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Franck Laloë, William J. Mullin

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Quantum non-local effects with Bose-Einstein condensates

F. Laloë and W.J. Mullin

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Abstract

We study the properties of double Bose-Einstein condensates in different spin states, represented quantum mechanically by double Fock states. We assume that individual measurements of the spins of the particles are performed in transverse directions (perpendicular to the spin quantization axis), giving access to the relative phase of the condensates. Initially, this phase is completely undefined, and the first measurements provide purely random results. But a fixed value of this phase rapidly emerges under the effect of the successive quantum measurements, giving rise to a quasi-classical situation where all spins have parallel transverse orientations. If the number of measurements reaches its maximum value (the number of particles), this classical phase disappears and quantum effects show up again. These effects can give rise to substantial violations of Bell type inequalities, comparable to those obtained with two spins in singlet state, even if the total number of particles involved is very large.

The notion of non-locality in quantum mechanics (QM) takes its roots in a chain of two theorems, the EPR (Einstein Podolsky Rosen) theorem [1] and its logical continuation, the Bell theorem. The EPR theorem starts from three assumptions (Einstein realism, locality, the predictions of quantum mechanics concerning some perfect correlations are correct) and proves that QM is incomplete: additional quantities, traditionally named λ , are necessary to complete the description of physical reality. The Bell theorem [2] then proves that, if the λ exist, the predictions of QM concerning other imperfect correlations can not always be correct. The ensemble of three assumptions: Einstein realism, locality, all predictions of QM are correct, is therefore self-contradictory; or, in other words, if Einstein realism is valid, then QM is non local. Bohr rejected Einstein realism because, in his view, the notion of reality could not correctly be applied to microscopic quantum systems, defined independently of the measurements apparatuses. Indeed, since EPR consider a system of two correlated microscopic particles, which can be “seen” only with the help of measurement apparatuses, the notion of their independent physical reality is open to discussion.

Nevertheless, it has been pointed out recently [5, 6] that the EPR theorem also applies to macroscopic systems, namely Bose-Einstein (BE) condensates in two different internal states. The λ introduced by EPR then corresponds to the relative phase of the condensates, in other words to macroscopic transverse spin orientations, that is physical quantities at a human scale; it then seems more difficult to deny the existence of their reality, even in the absence of measurement devices. This gives even more strength to the EPR argument and weakens Bohr’s refutation. On the other hand, no transposition of the Bell theorem to this case was obtained in [5].

This is precisely what we do in this article. We consider an ensemble of N_+ particles in a state characterized by an orbital state u and a spin state $+$, and N_- particles in the same state with spin orientation $-$, described quantum mechanically by a double Fock state:

$$|\Phi\rangle = \left[(a_{u,+})^\dagger \right]^{N_+} \left[(a_{u,-})^\dagger \right]^{N_-} |\text{vac.}\rangle \quad (1)$$

where $a_{u,+}$ and $a_{u,-}$ are the destruction operators associated with the two populated single particle states and $|\text{vac.}\rangle$ is the vacuum state. We introduce an appropriate sequence of transverse spin measurements that leads to quantum predictions that are in violation with the so called BCHSH [7, 8] Bell inequality. This is reminiscent of the work of Mermin [9], who finds exponential violations of local realist inequalities with N particle spin states, by taking an initial state that is maximally entangled and presumably difficult to produce. Here we assume that the initial state is the simple juxtaposition of two Fock states, possibly highly populated, and find violations of inequalities that are the same order of magnitude as with the usual singlet spin state (they may actually saturate the Cirel'son bound [10]), and also comparable to those observed in the experiments that have been performed so far.

We remain in this article at the level of discussing a thought experiment, without claiming to make a precise proposal for a feasible experiment. We just consider that, since double Fock states such as (1) exist in the quantum space of states, they can be reached physically; we also assume that a series of rapid transverse spin measurements can be performed and described by the usual postulate of measurement of QM. We do not worry about possible perturbations (for instance decoherence taking place between the measurements, breaking of the condensates into several phase independent domains, etc.). In other words, here we study the logical and physical content of QM for itself, without claiming that the experiment we consider is directly feasible in a laboratory. It is in this context that we will call the above double Fock state a "double BE condensate".

The operators associated with the local density and the local density of spins can be expressed as function of the two fields operators $\Psi_{\pm}(\mathbf{r})$ associated with the two internal states \pm as:

$$\begin{aligned} n(\mathbf{r}) &= \Psi_{+}^{\dagger}(\mathbf{r})\Psi_{+}(\mathbf{r}) + \Psi_{-}^{\dagger}(\mathbf{r})\Psi_{-}(\mathbf{r}) \\ \sigma_z(\mathbf{r}) &= \Psi_{+}^{\dagger}(\mathbf{r})\Psi_{+}(\mathbf{r}) - \Psi_{-}^{\dagger}(\mathbf{r})\Psi_{-}(\mathbf{r}) \\ \sigma_x(\mathbf{r}) &= \Psi_{+}^{\dagger}(\mathbf{r})\Psi_{-}(\mathbf{r}) + \Psi_{-}^{\dagger}(\mathbf{r})\Psi_{+}(\mathbf{r}) \\ \sigma_y(\mathbf{r}) &= i \left[\Psi_{-}^{\dagger}(\mathbf{r})\Psi_{+}(\mathbf{r}) - \Psi_{+}^{\dagger}(\mathbf{r})\Psi_{-}(\mathbf{r}) \right] \end{aligned} \quad (2)$$

while the spin component in the direction of plane xOy making an angle φ with Ox is:

$$\sigma_{\varphi}(\mathbf{r}) = e^{-i\varphi}\Psi_{\alpha}^{\dagger}(\mathbf{r})\Psi_{\beta}(\mathbf{r}) + e^{i\varphi}\Psi_{\beta}^{\dagger}(\mathbf{r})\Psi_{\alpha}(\mathbf{r}) \quad (3)$$

Consider now a measurement of this component performed at point \mathbf{r} and providing result $\eta = \pm 1$. The corresponding projector is:

$$P_{\eta=\pm 1}(\mathbf{r}, \varphi) = \frac{1}{2} [n(\mathbf{r}) + \eta \sigma_{\varphi}(\mathbf{r})] \quad (4)$$

and, because the measurements are supposed to be performed at different points (ensuring that these projectors all commute) the probability for a series of results $\eta_i \pm 1$ for a series of measurements at points \mathbf{r}_i of spins along directions φ_i can be written as:

$$\mathcal{P}(\eta_1, \eta_2, \dots, \eta_N) = \langle \Phi | P_{\eta_1}(\mathbf{r}_1, \varphi_1) \times P_{\eta_2}(\mathbf{r}_2, \varphi_2) \times \dots \times P_{\eta_N}(\mathbf{r}_N, \varphi_N) | \Phi \rangle \quad (5)$$

We now substitute (2) into (3), and then into (4) and (5), exactly as in the calculation of ref. [5], but with one difference: instead of assuming that the number of measurements is much smaller than N_{\pm} , we assume that it is equal to its maximum possible value $N = N_{+} + N_{-}$. In the product of projectors appearing in (5), because all \mathbf{r} 's are different, commutation of operators allows us to push all the field operators to the right, all their conjugate to the left; one can then easily see that each $\Psi_{\pm}(\mathbf{r})$ acting on $|\Phi\rangle$ can be replaced by $u(\mathbf{r}) \times a_{u,\pm}$, and similarly for the Hermitian conjugates. With the initial state (1), a non-zero result can be obtained only if exactly N_{+} operators $a_{u,+}$ appear in the term considered, and N_{-} operators $a_{u,-}$; a similar condition exists for the Hermitian conjugate operators. To express these

conditions, we introduce two additional variables. As in [5], the first variable λ ensures an equal number of creation and destruction operators in the internal states \pm through the mathematical identity:

$$\int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} e^{in\lambda} = \delta_{n,0} \quad (6)$$

The second variable Λ expresses in a similar way that the difference between the number of destruction operators in state $+$ and $-$ is exactly $N_+ - N_-$, through the integral:

$$\int_{-\pi}^{\pi} \frac{d\Lambda}{2\pi} e^{-in\Lambda} e^{i(N_+ - N_-)\Lambda} = \delta_{n, N_+ - N_-} \quad (7)$$

The introduction of the corresponding exponentials into the product of projectors (4) in (5) provides the expression (c.c. means complex conjugate):

$$\prod_{j=1}^N |u(\mathbf{r}_j)|^2 \frac{1}{2} \left[e^{i\Lambda} + e^{-i\Lambda} + \eta_j \left(e^{i(\lambda - \varphi_j + \Lambda)} + \text{c.c.} \right) \right] \quad (8)$$

where, after integration over λ and Λ , the only surviving terms are all associated with the same matrix element in state $|\Phi\rangle$ (that of the product of N_+ operators $a_{u,+}^\dagger$ and N_- operators $a_{u,+}^\dagger$ followed by the same sequence of destruction operators, providing the constant result $N_+!N_-!$). We can thus write the probability as:

$$\sim \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} \int_{-\pi}^{+\pi} \frac{d\Lambda}{2\pi} e^{i(N_+ - N_-)\Lambda} \prod_{j=1}^N \left\{ |u(\mathbf{r}_j)|^2 \frac{1}{2} \left[e^{i\Lambda} + e^{-i\Lambda} + \eta_j \left(e^{i(\lambda - \varphi_j + \Lambda)} + \text{c.c.} \right) \right] \right\} \quad (9)$$

or, by using Λ parity and changing one integration variable ($\lambda' = \lambda + \Lambda$), write $\mathcal{P}(\eta_1, \eta_2, \dots, \eta_N)$ as:

$$(2^N C)^{-1} \int_{-\pi}^{+\pi} \frac{d\Lambda}{2\pi} \cos[(N_+ - N_-)\Lambda] \int_{-\pi}^{+\pi} \frac{d\lambda'}{2\pi} \prod_{j=1}^N \left\{ \cos(\Lambda) + \eta_j \cos(\lambda' - \varphi_j) \right\} \quad (10)$$

The normalization coefficient C_N is easily obtained by writing that the sum of probabilities of all possible sequences of η 's is 1 (this step requires discussion; we come back to this point at the end of this article):

$$C_N = \int_{-\pi}^{+\pi} \frac{d\Lambda}{2\pi} \cos[(N_+ - N_-)\Lambda] [\cos(\Lambda)]^N \quad (11)$$

Finally, we just need to generalize this result to any number of measurements M that is smaller than N . We can always complete the sequence of M measurements by additional $N - M$ measurements, and take the sum over the possible results of these additional measurements. This provides:

$$\mathcal{P}(\eta_1, \eta_2, \dots, \eta_M) = (2^M C_N)^{-1} \int_{-\pi}^{+\pi} \frac{d\Lambda}{2\pi} \cos[(N_+ - N_-)\Lambda] [\cos \Lambda]^{N-M} \int_{-\pi}^{+\pi} \frac{d\lambda'}{2\pi} \prod_{j=1}^M \left\{ \cos(\Lambda) + \eta_j \cos(\lambda' - \varphi_j) \right\} \quad (12)$$

The Λ integral can be replaced by twice the integral between $-\pi/2$ and $\pi/2$, as can be seen by a change of Λ into $\pi - \Lambda$, which multiplies the result by $(-1)^{N_+ - N_- + N - M + M} = 1$.

This general formula includes different cases. If, for instance, if $M \ll N$, the large power of $\cos \Lambda$ in the first integral concentrates all its contribution around $\Lambda \simeq 0$, so that a good approximation is to assume

that Λ vanishes. We then recover the results of refs [5, 6], with a single integral over λ , corresponding to the relative phase of the condensates (Anderson phase). Initially, this phase is completely undetermined, and the first spin measurement provides a completely random result. But the phase rapidly emerges under the effect of a few measurements, and then remains constant; it takes a different value for each realization of the experiment, as if the experiment was revealing the pre-existing value of a classical quantity. Moreover, when $\cos \Lambda$ is replaced by 1, each term of the product over j remains positive (or zero), a situation where the Bell inequalities can be obtained, excluding any violation by the quantum results. On the other hand, when $N - M$ is small or even vanishes, $\cos \Lambda$ can take values that are smaller than 1, opening the possibility of Bell inequality violations. The additional variable Λ is, in a sense, the variable that controls the amount of quantum effects in the series of measurements.

We now discuss when these standard quantum predictions violate Bell inequalities resulting from local realism. We will need the value of the quantum average of the product or results, that is the sum $\mathcal{P}(\eta_1, \eta_2, \dots, \eta_M)$ over all possible values of the η 's, which according to (12) is given by:

$$E(\varphi_1, \varphi_2, \dots, \varphi_M) = (C_N)^{-1} \int_{-\pi}^{+\pi} \frac{d\Lambda}{2\pi} \cos[(N_+ - N_-)\Lambda] [\cos \Lambda]^{N-M} \int_{-\pi}^{+\pi} \frac{d\lambda'}{2\pi} \prod_{j=1}^M \cos(\lambda' - \varphi_j) \quad (13)$$

Consider a thought experiment where two condensates, each in a different spin state (two eigenstates of the spin component along the quantization axis Oz). We assume that the two condensates overlap in two remote regions of space \mathcal{A} and \mathcal{B} , where two experimentalists Alice and Bob measure the spins of the particles in arbitrary transverse directions (any direction perpendicular to Oz) at points of space where the orbital wave functions of the two condensates are equal. We assume that all measurements performed by Alice are made along the same direction φ_a , which plays here the usual role of the "setting" a , while all measurements performed by Bob are made with the same angle φ_b . To complete the analogy with the usual situation with two particles, we assume that Alice retains from all her measurements just their product A , while Bob retains only the product B of his results (these numbers are both ± 1 , the parities resulting from all the local measurements).

We now assume two possible orientations φ_a and φ'_a for Alice, two possible orientations φ_b and φ'_b for Bob. Within deterministic local realism, for each realization of the experiment, it is possible to define a two numbers A, A' , both equal to ± 1 , and associated with the two possible results that Alice will observe (more precisely product of results), depending of her choice of orientation; the same is obviously true for Bob, introducing B and B' . Within stochastic local realism [13], A and A' are the difference of probabilities associated to the difference of probabilities of Alice observing $+1$ and -1 , and these numbers have any value between $+1$ and -1 . In both cases, the following inequalities (BCHSH) are obeyed:

$$-2 \leq AB + AB' \pm (A'B - A'B') \leq 2 \quad (14)$$

In standard quantum mechanics, of course, "unperformed experiments have no results" [14], and several of the numbers appearing in (14) are undefined; only two of them are defined after the experiment has been performed with a given choice of the orientations. Consequently, while one can calculate from (13) the quantum average $\langle Q \rangle$ value of the sum of products of results appearing in the middle of (14), there is no special reason why $\langle Q \rangle$ should be limited between $+2$ and -2 . Situations where it is not the case will be considered as "quantum non-local situation".

There remains much flexibility on the possible experiments associated with (13): the choice of N_+ and N_- , the total number of measurements M , and how M it is shared between Alice (M_a measurements) and Bob (M_b measurements). Because the interesting quantum effects are due to the contribution of large values of Λ , we expect that the most interesting situations will occur when the cosines do not introduce their peaking effect around $\Lambda = 0$, that is when $N_+ = N_-$ (N is necessarily even) and the number of measurements M has its maximum value N . From now on, we assume that this is the case: then, for a given N , the only remaining choice is then how the number of measurements is shared between Alice and Bob.

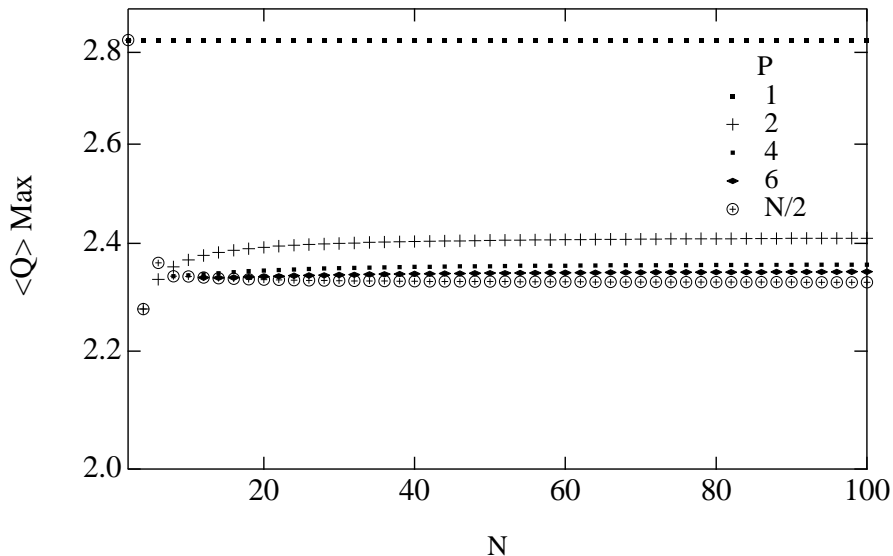


Figure 1: The maximum of the quantum average $\langle Q \rangle$ for Alice doing P experiments and Bob doing $N - P$, as a function of the total number of particles N . Local realist theories predict that this maximum can never exceed 2; large violations of this limit are obtained, even with macroscopic systems with $N \rightarrow \infty$. If $P = 1$, the violation saturates the Cirel'son limit for any value of N .

Assume first that $N_a = 1$ (Alice makes one measurement only) and therefore $N_b = N - 1$ (Bob makes all the others). Because N_+ and N_- are equal, the Λ integral in (13) disappears, and the λ integral only contains the product of $\cos(\lambda' - \varphi_a)$ by the $N - 1$ th power of $\cos(\lambda' - \varphi_b)$, which is straightforward and provides $\cos(\varphi_a - \varphi_b)$ times the normalization integral C_N . As a consequence, the quantum average associated to the product AB is merely equal to $\cos(\varphi_a - \varphi_b)$, exactly as the usual case of two spins in a singlet state. Then it is well-known that a violation of (14) occurs by a factor $\sqrt{2}$, which saturates the Cirel'son bound [10], when the angles form a “fan”¹ spaced by $\chi = \pi/4$. A similar calculation can easily be performed when Alice makes 2 measurements and Bob $N - 2$, and shows that the quantum average is now equal to $\frac{1}{2} \left[1 + \frac{1}{N-1} + \left(1 - \frac{1}{N-1} \right) \cos 2(\varphi_a - \varphi_b) \right]$, which is no longer independent of N . The maximum of the Bell quantity is, for this case no longer at the Cirel'son limit, but has value 2.28 at $N = 4$ and then *rises* to 2.41 as $N \rightarrow \infty$. It is possible to generate an integrated analytic expression for the generalization of the quantum average to any number P and $N - P$ of measurements by Alice and Bob, respectively. We find (with $\chi = \varphi_a - \varphi_b$)

$$E(\chi) = \frac{N!}{N!} \sum_{k=0}^{\{P/2\}} \frac{P!(N-2k)!}{k!(P-2k)!(N/2-k)!} \sin^{2k} \chi \cos^{P-2k} \chi \quad (15)$$

where $\{P/2\}$ is the integer part of $P/2$. The maximization of $\langle Q \rangle$ for this can be found using a numerical Mathematica routine. Results are shown for several values of P in Fig. 1. The angles maximizing the quantum Bell quantity always occur in the fan shape, although the basic angle changes with P and N .

In all but one of the curves in the figure, P is held fixed. All of these curves have a finite limit with increasing N . When N is large, for fixed P , all by the first term in Eq. (15) is negligible and

¹The term “fan” refers to the angles arranged as $\varphi_{ba} = \varphi_{a'b} = \varphi_{ab'}$ and $\varphi_{a'b'} = \chi$ where $\varphi_{ab} \equiv \varphi_a - \varphi_b$.

$E(\varphi_a, \varphi_b) \rightarrow \cos^P(\varphi_a - \varphi_b)$, which gives us limiting values of the Bell quantity ranging from 2.8 for $P = 1$ to 2.32 for P large. In the lowest curve we consider $P = N/2$, that is Alice and Bob each make half of the measurements. Even here there is a finite limit of 2.32 as the maximum violation of the Bell inequality in the limit of $N \rightarrow \infty$. Also in this case angle fan opening decreases as $1/\sqrt{N}$.

We can consider cases in which the number of measurements considered is $M < N$. These cases never violate the BCHSH inequality. That is, if Bob makes all his measurements, but ignores one or two of them (independently of the order of the measurements), when he correlates his results with Alice, he can not obtain violation. All measurements have to be taken into account to obtain violations. Furthermore if the number of particles in the two condensates do not satisfy $N_+ = N_- = N/2$ we find no inequality violation.

It is also possible to consider cases where we generalize the angles considered so that Carole makes measurements at φ_c and φ'_c and possibly David at φ_d and φ'_d . In every such case we find that maximization of $\langle Q \rangle$ reduces to the cases already studied where the new angles collapse to the previous angles $\varphi_a, \dots, \varphi'_b$. For the sake of simplicity, we have not yet discussed some general important issues that underlie our calculations. We now fill this gap, starting from a discussion of the so called “sample bias loophole” (also called “detection/efficiency loophole”). Expressions (2)-(5) are actually local densities (of spin, of probability, etc.) and must be integrated over position in a small volume $\Delta_{\mathbf{r}}$ surrounding \mathbf{r} to become real densities; this is done explicitly in the second of refs [5]. Each volume $\Delta_{\mathbf{r}}$ defines a small but finite “detection box”, all of them being spatially disconnected. It is then clear that ± 1 are not the only possible results of a spin measurement; actually, if the volumes are sufficiently small, most of the measurements find no particle at all in their volume. Therefore, strictly speaking, the normalization condition (11) is not correct; the particles have a much smaller chance to be detected, the average values are smaller and the violations of the inequalities do not occur. This is a well known difficulty, which already appears in the usual two-photon experiments [8], where most photons are missed by the detectors: local realism alone is not sufficient to put limits on the observed correlations, one has to assume in addition that the probability of detecting a particle at a particular point does not depend on the local setting of the apparatus (its orientation angle φ), a very plausible but not logically compulsory assumption.

Fortunately, if this loophole still raises a real experimental challenge, the difficulty can be completely resolved if we remain at the level of thought experiments. As noted by Bell [2], and Clauser and Shimony [8], it is sufficient to assume the presence of additional detectors, completely spin independent, which ensure the detection of one particle in each detector (see for instance [11] for a description of a thought experiment realizing this selection of initial conditions with veto detectors). Here we associate one of these preliminary detector to each “detection box”. In a given realization of the experiment, the chance that each preliminary detector will give the desired positive result may be very small, but this is not a problem: by repeating the experiment a sufficient number of times, one can preselect a sample of systems with appropriate initial conditions and then run the spin detectors. In other words, the role of the preliminary detectors is to select by the effect wave packet reduction on (1) a new state that contains only components where one particle exactly is in every detection box, which closes the “sample bias loophole”. The calculation of this state reduction can be performed explicitly and provides interesting results; in particular, it leads to the expression of a state for many distinguishable spins (spins contained in different boxes are indeed distinguishable) that violates the Bell inequalities. But, for lack of space, we postpone this discussion to another article [12].

A second important issue deals with the very existence of the local realist quantities A, B , etc.. For a pair of spins in the singlet state, the EPR local realist reasoning proves their existence from the perfect correlations observed when the two analyzers are parallel. Here, for two condensates, we have a slightly different situation: the local realist reasoning leads to the existence of a well defined phase between the condensates [5], not to individual properties associated with individual spins. Nevertheless, the emergence of this phase means that the probabilities associated with single particle measurements can be calculated locally as if each spin was polarized in a well defined direction; in other words, we replace the certainties of the usual EPR situation by an information on local probabilities. Fortunately, it is known [3, 8] that

local probabilistic information is sufficient to derive Bell inequalities (see also for instance [9] or appendix I of [13]). Here again, for lack of space, we can not discuss the question explicitly, but the absence of perfect correlations in the case we study does not prevent the logical construction of Bell inequalities.

In conclusion, we have shown that strong violations of local realist inequalities for large N particle systems, even if the N particle state is as simple as the simple juxtaposition of two Bose-Einstein condensates in different internal states. This does not mean that violations of the inequalities appear with the measurement of macroscopic variables. In all inequalities, we have assumed that the measured quantity is the product of many microscopic measurements, and not their sum or average, which would be clearly macroscopic. A sum of results is determined when most measurements have been performed, but a product of result remains sensitive to the last measurement, even after a long sequence of other measurements. Curiously, we have a case where the first few measurements are quantum, then become classical for many measurements, are become strongly quantum for the last one or two measurements, a sort of revival of quantum-ness of the system.

Laboratoire Kastler Brossel is “Unité de recherche associée du CNRS, de l’ENS, et de l’Université Pierre et Marie Curie”.

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