Attraction between electrons in a metallic dot capacitively coupled to a superconducting island

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(Dated: May 3, 2007)

We consider the hybrid set-up formed by a metallic dot, capacitively coupled to a superconducting island S connected to a bulk superconductor by a Josephson junction. Charge fluctuations in S act as a dynamical gate and screen the electronic repulsion in the metallic dot, yielding instead a net attraction. As the offset charge of the metallic dot is increased, positive steps ($+2e$) skipping charge numbers appear, followed by negative ones ($-e$) signaling the occurrence of a negative differential capacitance. A circuit set-up with a detection scheme is proposed, as well as potential applications in nanoelectronics.

PACS numbers: 73.23.Hk, 74.78.Na

The electronic Coulomb repulsion is central to the physics of nanostructures, as the source of single-electron charging effects. At low enough temperatures, when a small-capacitance grain is weakly coupled to a metallic reservoir, the average number of charges, n_N, in the grain increases one by one as the gate voltage V_{gN} is continuously varied, leading to a Coulomb blockade staircase [1]. Plateaus in the charging curve indicate an insulating-like regime, where the charge is stable, with zero differential capacitance $C_{diff} = e\frac{dn_N}{dV_{gN}}$. On the contrary, steps signal a metallic-like regime where two successive charge states are nearly degenerate, thus C_{diff} is very large. Such features have also been studied in superconducting islands where charging steps involve electron pairs if the superconducting gap in the island is larger than the charging energy [2], and single charges in the opposite case [3]. The study of charging patterns has been extended to double islands, coupled by a capacitive tunnel junction. The islands can be both normal metals [4, 5], or superconducting [6].

Hybrid metallic structures made of a superconductor and a normal metal have been little explored in the Coulomb blockade regime. The present Letter addresses the system made of a normal metallic dot (N), coupled to a superconducting island (S) by a large capacitance. The S island is connected to a superconducting reservoir by a Josephson junction (JJ), and acts as a Cooper pair box experiencing pair number fluctuations. The N dot is connected to a normal reservoir (Fig. 1). Here we assume that electron tunneling between S and N is negligible; therefore no proximity effect occurs in the N dot. We instead focus on the charging properties of the N grain, as its gate voltage is varied, under the influence of the S island which plays the role of an auxiliary (and, as we will see, nonlinear) gate. The main result of this Letter is that the Coulomb repulsion in N can be overscreened by the neighboring pair fluctuations in S, and an effective *local attraction* appears between electrons added into N. As a corollary, certain charge states are "skipped" as the N gate voltage is varied. Moreover, the charging curve becomes non-monotonous, displaying positive steps ($+2e$) followed by *negative* steps ($-e$). The latter signal a *negative* differential capacitance C_{diff} in the N dot. The attractive interaction is reminiscent of the so-called "negative-U" center in solids [8]. A related effect has been proposed by Averin and Bruder for providing a controlled coupling between two superconducting charge qubits [9]. Notice that if N instead was coupled to two reservoirs with a current flowing through it, our set-up would be similar to that of a Cooper pair box coupled to a single-electron transistor (SET). The latter has been studied in great detail as a read-out device for a superconducting (charge) qubit embodied in the S island [7]. In this case, contrary to ours, the coupling between N and S is chosen to be very small in order to minimize the decoherence due to backaction of the normal part of the device onto the superconducting one.

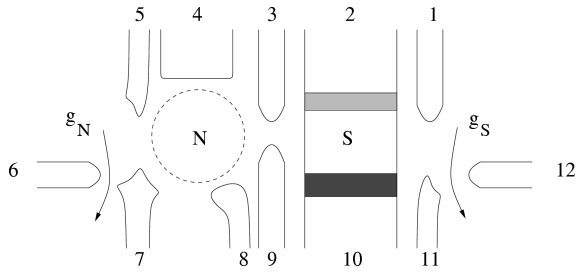

FIG. 1: Schematic view of a normal grain (2DEG) coupled to a Cooper pair box composed of a Josephson junction connecting superconducting reservoir 2 and island S gated by 10. For strong capacitive coupling (controlled by 3, 9), S imposes an attractive interaction among electrons tunneling between the normal island (N) and its reservoir (defined by 7, 8). Detection is made by sweeping the gate voltage (4) and measuring the island voltages using quantum point contacts for both N (5,6,7) and S (1,11,12).

The JJ connecting the S island to the reservoir has a Josephson energy E_J and capacitance C_J, and a gate imposes a charge offset $Q_S = 2eN_S = C_{gS}V_{gS}$, with $C_{gS} \ll C_J$. Symmetrically, the N island is connected to a normal reservoir by a tunnel junction, with one-electron tunneling rate Γ and capacitance C_N, and experiences a gate offset $Q_N = eN_N = C_{gN}V_{gN}$, with $C_{gN} \ll C_N$. Most importantly, the islands N and S are coupled together by a large capacitance $C_0 > C_N, C_J$. We take the gap in S to be larger than the

charging energy, so that only even charge number states $2\overline{n}_S$ occur in S, while all charge states n_N are a priori possible in N ($\overline{n}_S$ is the number of Cooper pairs in S). A low temperature allows us to neglect quasiparticle tunneling in S. Defining $C_{\Sigma S} = C_J + C_0 + C_{gS}$ and $C_{\Sigma N} = C_N + C_0 + C_{gN}$, $b = \frac{C_{\Sigma N}}{C_{\Sigma S}}$ and $r = \frac{C_0}{\sqrt{C_{\Sigma N} C_{\Sigma S}}}$, the total charging energy of the NS system can be written in a standard way as

$$E_C = E_{CN}[(n_N - N_N)^2 + 4b(\overline{n}_S - N_S)^2 + 4r\sqrt{b}(n_N - N_N)(\overline{n}_S - N_S)] \quad (1)$$

with $E_{CN} = \frac{e^2}{2C_{\Sigma N}(1-r^2)}$. Recall that N_N, N_S are continuous control parameters. Notice that the asymmetry parameter b and the coupling parameter $r < 1$ are not independent, as $r < \min(b, \frac{1}{\sqrt{b}})$. From Eq. (1), one can plot the charge stability diagram of the isolated NS system in the (N_N, N_S) plane. First, for a value N_S imposing an integer number of pairs in S, say $N_S = 1$, the charging number n_N increases monotonously with N_N. Next, consider a case where $\overline{n}_S$ fluctuates, for instance $N_S = 0.5$. For small r, as shown in Fig. 2(a), n_N is again a monotonous function of N_N: the sequence of charge states $(n_N, \overline{n}_S)$ as N_N increases reads $(0,\bar{0}), (0,\bar{1}), (1,\bar{0}), (1,\bar{1}), (2,\bar{0}), (2,\bar{1}), \ldots$ (notice the oscillation of $\overline{n}_S$). The result is very different if r is large. In Fig. 2(b), for $N_S = 0.5$, n_N increases with N_N but in a non-monotonous way, the charge state sequence being $(1,\bar{0}), (0,\bar{1}), (2,\bar{0}), (1,\bar{1}), (3,\bar{0}), (2,\bar{1})$, etc. The corresponding Coulomb staircases are plotted in inset.

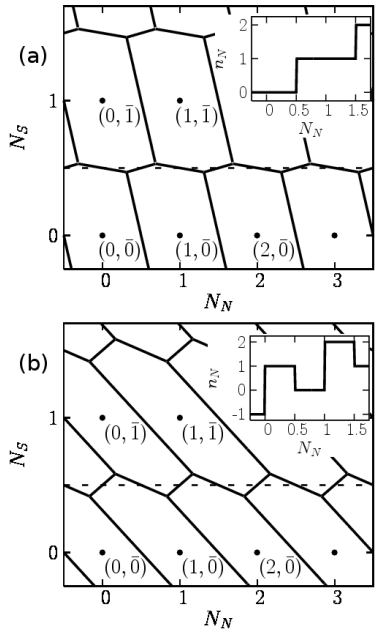

FIG. 2: (a) Charge stability (or honeycomb) diagram for $r = 0.2$, $b = 1$. The inset shows the charging curve for N. (b) Same for $r = 0.8$. The Coulomb staircase (inset) exhibits charge skipping effects.

One sees that the transition from $(n_N, \bar{1})$ to $(n_N + 2, \bar{0})$ at $N_N = n_N + 1$ "skips" the charge state $n_N + 1$ in the grain. This signals an attractive potential ("negative-U" [10]) in N which overcomes the Coulomb repulsion. After increasing by two units, n_N decreases by one unit, yielding a negative differential capacitance (NDCA) $C_{diff} = C_{gN}\frac{dn_N}{dN_N}$ at half-integer values of N_N. In addition to the already known "insulating" and "metallic" behaviors, this phenomenon signals an overscreening of the charge repulsion in N due to neighbouring pair fluctuations in S. Strikingly, the total number of steps, positive or negative, is doubled with respect to the usual case. Both "charge skipping" and NDCA effects occur above the dotted line indicated in the inset in Fig. 3 displaying a (b, r) diagram. This line can be determined by a simple magnetic analogy: defining charge pseudospins $\sigma_S^z = 2(\overline{n}_S - N_S)$ and $\sigma_N^z = n_N - N_N$ for $N_S = 0.5$ and N_N integer, E_C can be rewritten as $E_C = E_{CN}\left[(\sigma_N^z)^2 + 2r\sqrt{b}\,\sigma_N^z\sigma_S^z + b\right]$, thus with an anisotropy and an antiferromagnetic coupling between charge pseudospins . If $r\sqrt{b} > \frac{1}{2}$, the "antiferromagnetic" solution $\sigma_N^z = \pm 1$, $\sigma_S^z = \mp 1$ is favored, e.g. skipping the state $n_N = N_N$ ($\sigma_N^z = 0$).

To analyze the occurrence of an effective attraction (equivalently charge skipping) in the open NS system, let us now write its full Hamiltonian:

$$H = E_C + \sum_{k\sigma} \varepsilon_k\, c^\dagger_{kR,\sigma}\, c_{kR,\sigma} + \sum_{q\sigma} \varepsilon_q\, c^\dagger_{qN,\sigma}\, c_{qN,\sigma} + \sum_{kq\sigma} T_{k,q}\, c^\dagger_{kR,\sigma}\, c_{qN,\sigma} + H.c. - \frac{E_J}{2}\left(|\overline{n}_S + 1\rangle\langle\overline{n}_S| + H.c.\right), \quad (2)$$

where k (q) denotes electron states in the normal reservoir R (grain N), and the Coulomb interaction E_C is given by Eq. (1). The total charge in N is $n_N = \sum_{q\sigma} c^\dagger_{qN,\sigma}\, c_{qN,\sigma}$. Assuming constant densities of states in N and R, the single-electron transition rate from R to N is given in the golden rule approximation by $\Gamma^{(+1)} = \frac{\delta E_C^{(+1)}}{e^2 R_N}\left[\exp\left(\delta E_C^{(+1)}/k_B T\right) - 1\right]^{-1}$.

Considering first the case of small E_J, we perform a T-matrix calculation of the transition rates from $(0,\bar{1})$ to $(2,\bar{0})$ (close to $N_N = 1$) and from $(2,\bar{0})$ to $(1,\bar{1})$ (close to $N_N = 1.5$). For the first transition, we take into account three configuration paths involving higher-energy states: $(0,\bar{1}) \to (1,\bar{1}) \to (2,\bar{1}) \to (2,\bar{0})$, $(0,\bar{1}) \to (1,\bar{1}) \to (1,\bar{0}) \to (2,\bar{0})$, and $(0,\bar{1}) \to (0,\bar{0}) \to (1,\bar{0}) \to (2,\bar{0})$. For the second transition, only one excited state is involved: $(2,\bar{0}) \to (1,\bar{0}) \to (1,\bar{1})$ and $(2,\bar{0}) \to (2,\bar{1}) \to (1,\bar{1})$. The shape of each step is calculated at finite temperature by solving the master equation governing the dynamics of the probabilities $p(0,\bar{1})$, $p(2,\bar{0})$ for the positive step and $p(2,\bar{0})$, $p(1,\bar{1})$ for the negative one. The master equation reads as usual $\dot{p}(a) = \Gamma^{b\to a}p(b) - \Gamma^{a\to b}p(a)$ with $p(b) = 1 - p(a)$ for the main states a, b involved in the transition. Here, the probabilities of other states are neglected, e.g., close to $N_N = 1$ or $N_N = 1.5$. This is a valid assumption if the steps are sufficiently narrow. The calculated steps are shown in Fig. 3.

For the parameters indicated in the caption of Fig. 3, a positive step (where the charge number $n_N = 1$ is skipped) and a consecutive negative step are stabilized. Notice that contrary to the usual staircase, where all real transitions between n and $n \pm 1$ can be treated by the same master equation [11], here the rates are of higher order and the virtual states involved in

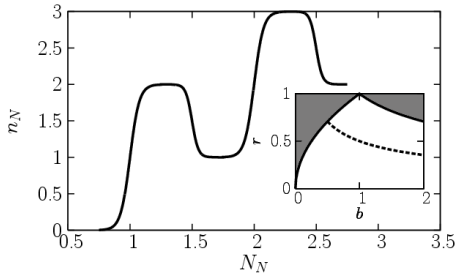

FIG. 3: Coulomb staircase in the small E_J regime for $r = 0.8$, $b = 1$, $E_J/E_{CN} = 0.5$, $k_BT/E_{CN} = 3 \cdot 10^{-2}$, $R_N/R_K = 10$. Charge skipping occurs for $r > 1/2\sqrt{b}$ (dotted line in the inset).

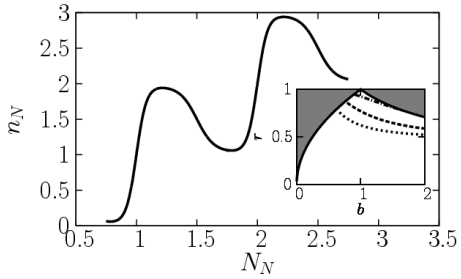

FIG. 4: Coulomb staircase in the adiabatic regime for $E_J/E_{CN} = 2$ (the other parameters are the same as in Fig. 3). The constraint for charge skipping depends on E_J. The inset shows the minimum r values for $E_J/E_{CN} = 1$ (dotted), $E_J/E_{CN} = 2$ (dashed), and $E_J/E_{CN} = 4$ (dash-dot).

one transition (positive step) become real states for the next (negative) one. A full treatment is beyond the scope of this Letter.

Let us now turn to the case of a large Josephson energy, $E_J > E_{CS} = \frac{e^2}{2C_{\Sigma S}(1-r^2)}$. Then one relies on an adiabatic assumption [9]: setting the phase difference to ϕ across the JJ, one can solve the Hamiltonian (2) neglecting the normal electron tunneling term. The adiabatic Hamiltonian $H_{ad} = E_C - E_J \cos\phi$ describes a Cooper pair box with an effective gate voltage, which is an adiabatic function of n_N. In the tight-binding limit $\frac{E_J}{E_{CN}} \gg b$, assuming that the junction dynamics is confined to the lowest Bloch band, one obtains:

$$H_{ad} = E_{CN}(1-r^2)(n_N - N_N)^2 - \Delta_0 \cos[2\pi(N_S - \frac{r}{2\sqrt{b}}(n_N - N_N))] , \quad (3)$$

where the bandwidth is given by

$$\Delta_0 = 16\sqrt{\frac{2}{\pi}}\, b\, E_{CN} \left(\frac{E_J}{2bE_{CN}}\right)^{3/4} e^{-\sqrt{8E_J/bE_{CN}}} . \quad (4)$$

The second term in H_{ad} represents an effective screening potential acting on the charge in N. Choosing N_S which controls the phase of the cosine term, one can achieve a negative curvature of H_{ad}, seen as an effective charging energy E_{CN}^{eff} for the gauged charge in N, $n_N - N_N$. A necessary condition for this is $\frac{\pi^2}{2}\frac{r^2}{b(1-r^2)}\Delta_0 > 1$, yielding the frontier lines in the inset in Fig. 4. Clearly, a large E_J puts a strong constraint on the coupling capacitance C_0, requiring values of r closer to one than for small E_J. If this is satisfied, one calculates the shape of the charge skipping and negative steps using a master equation based on transition rates between charge states $n_N = 0, 2$ or $n_N = 2, 1$, respectively. The adiabatic transition rates are given by $\Gamma_{ad} = \frac{\delta E_{CN}^{eff}}{e^2 R_N}\left[\exp\left(\delta E_{CN}^{eff}/k_BT\right) - 1\right]^{-1}$. The corresponding steps are shown in Fig. 4, and are flatter than in the small E_J case.

To operate in the Coulomb blockade regime, the temperature must be sufficiently low to suppress thermal excitations. The energy difference between two charge states depends on r. A larger r facilitates charge skipping, although a too strong coupling spoils it since the system virtually becomes one single island and the energy no longer depends on the location of the charge. An optimum r is close to 0.75 (for $b = 1$) for small Josephson energies. In this case, the requirement for Coulomb blockade is $k_BT < \frac{E_{CN}}{4}$. In the step calculations, the value $r = 0.8$ was used to accommodate for both the small and large Josephson energy cases. A temperature of $T \sim 30$ mK and a typical charging energy of $E_{CN} \sim 10^{-4}$ eV were used. For the symmetric case where $b = 1$, this charging energy gives $C_N = C_S \sim 2$ fF. Furthermore, if we assume, e.g., $C_{gN} = C_{gS} = 0.02$ fF, then the gate charges $N_S = 0.5$ and $N_N = 0.75 - 2.75$ correspond to $V_{gS} = 4$ mV and $V_{gN} = 6 - 22$ mV, respectively. The value of r chosen for the calculations corresponds to $C_0 = 4C_N = 8$ fF. The second requirement for Coulomb blockade is that the tunnel resistance R_N is larger than the resistance quantum $R_K = \frac{h}{e^2} \approx 25.8$ kΩ.The value $\frac{R_N}{R_K} = 10$ was used, yielding a first-order tunneling rate of $\Gamma \sim 10^9$ s^{-1}. The second- and third-order tunneling rates are 10^7 s^{-1} and $5 \cdot 10^3$ s^{-1}, respectively.

Let us briefly discuss the issue of phase coherence in S. As shown above, charge skipping only requires that pair tunneling occurs between the superconducting reservoir and the S island in order to screen the repulsive interaction in the normal grain. No phase coherence is needed, as shown by the first calculation where the Josephson tunneling is treated perturbatively. Moreover, as a backaction effect, charge fluctuations in N should strongly react upon S and reduce the phase coherence. A full treatment goes beyond the adiabatic approximation made in the large E_J case. One can anticipate that corrections to the adiabatic behavior can cause substantial fluctuations in the phase ϕ, renormalizing E_J to a smaller value, thus making the small-E_J case generic.

The relationship between charge skipping and proximity effect calls for a comment. The latter manifests the onset of pairing correlations in a metal, despite the absence of a pairing potential, due to Cooper pair diffusion. Here, in the absence of any tunneling of electrons between N and S, no phase coherence can be established whatsoever in N. Charge skipping indicates instead a local attractive (negative-U) potential capacitively induced in N. Adding to this a very small tunneling term T_{NS} between N and S opens the possibility of establishing a true phase coherence between states n_N, $n_N + 2$. Then such a proximity effect could be studied in a quite unusual regime, where $T_{NS} < |U|$. More generally, the occurrence

of an attraction in a metallic dot has interesting consequences, some of them having been theoretically explored in the context of molecules with polaronic behavior, like pair tunneling [10, 12], or the possibility of a charge Kondo effect [13] in the coherent regime of tunneling between N and R. Another application of the mechanism proposed in this Letter consists in inducing an attractive correlation between excess charges in two or more neighboring normal dots capacitively coupled to the same S island. Such a device could be useful in quantum information processing based on the charge [14] or spin [15] degree of freedom of individual electrons in normal quantum dots.

We now propose a scheme for detecting an induced attraction in a normal metallic grain. The goal is to detect the non-monotonous charging of the N grain. SETs or point contacts [16] provide very sensitive detection of the local change in the electrostatic potential (rather than the charge). In double-dot setups with weak mutual coupling, the potential fluctuations in each dot can be measured by a different neighboring point contact [17]. In the present case, placing a point contact close to N does not measure δn_N, but instead $\delta V_N = e(C^{-1})_{NN}(e\delta n_N) + 2e(C^{-1})_{NS}(\delta\overline{n}_S) = \frac{e}{C_{\Sigma N}(1-r^2)}\left[\delta n_N + 2r\sqrt{b}\,\delta\overline{n}_S\right]$. If $r\sqrt{b} > \frac{1}{2}$, doubling of the number of steps can be detected, but not the non-monotonous charging curve. To access the latter, it is suitable to measure $\delta V_S = \frac{e}{C_{\Sigma N}(1-r^2)}\left[r\sqrt{b}\,\delta n_N + 2b\,\delta\overline{n}_S\right]$ as well, with a second point contact close to S, and reconstruct $\delta n_N = \frac{C_{\Sigma N}}{e}\left[\delta V_N - \frac{r}{\sqrt{b}}\delta V_S\right]$. The parameters $C_{\Sigma N}, r, b$ can easily be measured from a honeycomb diagram obtained in the normal (non-superconducting) state in the presence of very weak tunneling between N and S. A setup adapting that of Ref. 17 is proposed in Fig. 1, involving a superconducting Nb strip with a Cooper pair box, coupled laterally to an $InAs$ 2DEG. Notice that the direction of charge transfer also can be measured [18], and that other experimental access to the correlation between charge fluctuation in N and S could be obtained from shot noise measurements, as in Ref. 19.

In conclusion, we have proposed a set-up inducing an electronic attraction in a metallic dot. We believe that it could be useful in view of more complex nanoelectronics devices. The authors are grateful to T. Martin, M. Fogelström, and G. Johansson for useful discussions. D. F. and A. Z. were partially supported by AC Ministère de la Recherche. The work of C. H. was supported by the Swedish Research Council (VR) under grant 621-2006-3072.

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