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Leaky modes of a left-handed slab

A. Moreau and D. Felbacq

April 3, 2007

Abstract

We show that there are two types of leaky modes in a left-handed slab (i) leaky slab modes which are always backward and (ii) leaky surface plasmons which may be forward or backward. The propagation direction of these modes are studied using complex plane analysis. The fact that slab modes are backward explains several previously reported results concerning lateral shifts. The complex plane analysis is shown to be particularly relevant for the study of this structure.

1 Introduction

Since it has been demonstrated that left-handed materials[1] could be made using meta-materials[2], they have attracted much attention. Many exciting applications have been suggested for the left-handed media. A slab of such a material could be used as a flat lens[1] allowing in some particular conditions to overcome the Rayleigh limit[3], a phenomenon in which the surface modes play a role.

The Goos-Hänchen shift experienced by a reflected beam[4, 5] has attracted a renewed interest too, especially in the case of multi-layered structures [6, 7]. Since left-handed materials were in the mood, the Goos-Hänchen shift has been studied for the reflection on such materials and has been found to be negative[8, 9]. It has been shown that the shift could even be huge for multilayered structures[10] because of the excitation of backward plasmons[11, 12]. Some large shift have been reported[13], wrongly attributed to such surface modes, in our opinion. Some other works report large shifts of the reflected beam on a left-handed slab[14] but these have not been interpreted in terms of the excitation of leaky guided modes.

The goal of this paper is to study all the leaky modes which can be supported by a left-handed slab. We will see that the slab does not only support leaky slab modes as for the right-handed slab. Leaky surface modes can be excited in this case - and the left-handed slab is the only structure for which these two types of leaky modes are observed. We will explain which modes are backward and why, using complex plane analysis. As we will see, leaky slab modes and leaky surface modes should not be confused[13], and leaky slab modes explain some large lateral shifts[14, 15].

2 Reflection coefficients

In this section, we will consider the reflection coefficient of a plane wave propagating in a right-handed medium on an interface with a right- (section 2.1) or a left- (section 2.2) handed medium and the relation between these two reflectivities. We will put emphasize on complex plane properties of these coefficients to introduce the complex plane tools and concepts.

2.1 Right-handed medium

Let us consider an interface between two right-handed media 1 and 2 characterized by their respective permittivity ϵ and permeability μ , all positive. The reflection coefficient of an incident plane $e^{i(\alpha x - \beta_1 y - \omega t)}$ wave can be written

$$r = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2}, \quad (1)$$

where $\kappa_i = \frac{\beta_i}{\mu_i}$ in TE polarization (or $\kappa_i = \frac{\beta_i}{\epsilon_i}$ in TM polarisation) with $\beta_i = \sqrt{\epsilon_i \mu_i k_0^2 - \alpha^2}$ and $k_0 = \frac{\omega}{c}$. The index of a medium is defined by $n = \sqrt{\epsilon \mu}$, and the propagation angle θ is defined by $\alpha = n k_0 \sin \theta$.

When total reflection occurs, which requires $n_2 < n_1$ and $\alpha > n_2 k_0$ then the lateral displacement of the reflected beam's barycenter along the interface is given by the formula

$$\delta = -\frac{d\phi}{d\alpha}, \quad (2)$$

where ϕ is the phase $r = e^{i\phi}$. This formula holds as long as the incident beam is spatially large enough (and hence spectrally narrow enough)[7].

This displacement is noticeable (a few wavelength) when the incidence angle is just above the critical angle and it is called the Goos-Hänchen shift[4].

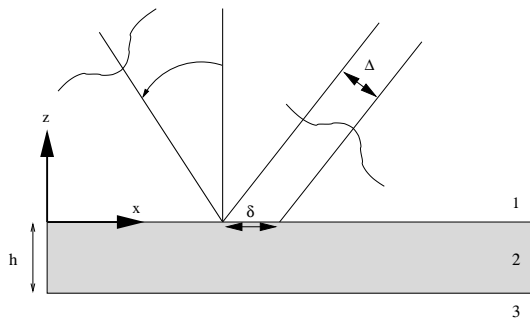


Figure 1: The asymmetric slab. The lateral displacement can be measured along the interface (δ) or in a direction perpendicular to the propagation of the reflected beam (Δ).

Since the square root can be continued on the complex plane, r can be continued as well. We choose to take $\sqrt{z} = \sqrt{r} e^{i\frac{\theta}{2}}$ with $z = r e^{i\theta}$ and $\theta \in] - \pi, \pi]$, as a definition of the square root. This means that we place the cut line on the negative part of the real axis and if x is a positive real, $\sqrt{-x} = i\sqrt{x}$. This defines the square root on the entire complex plane, to which we refer as the first Riemann sheet. When we will write the z is on the second Riemann sheet, it will mean that we choose to take the opposite of \sqrt{z} as defined above.

With this choice, we have (i) $\Re(\sqrt{z}) \geq 0$ (ii) $\sqrt{z^*} = \sqrt{z}^*$ for z on both sheets but *not on the cut line* (iii) if $\Im(z) < 0$, $\Im(\sqrt{z}) < 0$ and if $\Im(z) > 0$, $\Im(\sqrt{z}) > 0$ (iv) the function $\beta(z) = \sqrt{\epsilon \mu k_0^2 - z^2}$ has a cut line on the real axis (on $] - \infty, -n k_0] \cup [n k_0, +\infty[$ more precisely) and the function β on the real axis is continuous with the part of the complex plane which is *under the cut line* : when z passes through the cut line from the first Riemann sheet (coming from the lower part of the plane) to the second Riemann sheet, $\beta(z)$ is continuous. When a function which can be written using $\beta(z)$ presents a pole, it must be found either (i) for z on the first Riemann sheet and under the real axis (we will say that the pole itself is on the first Riemann sheet in this case) or (ii) for z on the second Riemann sheet but above the real axis.

2.2 Left-handed medium

Let us consider the case when medium 2 is left-handed. We should then distinguish between two cases : either $\alpha < n_2 k_0$ or $\alpha > n_2 k_0$.

If $\alpha < n_2 k_0$, then the outgoing wave condition is fulfilled only if $e^{i(\alpha x + \beta_2 y - \omega t)}$ is considered as an outgoing wave. In left handed materials, the Poynting vector is indeed opposite to the propagation vector. The reflection coefficient can thus be written

$$r = \frac{\kappa_1 + \kappa_2}{\kappa_1 - \kappa_2}. \quad (3)$$

Let us define $\kappa_2^+ = \frac{\beta_2}{|\mu_2|}$ in TE polarization (or $\kappa_2^+ = \frac{\beta_2}{|\epsilon_2|}$ in TM polarization). This would be the value of κ for a right-handed medium characterized by $|\epsilon_2|$ and $|\mu_2|$. Let us denote r^+ the reflection coefficient from a medium 1 on such a right-handed medium. Since $\kappa_2^+ = -\kappa_2$, we have $r = r^+$.

When $\alpha > n_2 k_0$, there is no need to use any outgoing wave condition so that, again,

$$r = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2}. \quad (4)$$

This time, we have $r = \frac{1}{r^+}$. While $|r| = 1$, this can be written $r^+ = r^*$ as in [9], which explains why the Goos-Hänchen shift is negative in this case.

For $\alpha > n k_0$ whatever the index n the guided modes of the interface can be found (surface modes). It has been shown[11, 12] that an interface between a right-handed medium (here, air with $\mu_1 = \epsilon_1 = 1$) and a left-handed medium 2 could support surface waves either forward (if $\epsilon_2 \mu_2 < 1$, $|\mu_2| < 1$ and $|\epsilon_2| > 1$ in TE polarization) or backward (if $\epsilon_2 \mu_2 > 1$, $|\mu_2| > 1$ and $|\epsilon_2| < 1$ in TE polarization). The surface mode has a propagation constant α_p which is given (in TE polarization and for $\epsilon_3 = \mu_3 = 1$) by

$$\alpha_p = k_0 \sqrt{\mu_2 \frac{\mu_2 - \epsilon_2}{\mu_2^2 - 1}}. \quad (5)$$

Since a guided mode does not need any incident wave to exist, the reflection coefficient diverges when $z = \alpha_p$. The coefficient is said to have a pole in $z = \alpha_p$. This is not the case for the right-handed slab : since $r^+ = \frac{1}{r}$, r^+ goes to zero when $z = \alpha_p$.

3 Leaky modes

A leaky mode of a given structure is a propagative mode which is attenuated because its energy leaks out. It can be considered as a guided mode which is coupled with an outgoing wave. The propagation constant α of such a mode thus has an imaginary part.

A mode (guided or leaky) being a solution without any incident wave, it always corresponds to a pole of the reflection coefficient (continued to the complex plane) of the structure. A *guided* mode is a pole on the real axis, that cannot be excited by an incident plane wave or beam. A *leaky* mode corresponds to a pole for $z = \alpha$ with $\Im(\alpha) \neq 0$. It can be excited by an incident beam and its behaviour is well studied[5]. Such a mode can be forward or backward. A forward mode is decreasing when propagating towards $+\infty$ so that the imaginary part of its constant propagation α is *positive*. The corresponding pole is hence above the real axis. A backward mode is decreasing when propagating towards $-\infty$ so that $\Im(\alpha) < 0$ and the pole belongs to the lower part of the complex plane. Until recently, backward leaky modes had been considered in some very particular cases only[5, 16].

When a leaky mode is excited by an incident beam, the reflected beam undergoes a very large displacement and it is generally distorted. In the case when the reflection coefficient has a modulus equal to one, the displacement of the beam's barycenter is given by equation (2). This means that the pole linked to the leaky mode produces a very swift variation of the phase of the reflection coefficient on the real axis. This is due to the fact that each pole is linked to a zero. When $|r| = 1$ on the real axis, the pole and the zero are even symmetrical with respect to the real axis. And the phase of r varies quickly on a line between the zero and the pole.

Let us stress that in this case the displacement is linked to the existence of a pole (a leaky mode) in the complex plane. We think that for this reason, this type of lateral shift should maybe not be called a Goos-Hänchen shift : the original Goos-Hänchen shift is due to the existence of a branch point and not to a pole of the structure[18].

Before discussing the case of the left-handed slab, let us now recall some well-known examples of leaky modes.

3.1 Leaky modes of the right-handed slab

Let us consider a right-handed slab of thickness h , as shown figure 1.

The reflection coefficient for this structure can be written

$$r = \frac{r_{23} e^{2i\beta_2 h} - r_{21}}{1 - r_{21} r_{23} e^{2i\beta_2 h}} \quad (6)$$

where

$$r_{ij} = \frac{\kappa_i - \kappa_j}{\kappa_i + \kappa_j}. \quad (7)$$

A pole can then be found in the complex plane when the condition

$$r_{23} r_{21} = e^{-2i\beta_2 h} \quad (8)$$

is verified. These modes have been partly investigated. Some have been studied experimentally[17], at least when there is a total internal reflection on the third medium. Figure 2 shows a leaky mode which leaks out in the upper and lower media.

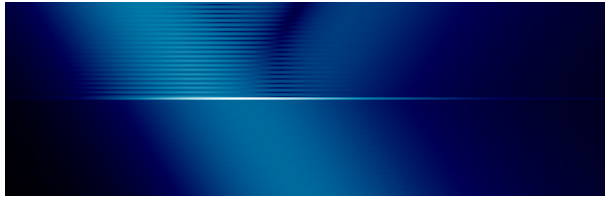


Figure 2: Modulus of the field for a symmetrical slab with $\epsilon_1 = \epsilon_3 = 9$, $\mu_1 = \mu_3 = 1$, $\epsilon_2 = 1.5$, $\mu_2 = 1$ and $h = 1.3 \lambda$ using a gaussian incident beam with a waist of 20λ and an incidence angle of $\theta = 22.78$.

3.2 Leaky surface plasmons

The prism coupler is used to excite surface plasmons, which are leaky in this case[18]. The leaky surface plasmons can be excited using two different configurations. The Kretschmann-Raether (KR) configuration corresponds to the case of a thin metallic slab between a high index medium (the prism) and a low index medium (air). The incident beam is propagating in the high index medium (medium 1). Above the critical angle between medium 1 and the air, it is possible to excite a leaky surface plasmon. But the metallic slab does not support slab modes, because the waves in media with negative permittivity are all evanescent.

The Otto configuration is the case when the upper medium is the high index medium, the lower medium the metallic one and when there is a small thickness of air between the two. In both configuration, the surface mode is localized at the interface between metal and air.

It has been shown that, in the case of the Otto configuration, with a left-handed medium instead of the metal, a leaky surface plasmon could be excited[10], leading to a large lateral displacement of the reflected beam.

4 Left-handed slab

The left-handed slab's reflectivity has the same expression as the right-handed slab's, which is

$$r = \frac{r_{23} e^{2i\beta_2 h} - r_{21}}{1 - r_{21} r_{23} e^{2i\beta_2 h}}. \quad (9)$$

The propagation direction of the energy must be taken into account when there is a semi-infinite left-handed medium like in section 2.2, but not in the case of a finite structure (inside the slab) when the real propagation direction does not matter.

The reflection coefficient r may present poles for two reasons. First, if the condition

$$r_{23} r_{21} = e^{-2i\beta_2 h} \quad (10)$$

is verified. Such poles correspond to leaky slab modes, like in the case of the right-handed slab. Secondly, if r_{23} presents a pole corresponding to a surface mode, then r will present a pole nearby too, corresponding to a leaky surface mode. This never happens with the right-handed slab, because the interface between two right-handed media does not support surface waves. We will, in the following, study both cases. Let us just underline that a left-handed slab presents *both types of leaky modes*, which is remarkable. These two different types of leaky modes cannot be confused.

4.1 Modulus of r_{ij}

We have

$$r_{ij} = \frac{\kappa_i - \kappa_j}{\kappa_i + \kappa_j}. \quad (11)$$

We will now prove that for z on the first Riemann sheet (but not on the cut line) we have $|r_{ij}(z)| > 1$ when media i and j are not both right-handed.

The modulus of r_{ij} reads as

$$|r_{ij}|^2 = \frac{(\kappa_i - \kappa_j)(\kappa_i^* - \kappa_j^*)}{(\kappa_i + \kappa_j)(\kappa_i^* + \kappa_j^*)} \quad (12)$$

$$= \frac{|\kappa_i|^2 + |\kappa_j|^2 - 2(\kappa_i' \kappa_j' + \kappa_i'' \kappa_j'')}{|\kappa_i|^2 + |\kappa_j|^2 + 2(\kappa_i' \kappa_j' + \kappa_i'' \kappa_j'')}, \quad (13)$$

$$(14)$$

where $\kappa = \kappa' + i \kappa''$.

Let us define x and y the real and imaginary part of $z = x + i y$ on the first Riemann sheet. Let us assume that $x > 0$. We have

$$\beta = \sqrt{n^2 k_0^2 - z^2} = \sqrt{n^2 k_0^2 - x^2 + y^2 - 2 i x y}. \quad (15)$$

If $y > 0$, then $x y > 0$ and thus $\Im(n^2 k_0^2 - z^2) < 0$ so that finally $\Im(\beta) < 0$. If $y < 0$, then $x y < 0$ so that $\Im(\beta) > 0$. Since $\beta(-z) = \beta(z)$ the result will hold for $x < 0$ too and for $x = 0$, $\beta(z)$ is real and positive so that the result obviously holds. Hence the imaginary part of $\beta(z)$ is thus positive (resp. negative) when the imaginary part of z is negative (resp. positive).

For any right-handed medium, κ has the same property than β . For a left-handed medium, since $\kappa = \frac{\beta}{\mu}$ or $\kappa = \frac{\beta}{\epsilon}$ depending on the polarization, the imaginary part of κ has the sign of $\Im(z)$. Since i and j are not both right-handed, then κ_i'' and κ_j'' have not the same sign and the product $\kappa_i'' \kappa_j''$ is always negative. Since $\Re(\sqrt{z}) > 0$ for all z on the first Riemann sheet then $\kappa_i' \kappa_j'$ is always negative too.

Finally, since $\kappa_i' \kappa_j' + \kappa_i'' \kappa_j'' < 0$, we have $|r_{ij}| > 1$ for all z except on the real axis.

On the real axis, whether $\alpha > \max(n_1, n_2, n_3) k_0$ or $\alpha < \max(n_1, n_2, n_3) k_0$ it is easy to see that $|r| > 1$. Let us just stress that r has two cut lines (one for β_1 and the other for β_3), but there is no cut line for β_2 since changing β_2 into $-\beta_2$ does not change r_{ij} . The modulus of $|r_{ij}|$ is equal to 1 only if κ_1 is real and κ_2 is imaginary (or the contrary), which occurs for the total internal reflection.

4.2 Fundamental relation

If the medium j is a left-handed medium characterized by $\epsilon_j < 0$ and $\mu_j < 0$, then $\kappa_j = -\kappa_j^+$, κ_j^+ being, as in section 2.2, the κ coefficient for a right-handed medium with $|\epsilon_j|$ and $|\mu_j|$. We will denote r_{ij}^+ the reflection coefficient r_{ij} when the medium j is this particular right-handed medium.

It is easy to see that we have

$$r_{ij}^+ = \frac{1}{r_{ij}}. \quad (16)$$

Let us underline that, as we just have seen section 2.2, the coefficient r_{ij} is not exactly the reflection coefficient on a left-handed medium. That is

why its modulus is superior to one in all the complex plane, as shown just above. Relation (16) allows to conclude that $|r_{ij}^+| < 1$ in all the complex plane (except on the cut line).

The reflection coefficient r can be written

$$r = \frac{\frac{e^{2i\beta_2 h}}{r_{23}^+} - \frac{1}{r_{21}^+}}{1 - \frac{e^{2i\beta_2 h}}{r_{21}^+ r_{23}^+}} \quad (17)$$

$$= \frac{r_{23}^+ e^{-2i\beta_2 h} - r_{21}^+}{1 - r_{21}^+ r_{23}^+ e^{-2i\beta_2 h}} \quad (18)$$

$$(19)$$

Since $\sqrt{z^*} = \sqrt{z}^*$ except when z is on the cut line, then $\beta(z^*) = \beta(z)^*$ and hence $r_{ij}^+(z)^* = r_{ij}^+(z^*)$ so that

$$r(z)^* = \frac{r_{23}^+(z^*) e^{2i\beta_2(z^*) h} - r_{21}^+(z^*)}{1 - r_{21}^+(z^*) r_{23}^+(z^*) e^{2i\beta_2(z^*) h}}, \quad (20)$$

which can simply be written

$$r(z)^* = r^+(z^*), \quad (21)$$

where r^+ is the coefficient reflection of a right-handed slab with $|\epsilon_j|$ and $|\mu_j|$. There is a tight connexion between the left-handed slab's reflectivity and the right-handed slab's. Note that this relation does not hold on the cut line, but that it holds for the two Riemann sheets.

The main consequence of relation (21) is that the zeros and the poles of r are symmetrical of those of r^+ in the complex plane. As we will see, this does not mean that the left-handed slab's leaky modes can be excited for the same angle of the incident beam as for the right-handed slab : the relation does not hold on the cut line, which is a part of the real axis here.

Figure 3 shows the phase of the reflection coefficient when z is on the first Riemann sheet. It has been obtained for $h = 2\lambda, \epsilon_1 = 9, \mu_1 = \epsilon_3 = \mu_3 = 1, \epsilon_2 = -5$ and $\mu_2 = -1$. A zone where the phase varies quickly can be seen between each pole and the corresponding zero. The real axis is continuous with the lower part of the complex plane (ie $r(z)$ is continuous when z arrives on the real axis coming from the lower part of the complex plane), so that the zeros which are above the cut line (and which correspond to poles on the other Riemann sheet) have no effect on it. These zeros are the symmetrical

of the zeros of right-handed slab's leaky modes. On the contrary, the poles which are under the real axis are “seen” by the real axis. The corresponding zeros are on the other Riemann sheet.

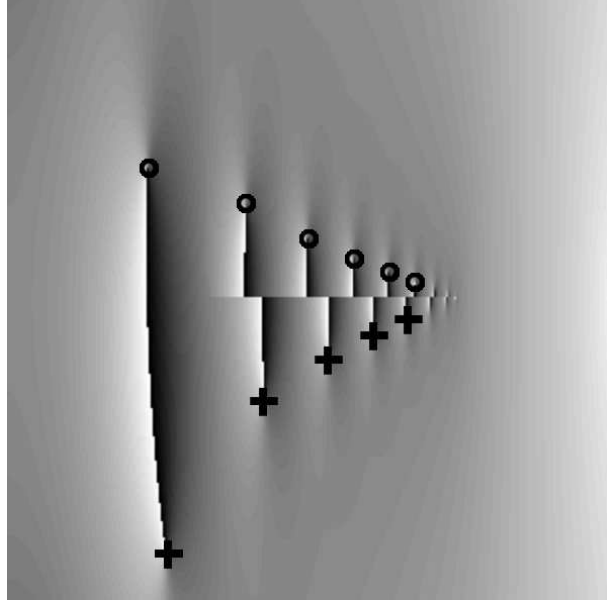


Figure 3: The phase of the reflection coefficient in the complex plane. Between a pole (represented by a cross) and a the corresponding zero (represented by a circle), a zone where the phase varies quickly can be found. The imaginary range is $[-\frac{k_0}{\pi}, \frac{k_0}{\pi}]$ and the real range is $[0, n_1 k_0]$ and $n_1 > n_2 > n_3$ so that a cut line is clearly visible.

4.3 Leaky slab modes

There is a pole of r in the complex plane when relation (10) is verified. This relation gives

$$|r_{23} r_{21}| = e^{2\beta_2'' h}. \quad (22)$$

Since $|r_{ij}| > 1$ if one of the media is left-handed, relation (10) can be verified only for $\beta_2'' > 0$, which happens, as seen in section 2.1, for $\alpha'' < 0$ and hence only in the part of the complex plane which is under the real axis. The fact that $r_{ij} > 1$ is thus the main reason why the poles of r are under the axis and hence that the leaky slab modes are backward.

Now we have to distinguish between two types of leaky modes.

When $\Re(\alpha) < \min(n_1, n_2, n_3)k$ then the mode leaks out both in the upper and lower right-handed media. We will call these modes S_2 modes. Since there is no cut line on the real axis for these modes, and since $r \neq 1$ on the real axis, the corresponding pole and zero are both on the first Riemann sheet and are not symmetrical with respect to the real axis. The pole is located under the real axis, so that the S_2 modes are always backward. Figure 4 shows what happens when the leaky mode is excited by an incoming beam.

Since $r^+(z)^* = r(z^*)$ then the right-handed slab of the same thickness presents a pole too, which corresponds to a forward leaky mode and can be excited for the same angle of incidence of the incoming beam. Figures 4 and 2 correspond in fact to poles which are symmetrical with respect to the real axis.

When the leaky slab modes are excited, both the reflected and the transmitted beam undergo a large negative lateral shift. Although they have not been interpreted, large negative shifts have been reported in a previous work[14]. Our work allows to interpret these shifts as the excitation of leaky S_2 modes, which have not been much studied even for the right-handed slab. The small positive shifts reported for the left-handed slab[14] are not due to the excitation of leaky modes.



Figure 4: Modulus of the field for a symmetrical slab with $\epsilon_1 = \epsilon_3 = 9$, $\mu_1 = \mu_3 = 1$, $\epsilon_2 = -1.5$, $\mu_2 = -1$. and $h = 1.3\lambda$ using a gaussian incident beam with a waist of 20λ and an incidence angle of $\theta = 22.78$. The pole corresponding to the leaky mode is located at $\alpha_p = (1.16823 - 0.01125i)k_0$

When $n_3 k_0 < \Re(\alpha) < \min(n_1, n_2)k_0$ then the mode is leaky only in the upper medium. The beam undergoes a total internal reflection when reflecting on medium 3. We will call such modes S_1 . For such modes, there is a cut line on the real axis, so that the upper part of the complex plane and the lower one are not continuous any more. We have demonstrated above that all the poles in the complex plane are located under the real axis. This

does not mean that there cannot be any pole on the second Riemann sheet above the real axis, which would then have an effect on the real axis. Indeed, $|r_{23}| < 1$ on the second Riemann sheet, so that the product $r_{23} r_{21}$ has a modulus which is greater than 1 on the real axis, but not necessarily in all the second Riemann sheet. The existence of a forward S_1 mode cannot, using this argument, be completely excluded. It is highly improbable though.

Let us stress that S_1 modes of the right-handed slab cannot be excited using the same angle of incidence as for the left-handed slab, as it has been explained above.

4.4 Leaky surface modes

The leaky surface modes (or leaky plasmons) corresponds to poles for which $\max(n_2, n_3) k_0 < \Re(\alpha) < n_1 k_0$, so that they cannot be confused with the leaky slab modes. These poles can be linked to the fact that coefficient r_{23} has a pole on the real axis in the vicinity. We will now assume that $\epsilon_3 = \mu_3 = 1$.

The *backward* leaky plasmon is expected to be found for $\mu_2 < 1$ and $\mu_2 \epsilon_2 > 1$. We have indeed found that there is a pole of r under the real axis. The corresponding zero is located on the other Riemann sheet (for both β_2 and β_3 this time). We have studied the location of this pole in the complex plane when the thickness of the slab varies, and more particularly for small thicknesses. The location of the pole is shown figure 5. Even when the thickness of the left-handed material is very small, the mode remains backward.

In the case when $\epsilon_2 \mu_2 < 1$ and $\mu_2 > 1$, the leaky surface mode is expected to be forward. There is indeed a zero on the first Riemann sheet, under the axis. The corresponding pole is on the other sheet. Its location is shown figure 6 for different thicknesses. The behaviour of the pole is very different from the case of the KR configuration with a metallic slab[18]. Figure 7 shows what happens when a forward leaky plasmon is excited. Let us underline that this kind of leaky mode is the only one which is forward for a left-handed slab.

Since $r^+(z)^* = r(z^*)$ the poles corresponding to leaky surface modes exist for the right-handed slab too, but they are located on the wrong side of the real axis so that they have no effect on it.

Finally, our study firmly assesses that the large negative displacements reported in [13] are due to the excitation of leaky slab modes and not of

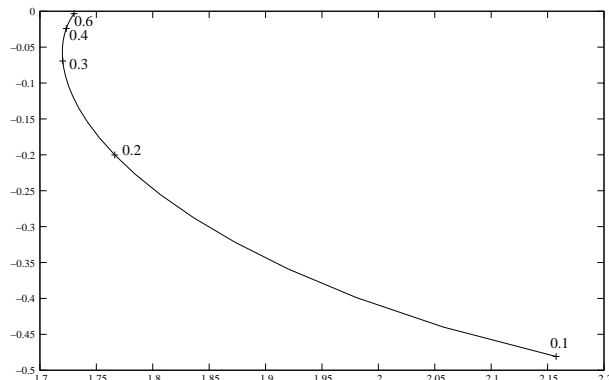


Figure 5: Location ($\frac{\alpha}{k_0}$) of the pole corresponding to the *backward* leaky surface plasmon for different thicknesses of the left-handed slab. The thickness is given as a fraction of the wavelength.

leaky surface plasmons. They actually present propagation constants such that $\Re(\alpha) < n_2 k$. In these conditions, it is not possible to excite any leaky surface plasmon.

5 Grounded left-handed slab

The case of the grounded left-handed slab can easily be treated with the tools that have been used above. The reflection coefficient of the grounded slab is given by (9) with $r_{23} = -1$ for the TE polarization and $r_{23} = 1$ for the TM polarization. This means that the relation $r^+(z)^* = r(z^*)$ holds. For such a grounded slab, no leaky surface mode can be excited, so that all the leaky modes are leaky slab modes. They can be found with $\Re(\alpha) < n_2 k$ so that there is no cut line problem for these modes. As a consequence, the right-handed grounded slab and the left-handed grounded slab present leaky modes for the same incidence angle.

For a leaky mode, the relation

$$|r_{21}| = e^{2i\beta_2'' h} \quad (23)$$

is verified, in TM or TE. For the same reasons as above, this can be verified for $\Im(\alpha) < 0$ only, so that all the leaky modes of the left-handed grounded slab are backward (the corresponding poles being under the real axis on the complex plane).

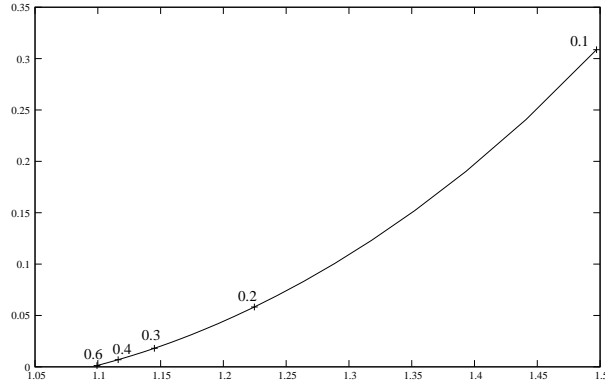


Figure 6: Location ($\frac{\alpha}{k_0}$) of the pole corresponding to the *forward* leaky surface plasmon for different thicknesses of the left-handed slab. The thickness is given as a fraction of the wavelength.



Figure 7: Modulus of the field for a left-handed slab with $\epsilon_1 = 9, \epsilon_3 = \mu_1 = \mu_3 = 1, \epsilon_2 = -0.5, \mu_2 = -1.5$ and $h = 0.6\lambda$ using a gaussian incident beam with a waist of 20λ and an incidence angle of $\theta = 21.496$. The pole corresponding to the leaky mode is located at $\alpha_p = (1.0993 + 0.001267i)k_0$.

6 Conclusion

We have studied the leaky modes of a left-handed slab, using a complex plane analysis of the slab's reflection coefficient. First, we have shown that there is a fundamental relation between the reflection coefficients of the left-handed slab and of the right-handed slab which is true for all the complex plane, except a part of the real axis.

The leaky modes can be classified into two main types : the leaky slab modes (usually supported by right-handed slabs) and the leaky surface modes (usually supported by metallic slabs). The left-handed slab can support both types and we have explained why they cannot be confused : the field is propagative in the left-handed medium for slab modes and evanescent in this medium for leaky surface modes, so that the leaky surface modes are excited

for higher incidence angle than all the leaky slab modes. More precisely, the slab modes poles are due to the form of the reflection coefficient for the whole structure, while the leaky modes are linked to a pole of the reflection coefficient of one interface only. This discussion on the nature of a resonance using complex plane analysis is useful even for other structures[19]

We have explained why the leaky modes of the slab are backward : they correspond to poles of the reflection coefficient which are located under the real axis. We have thoroughly studied why this is the case using complex plane analysis.

The leaky slab modes present different properties if they are leaky in both the upper and the lower medium (S_2 modes) or if they are totally reflected on the lower medium (S_1 modes). The S_2 modes of the left-handed slab can be excited for the same incidence angle as for a right-handed slab with opposite permittivity and permeability and the same thickness. This is not the case for the S_1 mode.

Then we have studied the behaviour of the poles corresponding to leaky surface plasmons when the thickness of the slab varies. We have found that the modes were backward (resp. forward) when the plasmons on the interface alone is expected to be backward (resp. forward), whatever the thickness.

Finally, we have extended our results to the grounded left-handed slab.

We hope that this paper will help to interpret the lateral shifts of a beam reflected by a left-handed slab (already reported) as the excitation of backward leaky modes.

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