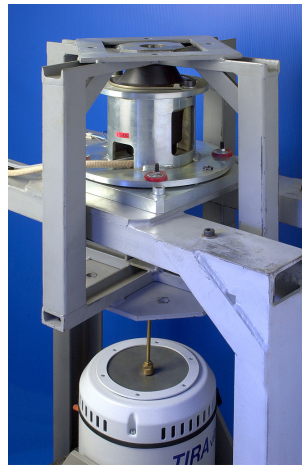


Adaptive rejection of unknown finite band disturbances

Theory and Applications

I.D. Landau, A. Constantinescu

Laboratoire d'Automatique de Grenoble(INPG/CNRS), France



Professor Tomizuka's 60th birthday Symposium

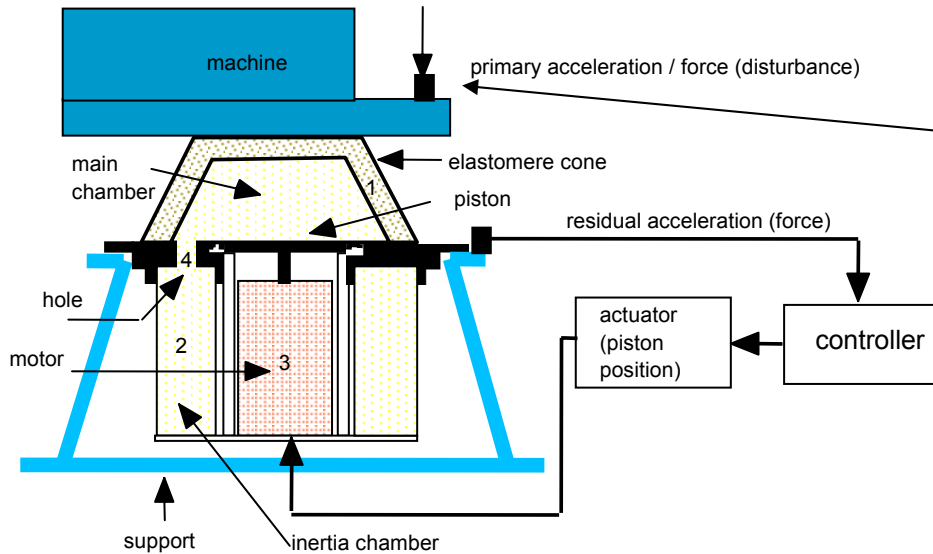
June 19, 2006, Berkeley

Outline

- Introduction
- Rejection of unknown narrow band disturbances in active vibration control. *Real-time results*
- Rejection of unknown finite band stationary disturbances. *Basic facts*
- Adaptive control solutions (indirect/direct)
- Further experimental results
(*comparison direct/indirect adaptive control*)
- How about rejection of unknown periodic disturbances ?
- Conclusions

Rejection of unknown narrow band disturbances in active vibration control

The Active Suspension System



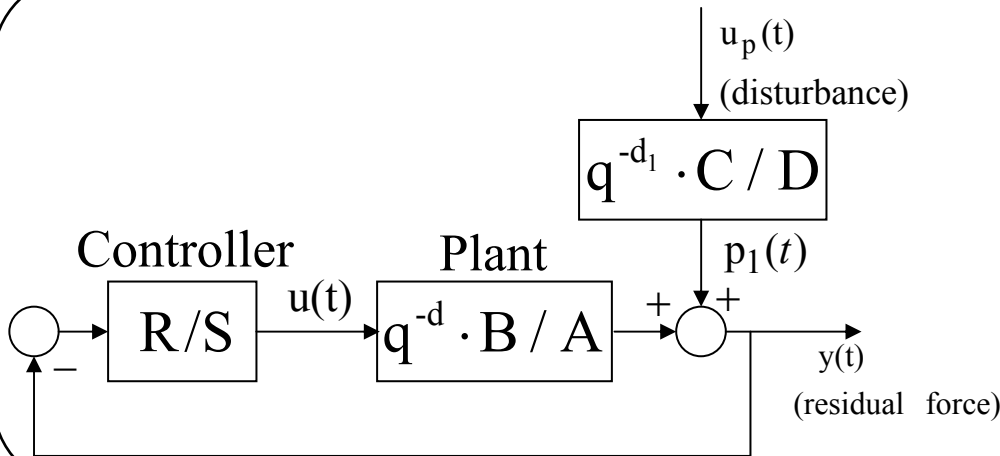
Objective:

- Reject the effect of unknown and variable narrow band disturbances
- Do not use an additional measurement

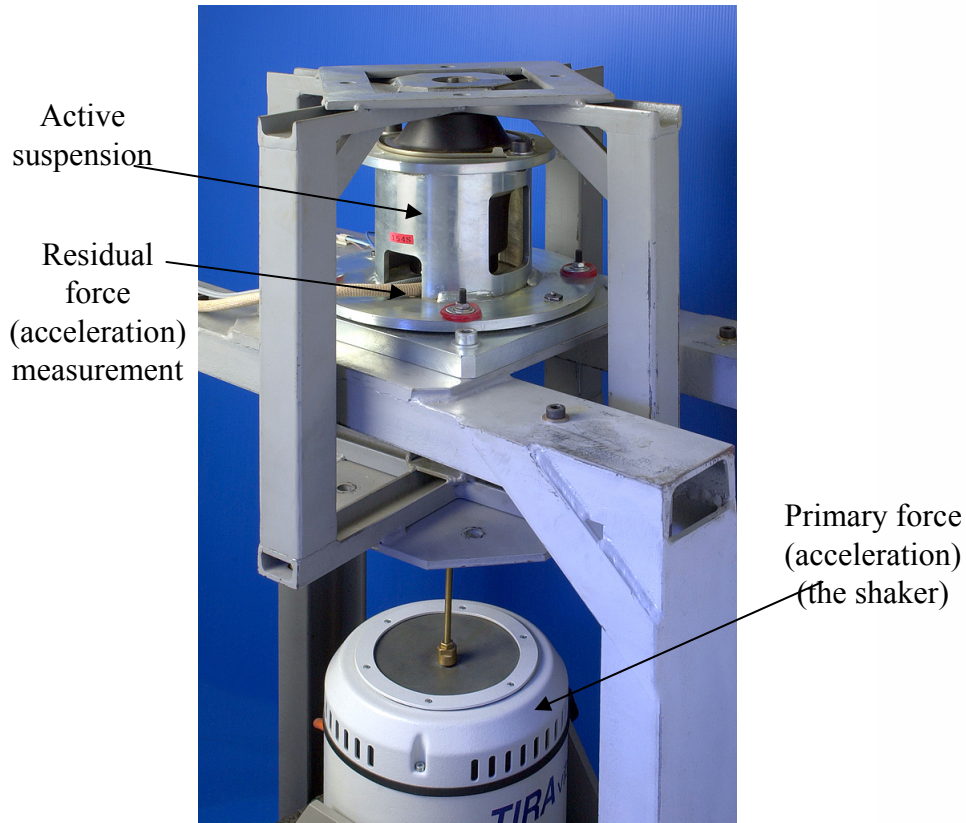
Two paths :

- Primary
- Secondary (double differentiator)

$$T_s = 1.25 \text{ ms}$$



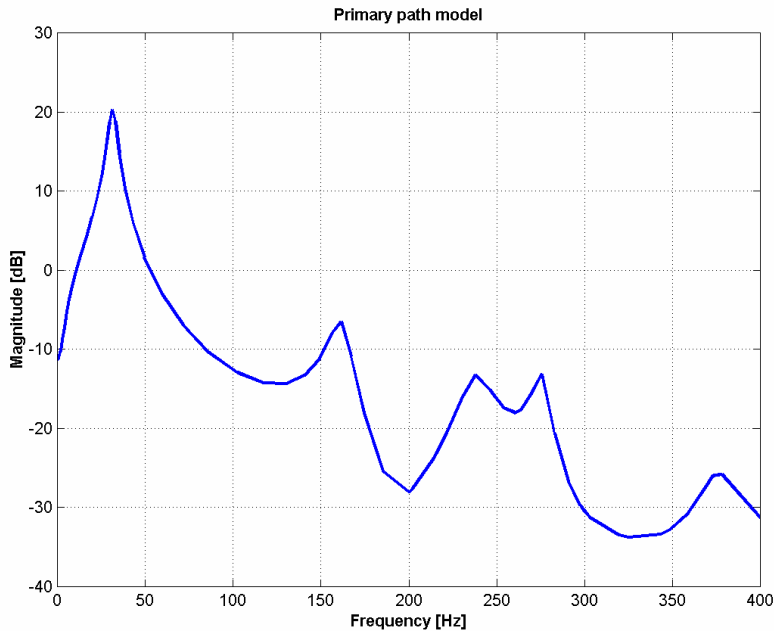
The Active Suspension



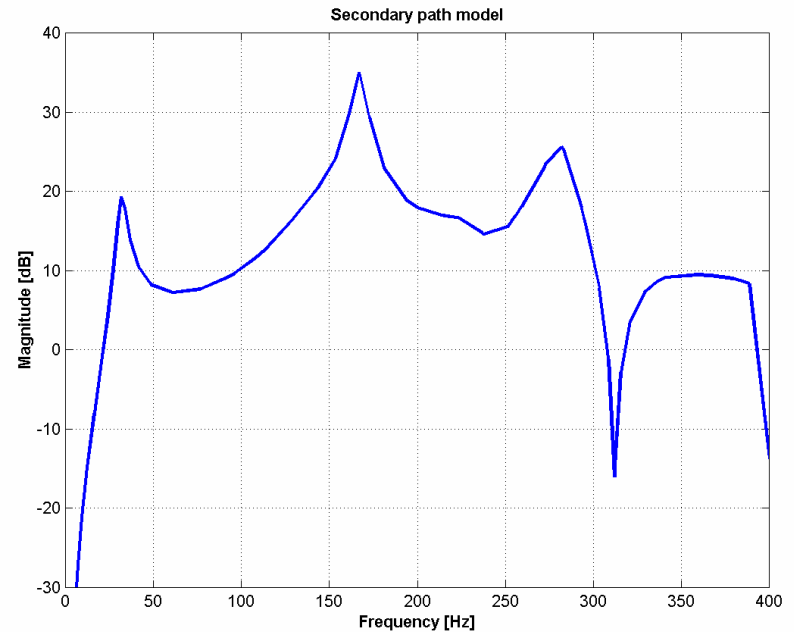
Active Suspension

Frequency Characteristics of the Identified Models

Primary path



Secondary path



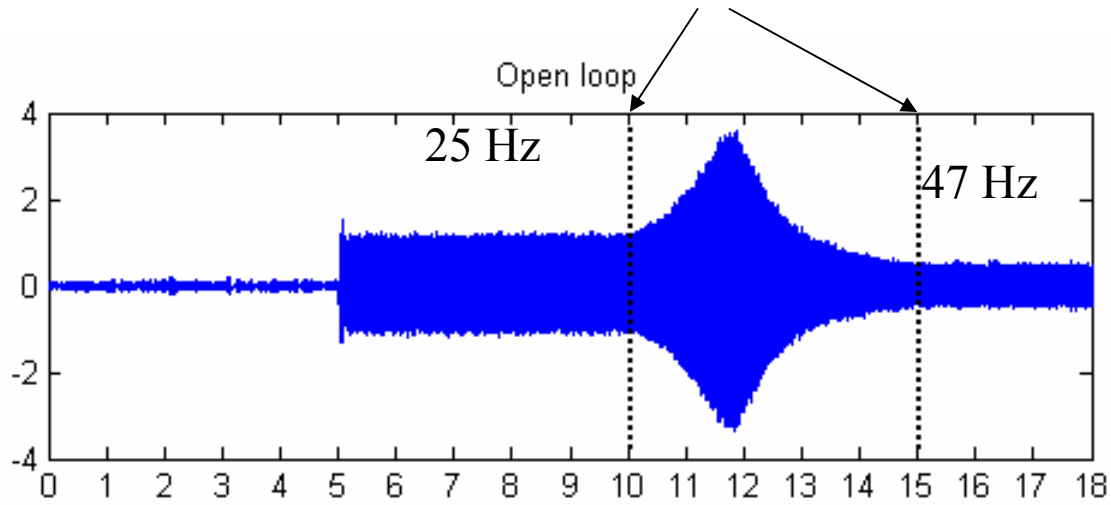
$$n_A = 14 ; n_B = 16 ; d = 0$$

Further details can be obtained from : http://iawww.epfl.ch/News/EJC_Benchmark/

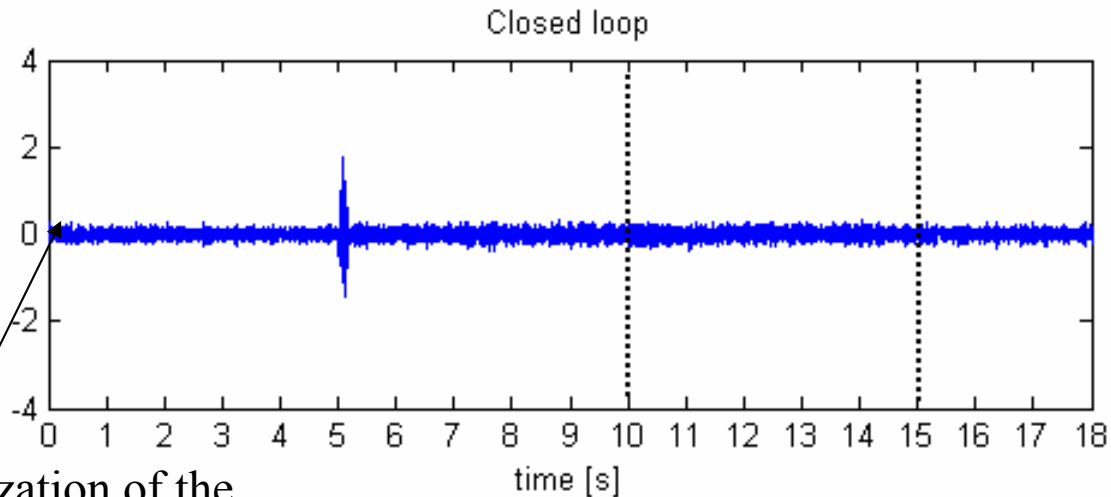
Direct Adaptive Control : disturbance rejection

Disturbance : Chirp

Open loop



Closed loop



Initialization of the adaptive controller

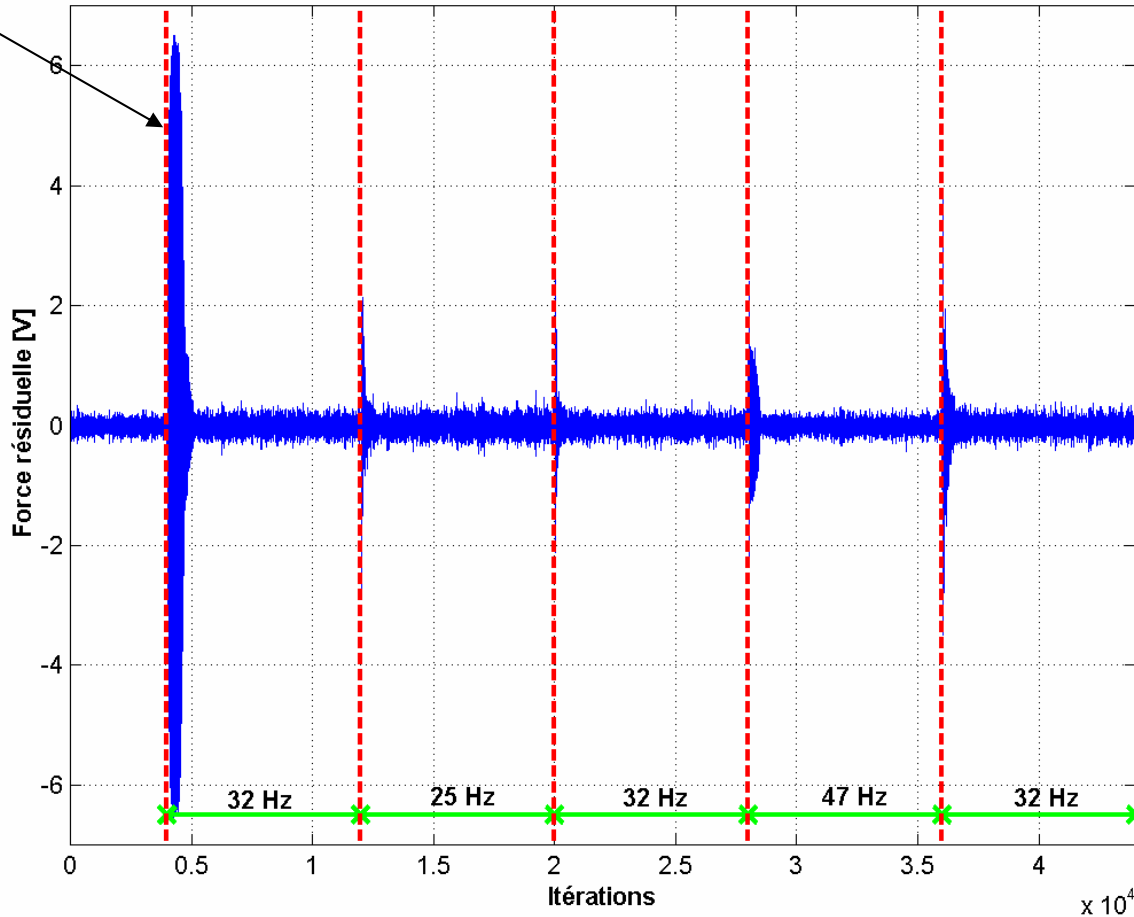
Time Domain Results

Adaptive Operation

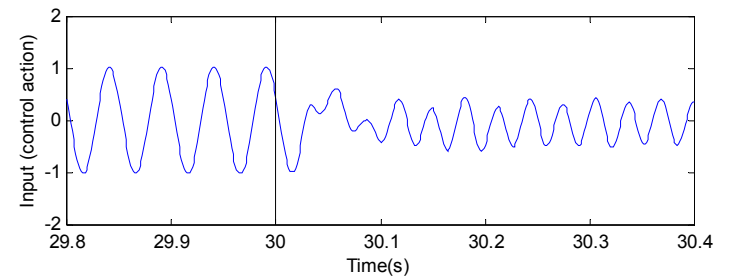
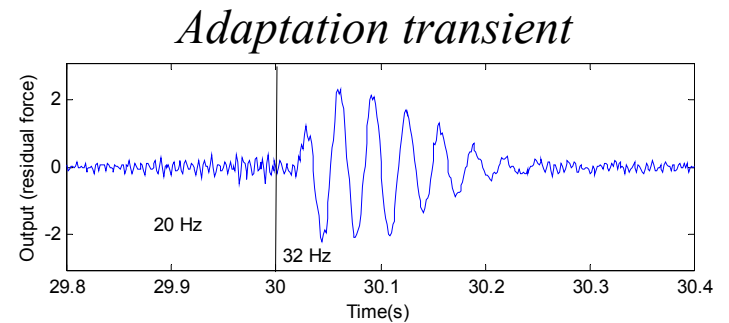
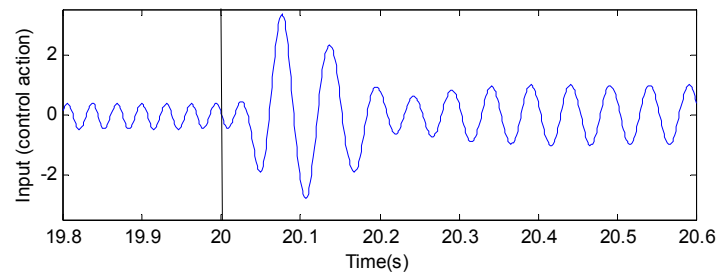
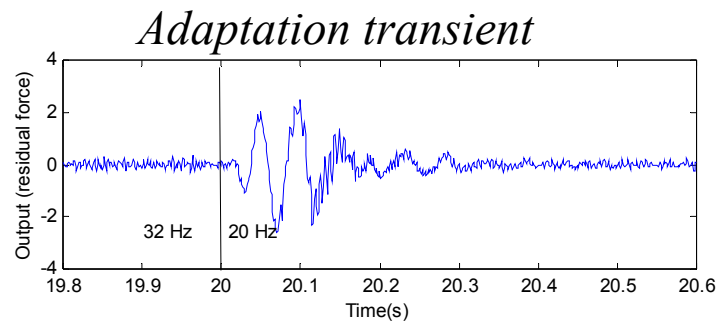
Simultaneous controller initialization and disturbance application

Direct adaptive control

Commande adaptative directe en adaptatif



Direct Adaptive Control



Rejection of unknown finite band disturbances

- **Assumption:** Plant model almost constant and known (obtained by system identification)
- **Problem:** Attenuation of unknown and/or variable stationary disturbances without using an additional measurement
- **Solution:** Adaptive feedback control
 - Estimate the model of the disturbance (indirect adaptive control)
 - Use the internal model principle
 - Use of the Youla parameterization (direct adaptive control)

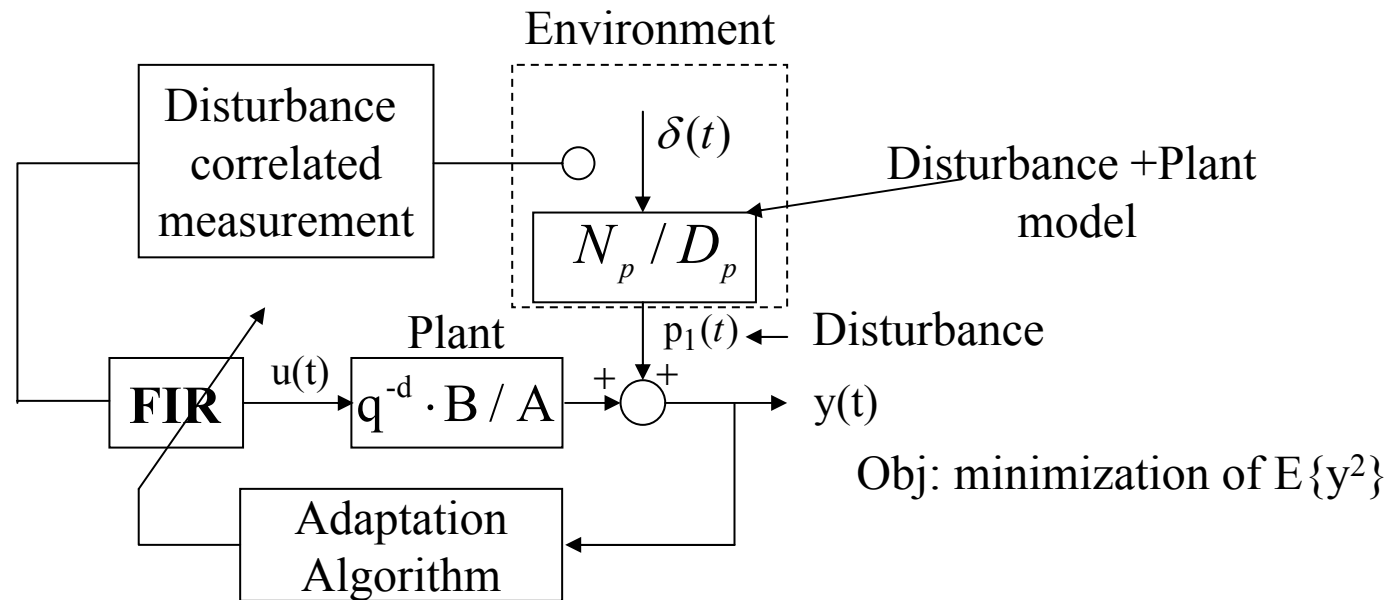
A robust control design should be considered assuming that the model of the disturbance is known

A class of applications: suppression of unknown vibration (active vibration control)

Attention: The area is “dominated “ by adaptive signal processing solutions (Widrow’s adaptive noise cancellation) which require an additional transducer

Remainder : Models of stationary disturbances have poles on the unit circle

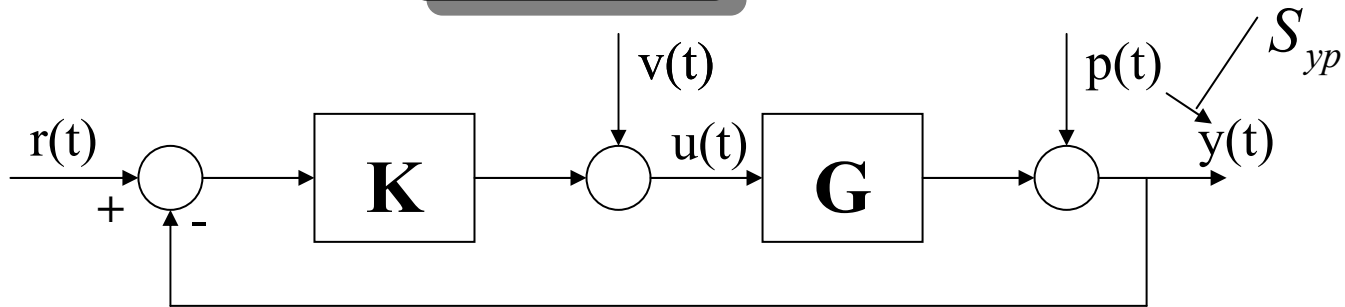
Unknown disturbance rejection – classical solution



Disadvantages:

- requires the use of an additional transducer
- difficult choice of the location of the transducer
- adaptation of many parameters

Notations



$$G(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$

$$K(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})} = \frac{R'(q^{-1})H_S(q^{-1})}{S'(q^{-1})H_R(q^{-1})}$$

H_R and H_S are pre-specified

Output Sensitivity function :

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S'(z^{-1})H_S(z^{-1})}{P(z^{-1})}$$

Closed loop poles :

$$P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$$

The gain of S_{yp} is zero at the frequencies where $S_{yp}(e^{j\omega})=0$ (perfect attenuation of a disturbance at this frequency)

Disturbance model

Deterministic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance}$$

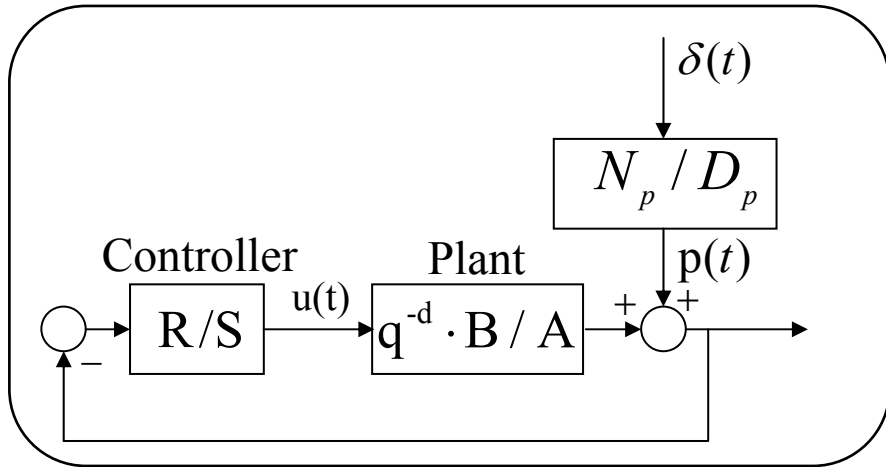
$D_p \rightarrow$ poles on the unit circle; $\delta(t) =$ Dirac

Stochastic framework

$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot e(t) : \text{stochastic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $e(t) =$ Gaussian white noise sequence $(0, \sigma)$

Closed loop system. Notations



$$p(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) : \text{deterministic disturbance}$$

$D_p \rightarrow$ poles on the unit circle; $\delta(t) =$ Dirac

Controller :

$$R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$$

$$S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$$

Internal model principle: $H_S(z^{-1}) = D_p(z^{-1})$

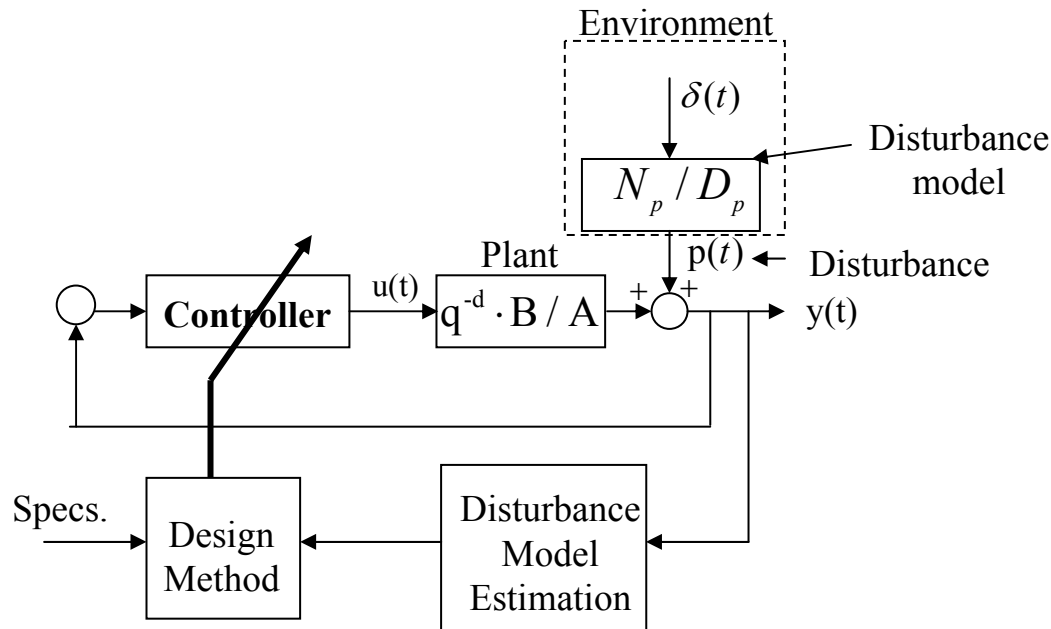
$$\text{Output : } y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) \rightarrow y(t) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})N_p(q^{-1})}{P(q^{-1})} \cdot \frac{1}{D_p(q^{-1})} \cdot \delta(t)$$

$$\text{CL poles : } P(q^{-1}) = A(q^{-1})S(q^{-1}) + z^{-d}B(q^{-1})R(q^{-1})$$

Indirect adaptive control

Two-steps methodology:

1. Estimation of the disturbance model, $D_p(q^{-1})$
2. Computation of the controller, considering $H_S(q^{-1}) = \hat{D}_p(q^{-1})$



It can be time consuming (if the plant model B/A is of large dimension)

Indirect adaptive control

Step I : Estimation of the disturbance model

ARMA identification (Recursive Extended Least Squares)

Step II: Computation of the controller

Solving Bezout equation (for S' and R)

$$H_S = \hat{D}_p$$

$$A\hat{D}_p S' + q^{-d} BR = P$$

$$S = D_p S'$$

Remark:

It is time consuming for large dimension of the plant model

Internal model principle and Q-parametrization)

Central contr: $[R_0(q^{-1}), S_0(q^{-1})]$.

CL poles: $P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t)$

Control: $S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t) - Q(q^{-1}) w(t)$,

where $w(t) = A(q^{-1}) y(t) - q^{-d}B(q^{-1}) u(t)$.

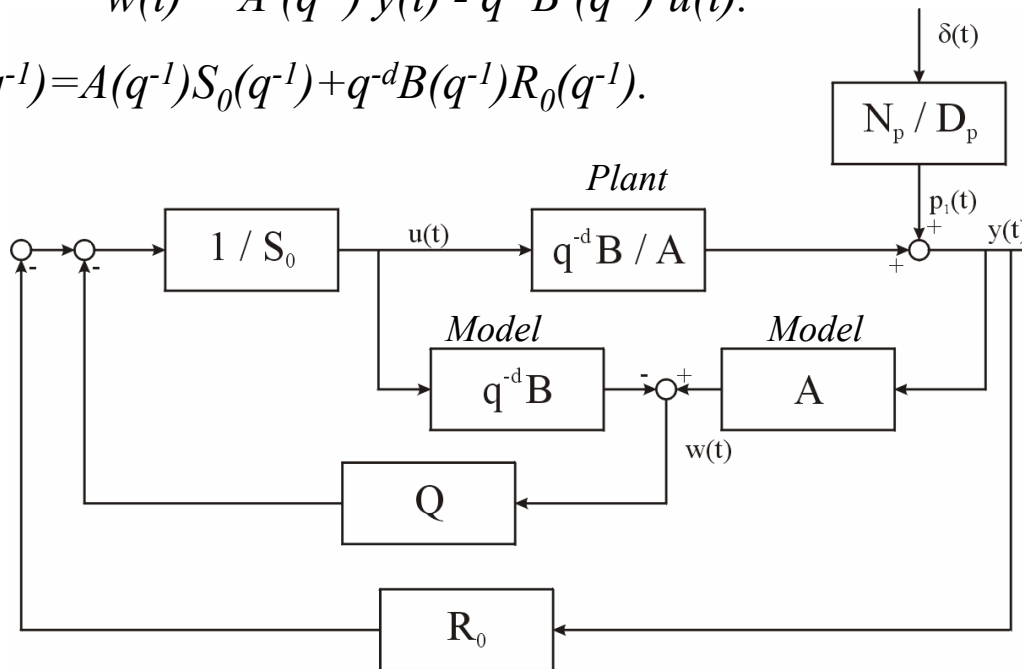
CL poles: $P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d}B(q^{-1})R_0(q^{-1})$.

Q-parameterization :

$R(z^{-1}) = R_0(q^{-1}) + A(q^{-1})Q(q^{-1});$

$S(q^{-1}) = S_0(z^{-1}) - q^{-d}B(q^{-1})Q(q^{-1}).$

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + \dots + q_{n_Q} q^{-n_Q}$$



Model = Plant
 $w = Ap$

Internal model principle and Q- parameterization

Central contr: $[R_0(q^{-1}), S_0(q^{-1})]$.

CL Poles: $P(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d}B(q^{-1})R_0(q^{-1})$.

Control: $S_0(q^{-1}) u(t) = -R_0(q^{-1}) y(t)$

Q-parameterization :

$R(z^{-1}) = R_0(q^{-1}) + A(q^{-1})Q(q^{-1});$

$S(q^{-1}) = S_0(z^{-1}) - q^{-d}B(q^{-1})Q(q^{-1}).$

Closed Loop Poles remain unchanged

Internal model assignment on Q

$$S = S_0 - q^{-d} BQ = MD_p \quad \longrightarrow \quad \text{Solve: } \begin{matrix} MD_p + q^{-d} BQ = S_0 \\ \uparrow \quad \quad \quad \downarrow \\ ? \quad \quad \quad ? \end{matrix}$$

$Q(q^{-1})$ computed such as $[S(q^{-1})]$ contains the internal model of the disturbance

Will lead also to an « indirect adaptive control solution »

BUT:

Q can be used to “directly” tune the internal model without changing the closed loop poles(see next)

Direct Adaptive Control (unknown D_p)

(Based on an idea of Y. Z. Tsytkin)

Hypothesis: Identified (known) plant model (A, B, d) .

Goal: minimize $y(t)$ (according to a certain criterion).

Consider $p_1(t) = \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t)$: deterministic disturbance.

$$y(t) = \frac{A(q^{-1}) \left[S_0(q^{-1}) - q^{-d} B(q^{-1}) Q(q^{-1}) \right]}{P(q^{-1})} \cdot \frac{N_p(q^{-1})}{D_p(q^{-1})} \cdot \delta(t) = \frac{\left[S_0(q^{-1}) - q^{-d} B(q^{-1}) Q(q^{-1}) \right]}{P(q^{-1})} w(t)$$

Define:

$$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d} B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t).$$

Let $\hat{Q}(t, q^{-1})$ be an estimated value of $Q(q^{-1})$

We can show that

Leads to a direct adaptive control \rightarrow

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1, q^{-1})] \cdot \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + v(t+1)$$

$(v(t+1) = \text{disturbance term} \rightarrow 0)$

The Algorithm

A priori adaptation error : $\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t);$

A posteriori adaptation error : $\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t),$

where

$$\hat{\theta}^T(t) = [\hat{q}_0(t) \quad \hat{q}_1(t)]; \quad \phi^T(t) = [w_2(t) \quad w_2(t-1)], \quad (\text{for } n_Q = 1)$$

and

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1);$$

$$w_2(t) = \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t);$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d} B^*(q^{-1}) \cdot u(t);$$

$$B(q^{-1}) \cdot u(t+1) = B^*(q^{-1}) \cdot u(t).$$

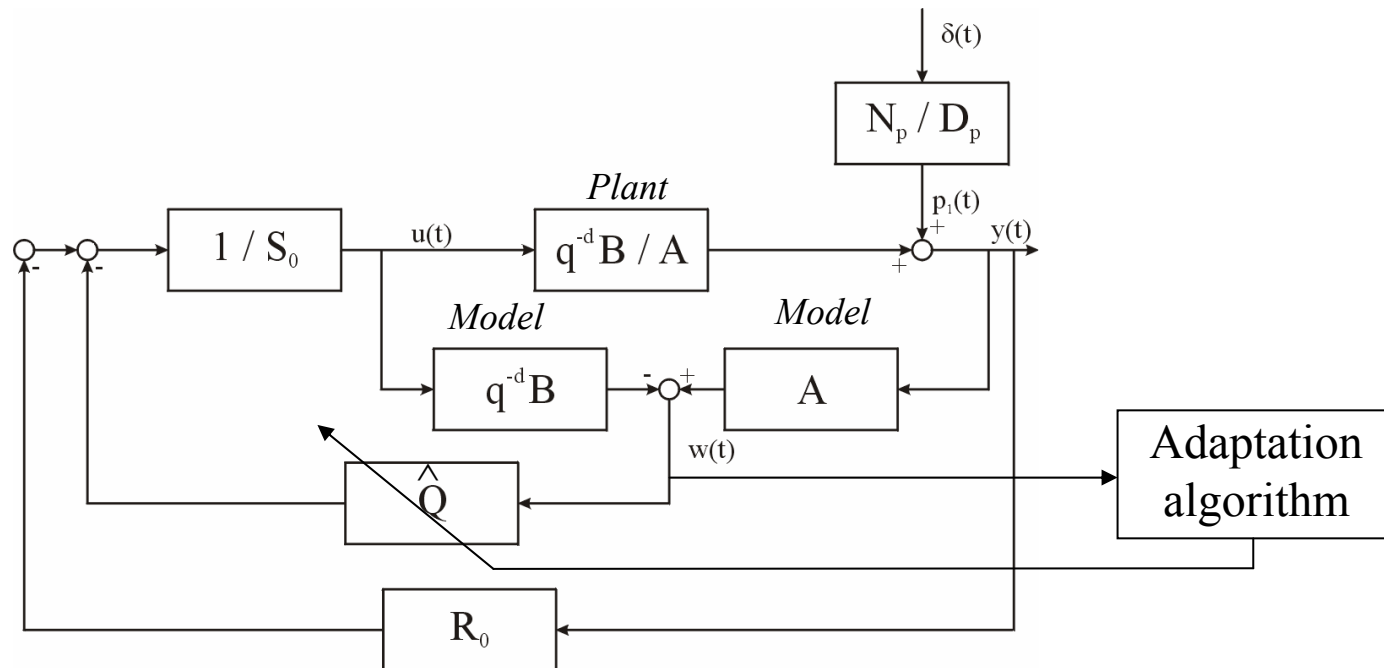
$$n_Q = n_{D_P} - 1$$

Parameter adaptation algorithm:

$$\begin{cases} \hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)\phi(t)\varepsilon^0(t+1); \\ F^{-1}(t+1) = \lambda_1(t)F^{-1}(t) + \lambda_2(t)\phi(t)\phi^T(t). \end{cases}$$

Various choices possible for λ_1 and λ_2

Direct adaptive rejection of unknown disturbances



- *The order of the Q polynomial depends upon the order of the disturbance model denominator (D_p) and not upon the complexity of the plant model*
- *Less parameters to estimate than for the identification of the disturbance model*
- *Operation in “self-tuning” mode (constant unknown disturbance) or “adaptive” mode (time varying unknown disturbance)*

Further experimental results on the active suspension
Comparison between direct/indirect adaptive control

Real-time results – Active Suspension (continuation)

Narrow band disturbances = variable frequency sinusoid $\Rightarrow n_Q = 1$
Frequency range: 25 ÷ 47 Hz

Evaluation of the two algorithms in real-time

Nominal controller $[R_0(q^{-1}), S_0(q^{-1})]$: $n_{R0}=14$, $n_{S0}=16$

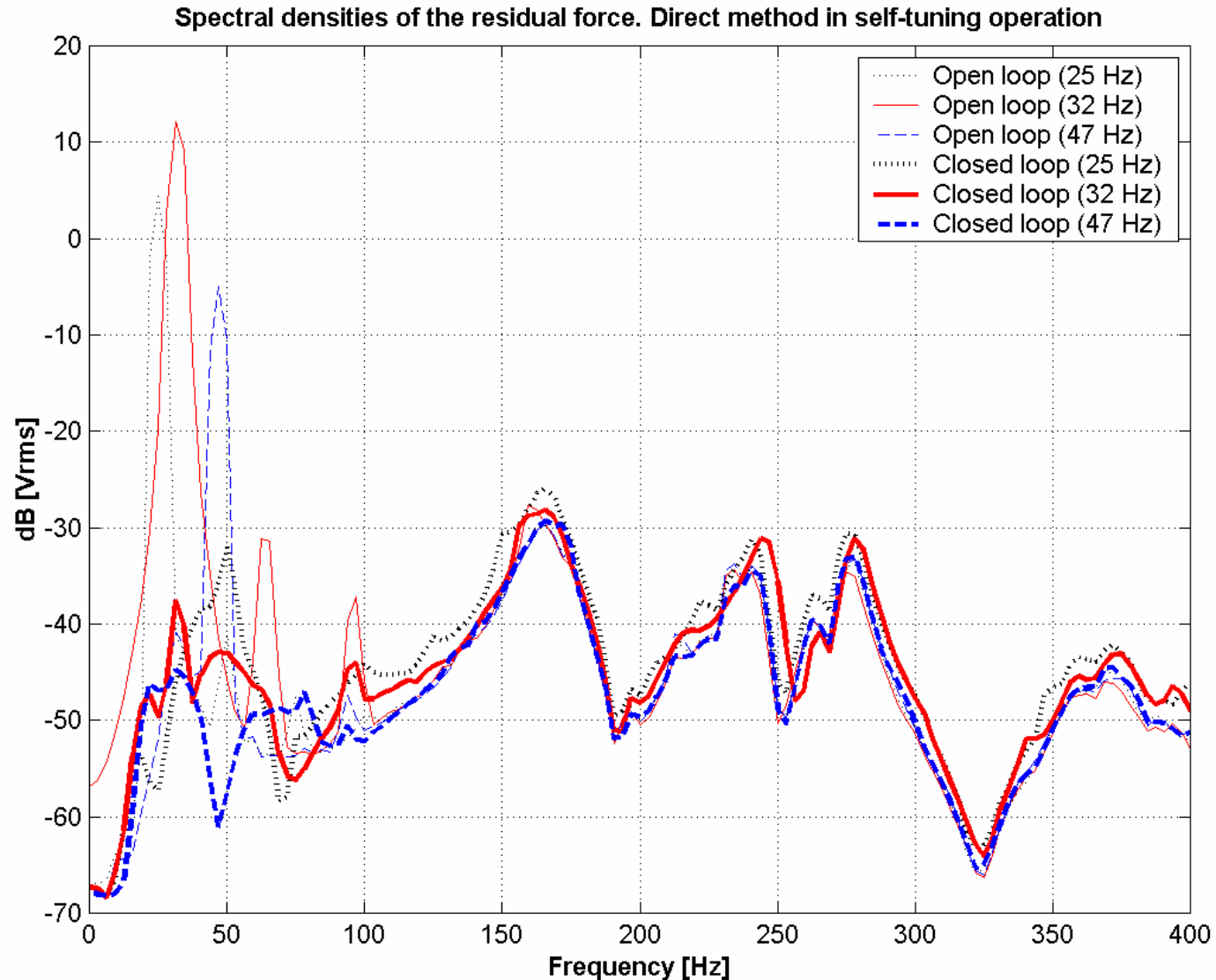
Implementation protocol 1: Self-tuning

- The algorithm stops when it converges and the controller is applied.
- It restarts when the variance of the residual force is bigger than a given threshold.
- As long as the variance is not bigger than the threshold, the controller is constant.

Implementation protocol 2: Adaptive

- The adaptation algorithm is continuously operating
- The controller is updated at each sample

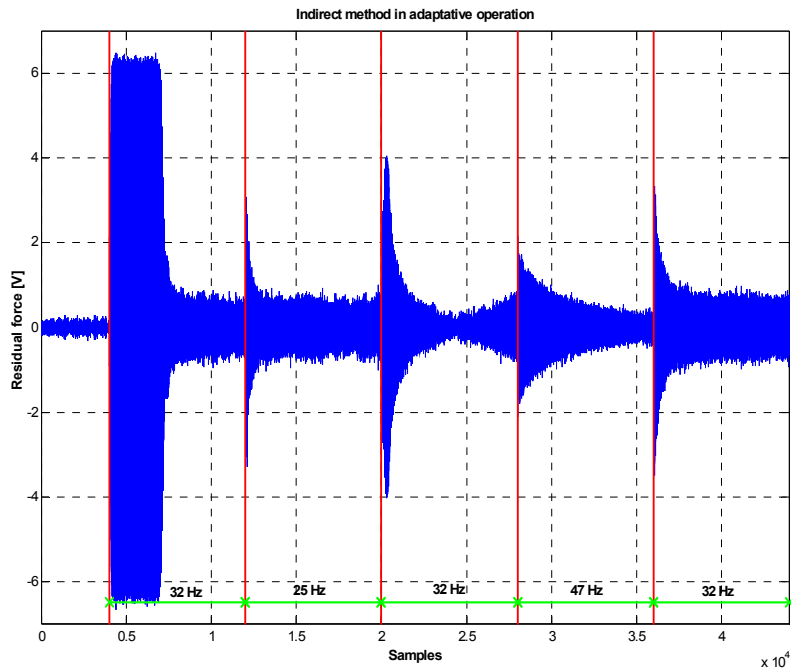
Frequency domain results – direct adaptive method



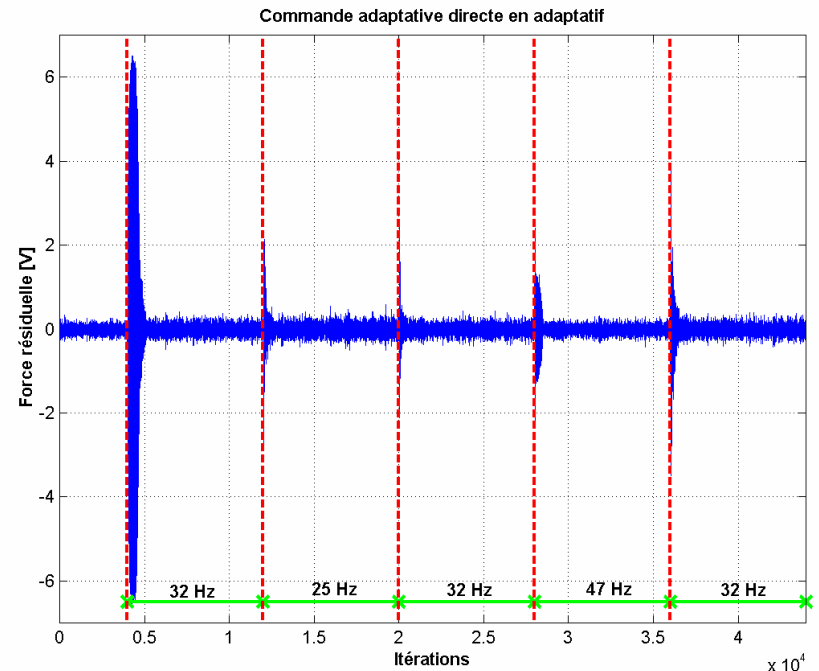
Time Domain Results

Adaptive Operation

Indirect adaptive method



Direct adaptive method

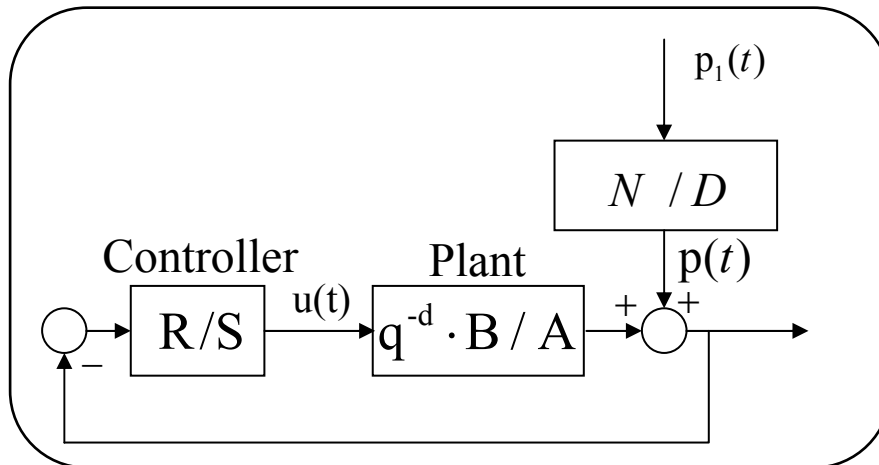


Direct adaptive control leads to a much simpler implementation and better performance than indirect adaptive control

Beyond rejection of narrow band disturbances
Simulation results

Types of Disturbances

- One sinusoid + dc disturbance ($n_Q = 2$)
- Two sinusoids ($n_Q = 3$)
- n sinusoids ($n_Q = 2n - 1$)
- Fundamental + (n_H)harmonics ($n_Q = 2(1 + n_H) - 1$)



Plant:
$$\frac{1}{1 + 0.01s}$$

$T_s = 0.004s$ ($f_s = 250\text{Hz}$)

$(N/D) = 1$ *Disturbance on the output*

$(N/D) = q^d B/A$ *Disturbance on the input*

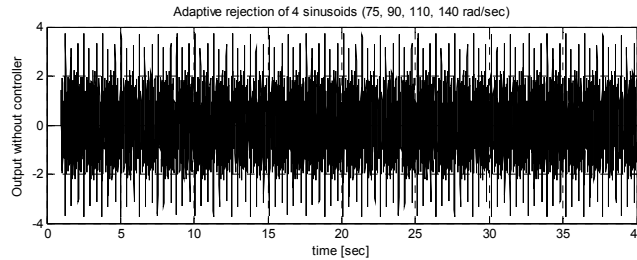
Adaptation algorithm : variable forgetting factor + constant trace

Time domain results - *Adaptive operation*

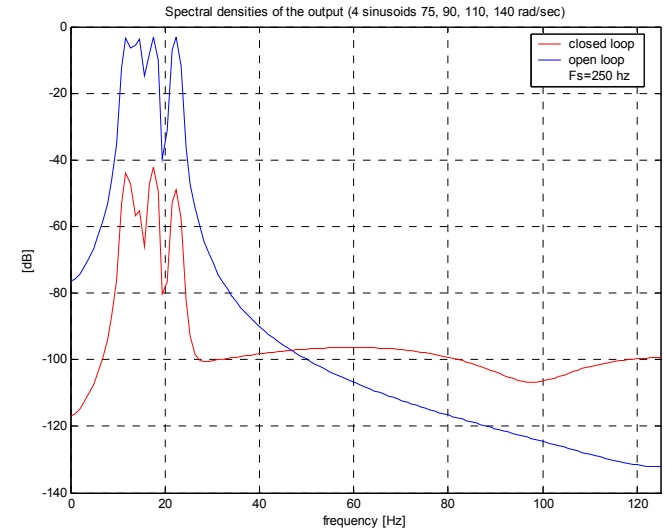
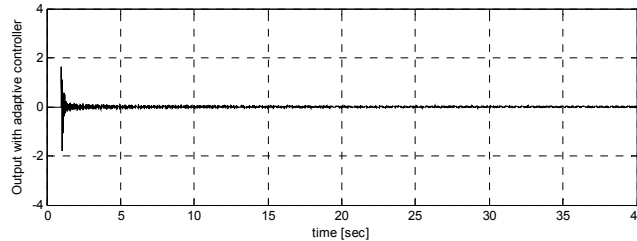
Disturbance : 4 sinusoids on the output

75, 90, 110, 140 rad/s

Open loop

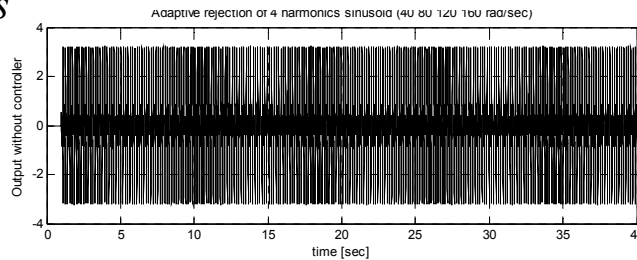


Closed loop

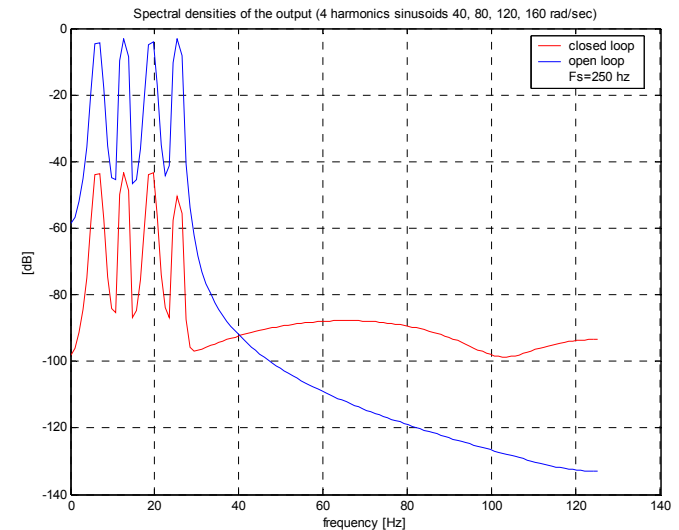
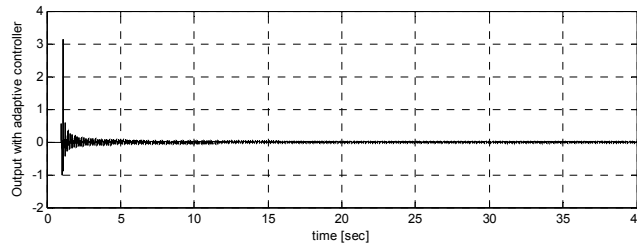


40, 80, 120, 160 rad/s

Open loop



Closed loop



Time domain

Spectral density

Rejection of unknown periodic disturbances

Typical model : $\frac{1}{1 - q^{-N}}$ *N may be very large and may change !*

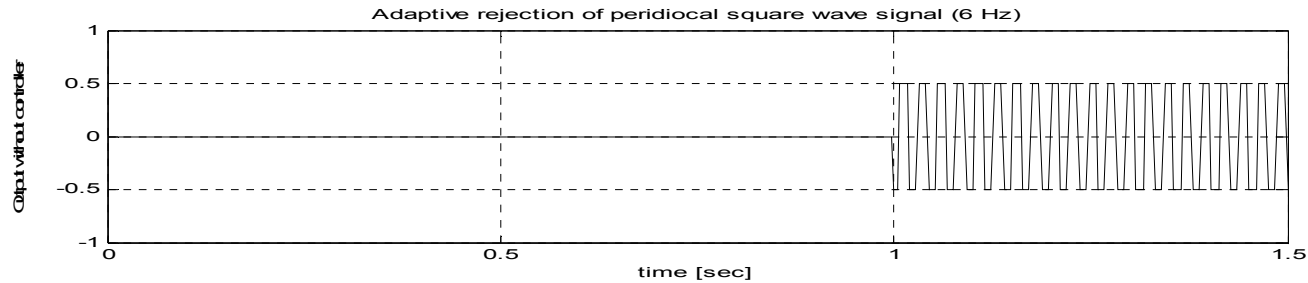
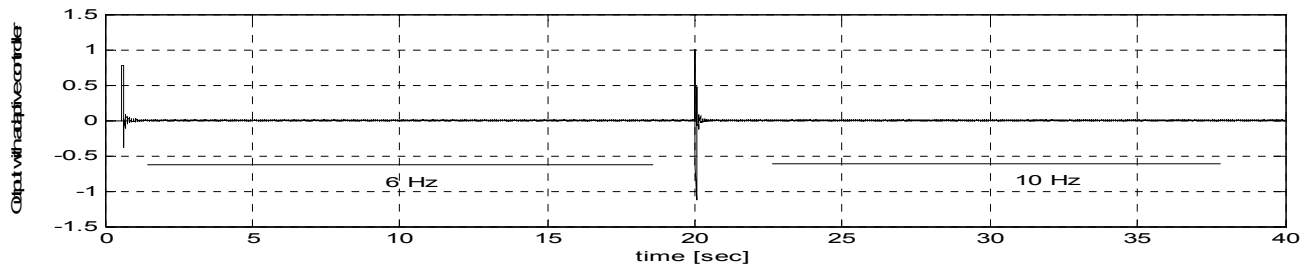
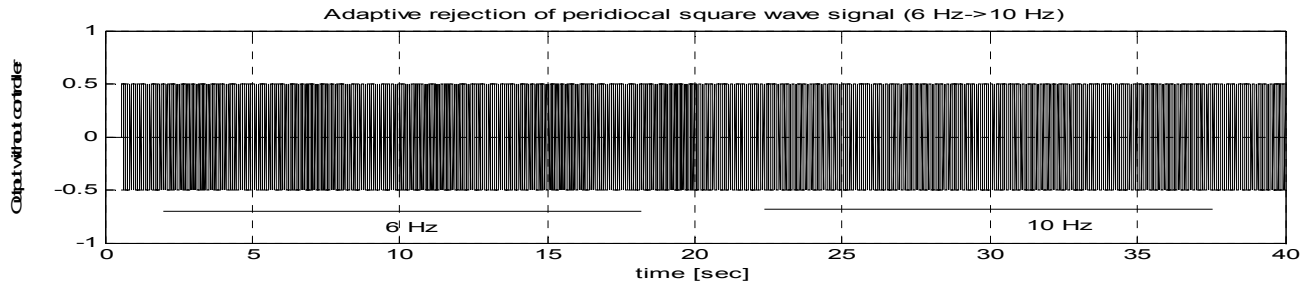
- Periodic disturbances have a Fourier's series development
- One can get good results if we attenuate the fundamental and a number of harmonics.

Next slide will show the results for the case of a step periodic disturbance.

Only the fundamental and the first three harmonics are considered ($n_Q=7$)

Rejection of unknown periodic disturbances

Square wave disturbance on the output: 6 Hz \rightarrow 10 Hz



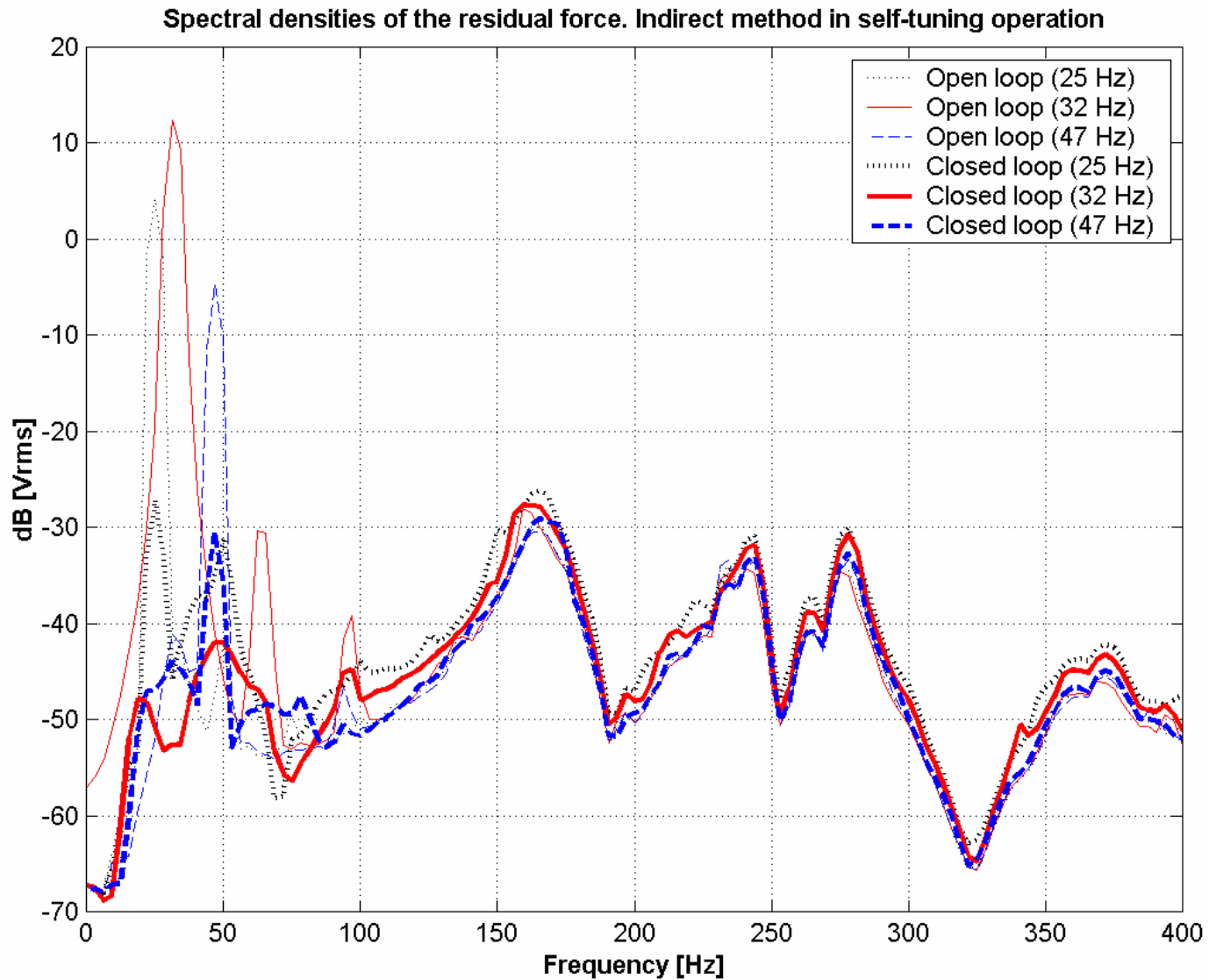
Conclusions

- Using internal model principle, adaptive feedback control solutions can be provided for the rejection of unknown disturbances
- Both direct and indirect solutions can be provided
- Two modes of operation can be used : self-tuning and adaptive
- Direct adaptive control is the simplest to implement**
- Direct adaptive control offers better performance**
- The methodology has been extensively tested on an active suspension system

Open problems:

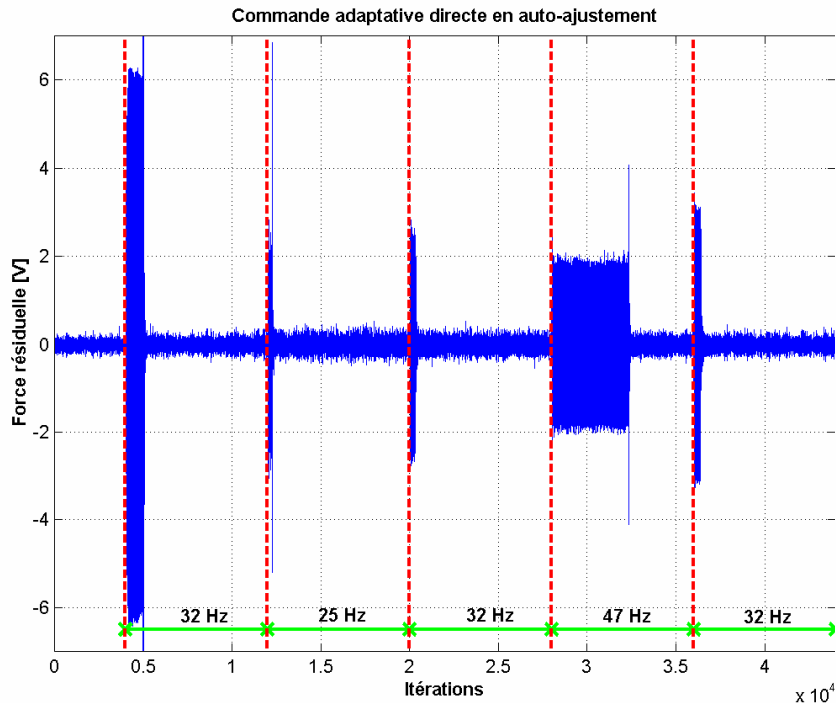
- Robustness with respect to plant model error (theory)
- Connections with adaptive repetitive control

Frequency domain results – indirect adaptive method

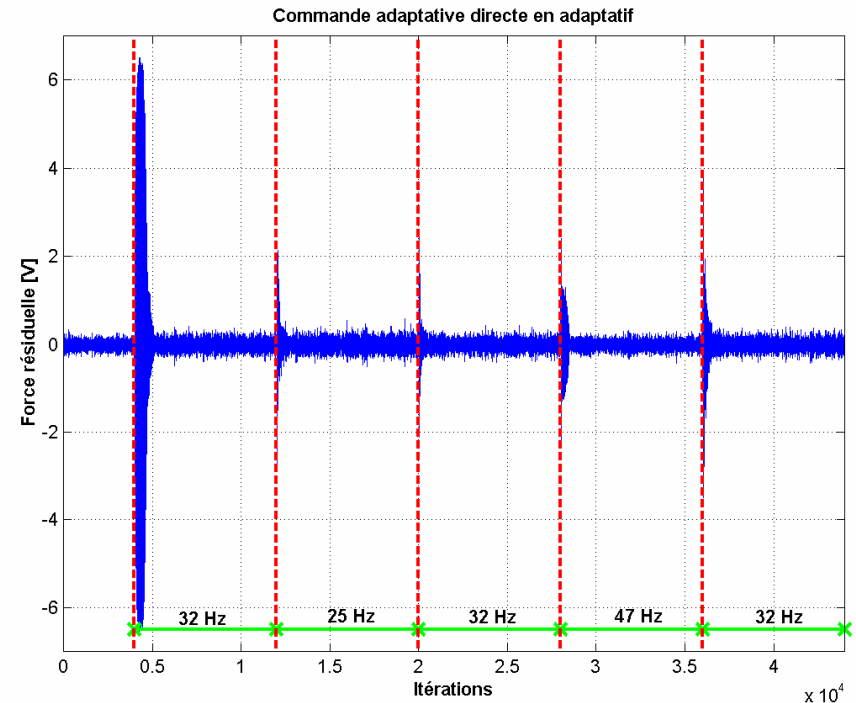


Direct Adaptive Control

Self-tuning Mode



Adaptive Mode



- *Direct adaptive control in adaptive mode operation gives better results than direct adaptive control in self-tuning mode*
- *Direct adaptive control leads to a much simpler implementation and better performance than indirect adaptive control*

Direct Adaptive Control (unknown D_p)

$$\varepsilon(t) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t) - \frac{q^{-d} B(q^{-1})}{P(q^{-1})} Q(q^{-1}) \cdot w(t). \quad (*)$$

We need to express $\varepsilon(t)$ as:

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1, q^{-1})] \Psi(t)$$

Using: $MD_p + q^{-d} BQ = S_0$, (*) becomes *Vanishing term*

$$\varepsilon(t+1) = [Q(q^{-1}) - \hat{Q}(t+1, q^{-1})] \cdot \frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) + \frac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})} p(t+1)$$

Instead of solving $MD_p + q^{-d} BQ = S_0$ search recursively for :

$$\hat{Q}(t, q^{-1})^* = \arg \min_{\hat{Q}} \sum_{i=0}^t \varepsilon^2[i, \hat{Q}]$$

Details:

$$\frac{M(q^{-1})D_p(q^{-1})}{P(q^{-1})} p(t+1) = \frac{M(q^{-1})N_p(q^{-1})}{P(q^{-1})} \delta(t+1)$$

$$\frac{q^{-d} B^*(q^{-1})}{P(q^{-1})} \cdot w(t) = \frac{q^{-d} B(q^{-1})}{P(q^{-1})} \cdot w(t+1)$$