



HAL
open science

Application of regularisation method of quasi-singular integrals to compute eddy-current distribution near cracks

Philippe Beltrame, Noël Burais

► **To cite this version:**

Philippe Beltrame, Noël Burais. Application of regularisation method of quasi-singular integrals to compute eddy-current distribution near cracks. *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, 2002, 21 (4), pp.519-533. 10.1108/03321640210437789 . hal-00140341

HAL Id: hal-00140341

<https://hal.science/hal-00140341>

Submitted on 26 Apr 2007

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Application of regularization method to compute eddy-current distribution near cracks

Ph. Beltrame, N. Burais

CEGELY – UMR CNRS 5005 - Ecole Centrale de Lyon – BP 163 – 69131 Ecully cedex – France

e-mail: beltrame@eea.ec-lyon.fr

Abstract— For thin cracks, in Eddy-current Testing (ECT), the field-flaw interaction is equivalent to a current dipole layer on its surface. The dipole density is solution of an integral equation with a non-integrable kernel. A general algorithm to solve it is developed in [1]. The variation of coil impedance is directly obtained from this density. In order to complete the ECT simulation, a numerical code is developed to compute eddy-current distribution near cracks. We present two examples: a coil above one crack or two cracks in a plate.

INTRODUCTION

In order to avoid a full 3D computation of the scattered field, we have used the elementary Green's function. The crack is equivalent to a dipole currents surface [2], which is solution of an integral equation only on the crack surface. The integral is to be interpreted as the finite part of Hadamard. A regularization method is required to evaluate it [3].

The scattered field in the tested piece is given by integration on the crack surface of the dipole density. This integral is regular. But, in order to evaluate the current density near the crack, from the numerical point of view, the integral is like a singular one. A similar regularization can be developed to overcome this last problem.

In a first part, we present the effectiveness of the use of regularization method, and, in a second part, we give examples of eddy current distribution for one or two thin crack(s) in a plate.

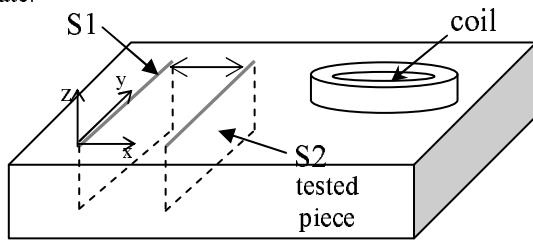


Fig. 1: Schematic configuration for two cracks detection.

FORMULATION AND NUMERICAL COMPUTING

We consider a coil and a tested piece with the same parameters as in [2]. Let suppose the skin depth is small in comparison with the thickness of the plate. The slot S_1 ($l_y \times l_z$) is in the plane $x=0$, and the slot S_2 is in the plane $x = \frac{\delta}{2}$ with same sizes (Fig. 1).

One crack in the plate: the dipole density p_1 is solution of an integral equation [1]. We have used a collocation method and 2^{nd} order shape functions to solve it. The perturbed current density, \mathbf{J}_p , at a point \mathbf{r}' is:

$$\mathbf{J}_p(\mathbf{r}') = \mathbf{J}_o(\mathbf{r}') + \int_{S_1} \overline{\mathbf{G}}(R_{11}) \mathbf{p}_1(\mathbf{r}_1) dS_1 \quad (1)$$

where \mathbf{J}_o is the incident current, $R_{11} = |\mathbf{r}_1 - \mathbf{r}'|$ and $\overline{\mathbf{G}}$ is the dyadic electrical Green function of an half space conductor.

Two cracks S_1 and S_2 in the plate: the respective dipoles densities p_1 and p_2 are solution of two integral equations:

$$\begin{aligned} J_{0n}(\mathbf{r}') &= -PF \int_{S_1} \left(\overline{\mathbf{G}}(R_{11}) \mathbf{p}_1(\mathbf{r}_1) \right) \cdot \mathbf{n} dS_1 - PF \int_{S_2} \left(\overline{\mathbf{G}}(R_{12}) \mathbf{p}_2(\mathbf{r}_2) \right) \cdot \mathbf{n} dS_2 \\ J_{0n}(\mathbf{r}') &= -PF \int_{S_1} \left(\overline{\mathbf{G}}(R_{21}) \mathbf{p}_1(\mathbf{r}_1) \right) \cdot \mathbf{n} dS_1 - PF \int_{S_2} \left(\overline{\mathbf{G}}(R_{22}) \mathbf{p}_2(\mathbf{r}_2) \right) \cdot \mathbf{n} dS_2 \end{aligned} \quad (2)$$

where $R_{ij} = |\mathbf{r}_i - \mathbf{r}'|$. The system (2) is solved with the same method as previously. The electrical perturbed field is:

$$\mathbf{J}_p(\mathbf{r}') = \mathbf{J}_o(\mathbf{r}') + \int_{S_1} \overline{\mathbf{G}}(R_{11}) \mathbf{p}_1(\mathbf{r}_1) dS_1 + \int_{S_2} \overline{\mathbf{G}}(R_{12}) \mathbf{p}_2(\mathbf{r}_2) dS_2 \quad (3)$$

Numerical difficulties: Integrals in (1) and (3) are regular if \mathbf{r}' is not on a crack surface. Nevertheless the integrand contains terms in $\frac{1}{R_{ij}^3}$ which can take high values when R_{ij} is small, i.e. when \mathbf{r}' is close to a crack.

In order to point the numerical difficulties of the accurate evaluation of these integrals, we are using, on one hand, two classical quadrature schemes: an adaptive recursive Newton Cotes 8 panel formula and an adaptive recursive Simpson's rule, on the other hand, a regularization method (Table I). We compare the time CPU to a numerical integration of a regular function. The Newton formula needs the same CPU time as a regular function, but for a distance $d \cong \delta/2$ (δ is the skin depth) from crack the integral diverges. The Simpson's rule is accurate for little values of d (up to $\delta/100$), but it needs a long CPU time ($\times 220$). From $d \cong 5.10^{-3} \delta$ this method diverges. Only a specific algorithm [3] allows to accurately compute the integral without increasing a lot the CPU time ($\times 5$) for any distances d .

In the next section, the current density is computed by a regularization method if the distance to crack is less than δ , otherwise, by Newton Cotes 8 panel formula.

x/δ Adimensional distance $x = d/\delta$ of the crack	Regularized integral Relative time	Simpson's Rule		Newton-Cotes Formulas	
		Relative error %	Relative time	Relative error %	Relative time
$x = 10$	$\times 1$	0.02 %	$\times 9.5$	0.46 %	$\times 1$
$x = 2$	$\times 1.5$	0.02 %	$\times 12$	0.12 %	$\times 0.9$
$x = 1$	$\times 1.5$	0.05 %	$\times 25$	3.4 %	$\times 0.9$
$x = 0.6$	$\times 2$	0.07 %	$\times 41$	13.2 %	$\times 1$
$x = 0.1$	$\times 3.5$	0.10 %	$\times 295$	$\gg 100$ %	$\times 1$
$x = 0.01$	$\times 4$	0.09 %	$\times 257$	$\gg 100$ %	$\times 1$
$x = 6e-3$	$\times 4$	7.48 %	$\times 224$	$\gg 100$ %	$\times 1$
$x = 5e-3$	$\times 4$	37.3 %	$\times 226$	$\gg 100$ %	$\times 1$
$x = 4e-3$	$\times 4$	192 %	$\times 224$	$\gg 100$ %	$\times 1$

TABLE I. DIFFERENT ALGORITHMS TO COMPUTE THE x COMPONENT OF CURRENT DENSITY AT THE POINT $y = 2.2$ MM AND $z = -4.2$ MM

EDDY-CURRENT DISTRIBUTION

One crack in the plate: Fig. 2 shows eddy-current distribution in the plane of the crack surface. The bold lines are the limits between eddy-current, which flows under (39%) or round (left: 25%/right: 36%) the crack. Fig. 3(a) shows eddy-currents in the plane $z = -1.4\delta$.

Two cracks S_1 and S_2 in the plate: Fig. 3(b) represents eddy-current in the same plane z . Between the both cracks, eddy currents stay inside and have a weak modulus comparing to the field outside the cracks. The eddy-current on the outside face of S_1^- ($x=0^-$) is like the previous case (25% flows under, 34% flows around left and 42% around right). But, on the opposite face S_1^+ ($x=0^+$), the distribution is quite different: except the crack edges, the modulus of scattered field is negligible comparing the other face (Fig 4).

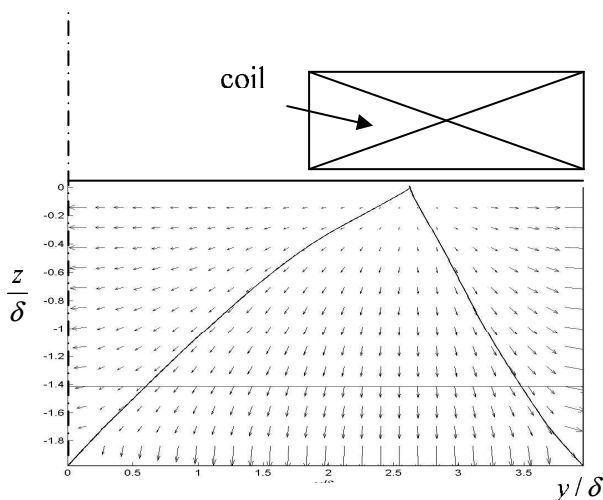


Fig. 2. Eddy-current(arrows) distribution on the crack face $x = 0$ and demarcation lines (thick) for the problem with one slot

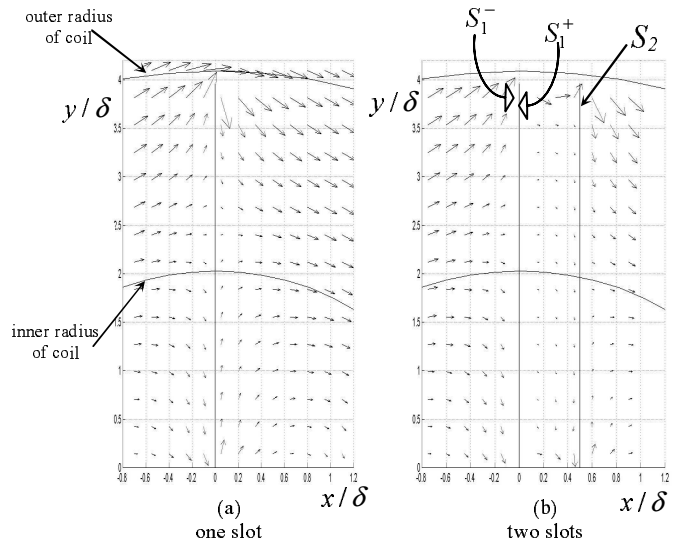


Fig. 3. Eddy-current distribution in the plane $z = -1.4\delta$ for 1 or 2 slots

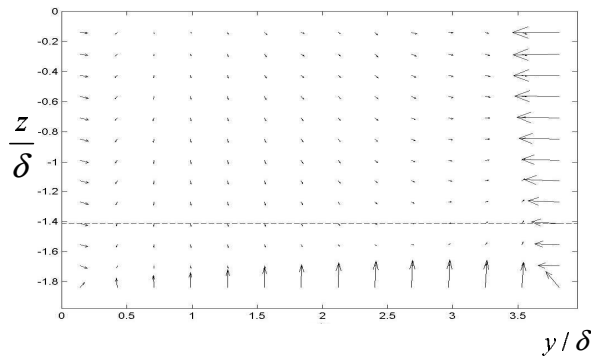


Fig. 4. Eddy current on the inside crack face S_1^+

CONCLUSION

The current dipole model has been used to solve the ECT problem of several thin cracks in a plate conductor. A regularization method is used to overcome the singularity problem without increasing a lot the CPU time. We can represent the eddy-current in all cases. Thus, with [1] we have developed a complete tool to simulate the crack problem.

Furthermore, the method can be extended to other geometries of a tested piece: for example a plate of finite width because the singular part of dyadic Green function is the same, only reflection terms change.

REFERENCES

- [1] Ph. Beltrame, N. Burais, "Computing method of hypersingular integral applied to eddy currents testing for crack problem", *Compumag* 2001.
- [2] J. Bowler, "Eddy-current interaction with an ideal crack. The forward problem," *In J. Appl. Phys.*, vol. 75(12), pp. 8128-8137, 1994.
- [3] M. Guiggiani, G. Krishnasamy, J.T. Rudolph and F.J. Rizzo, "A general algorithm for the numerical solution of hypersingular boundary integral equations," *ASME J. Appl. Mech.*, 59, pp. 604-614, 1992.