mm. Fig. 7 shows the current at the center of the antenna. Results are in excellent agreement with those presented in [5]. The modification of the ratio L/a shows a slight influence on the magnitude of the current and on its frequency.

TABLE I
COMPARISON OF THE DISCRETIZATION PARAMETERS AND CPU TIME ON HP9000/700 FOR BOTH EXAMPLES PRESENTED IN SECTION A AND B

Example	Radius (mm)	Δl (mm)	Δt (ns)	Number of nodes	Cpu time (s)
A	6.7	33.	0.110	31	30
B	6.74	33.7	0.1123	31	35
B	4.166	20.83	0.0694	51	100

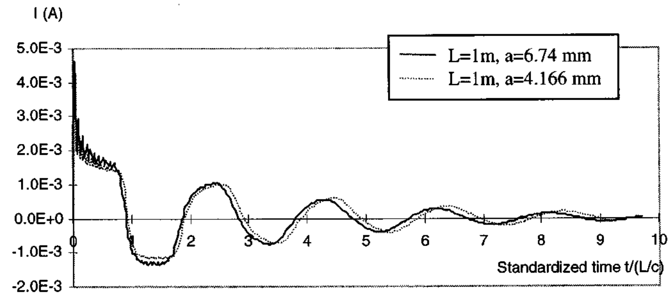

Fig. 7. Current at the center of a 1 m wire antenna for two different radii. The antenna is fed by a 1 V voltage step.

VI. CONCLUSION AND PERSPECTIVES

A time-domain integral formulation to evaluate induced current in thin wires when illuminated by transient wave has been presented. The Electric Field Integral Equation is solved by the Method of Moments and an iterative time-stepping procedure. The delta function is chosen as testing function for point-matching. It is shown that a first-order interpolation scheme can be used to expand the unknown currents. This new model is validated by determining the space and time discretization rules. Its accuracy is demonstrated using two examples of different nature.

The real interest of such a formulation is actually the modeling of EMC problems. By inserting non linear components, including generators, and multiple junctions, both conducted and induced phenomena may be modeled, so that the analysis of electronic device in normal running is made possible [9].

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