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Endogenous distributions in multi-agents models: the example of endogenization of ends and time constants

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Abstract: Multi-agents modelers recurrently face the problem of the choice of their parameters' values while most of them are exogenous. In this paper we address the issue of endogenization of these parameters when it makes sense in a social learning perspective within the formalism of metamimetic games. We first show how its is possible to endogeneize the agents' ends distribution with a spatial prisoner's dilemma as case study. Then we apply the method to endogenization of time constants in the model, each agent having its own subjective perception of time. In this perspective, the values of endogenous parameters are the outcome of a dynamical process characterized by agent's cognitive capacities and environmental constraints.

Keywords: parameters endogenization, endogenous distributions, spatial games, time constants, evolution of cooperation, metamimetic games.

Introduction

Social systems modelers generally represent agents as a hierarchies of rules determining dynamics on individual traits. At each level, traits and rules evolve under the dynamics defined by their rule and meta-rules if any. Depending on the level, these rules can be interpreted as behavioral rules, rules for decision making, rules for cultural or genetic transmission (*cf.* Fig. 1). The emergence of patterns at the collective level can, thus, be understood as the selection of a particular distribution on the set of possible traits, rules and meta-rules.

Roughly speaking, the modeler faces two main questions:

1. Which are the traits and rules to be considered in a model?
2. How should be determined the distribution of traits and rules at the different levels ?

In the following we will address only the second issue which is strongly linked to the origin of social differentiation. Since in a hierarchical organization of

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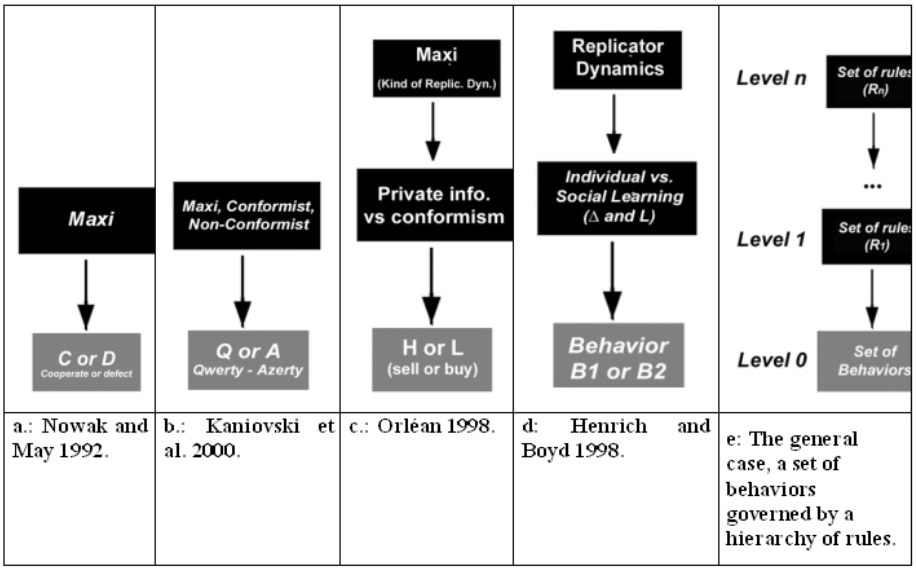

Figure 1: Modelers generally represent agents as a hierarchies of rules

traits, rules and meta-rules, the dynamics at each level is defined by the rules of the above level, intermediate levels are mainly concerned with the issue of initial conditions' settings.

Things are different for the last meta-level that might be said to represent (long term) ends of the agents (payoffs maximization, reproductive success, altruism, etc.). Usually, these ends are given top-down by the modeler. Most of the time individual's ends share the following characteristics:

- they are assigned to the agents prior to their social activity and are immutable thereafter,
- in theory these ends could be heterogeneous but in practice they are often all the same i.e. all the agents share the same goals or payoffs' function or these payoffs function evolve under the same selection pressure (which is formally equivalent). In case of true heterogeneity, the justification of its origin is often skipped which renders interpretation of results difficult.

Hidden behind most of socio-economic models we consequently find something equivalent to a social teleology (everybody pursue the same goals, wants to maximize the same function or the whole system dynamics minimize a given potential function) which is paradoxically close to the adhesion to a holistic principle.

When it comes to understand or reconstruct stylized facts relatively to socio-economic dynamics, this position is questionable since it is hard to find an end that everybody should pursue and there is no reason to think that such particular ends exist. Think for example to a person who buys a car. Does she worked to able to buy this car or does she buys this car to be able to go to work ? Indeed, it could be none of these reasons. This crucial point was already raised by Hayek [Hayek(1978)] about economic modeling:

"I now find somewhat misleading the definition of the science of economy as 'the study of the disposal of scarce means towards the realization of given ends' (...) the reason is that the ends which a Catallaxy [1] *serves are not given in their totality to anyone, that is, are not known either to any individual participant in the process or to the scientist studying it."*

This remark changes radically the perspective of social system modeling. If ends are multiple what can say the modeler about social dynamics ? It is clear that we are not going to reconstruct the immense diversity of human ends. What a model can help us to think is rather the factors that influence their evolution and what is the relation between cognitive capacities at the individual level and the distribution of ends in the population. The central question is then: *Is it possible to endogenize the distribution of ends or meta-rules of behavior such that this distribution becomes the outcome of the dynamics it defines ?*

As we shall see, answering this questions not only gives an insight into endogenization of rules' distribution but opens new horizons for parameters endogenization in multi-agents models. Nevertheless this question raises first of all some tricky issues, among others because it requires some kind of self-reference in the definition of the dynamics. We explored this perspective in previous work with the *metamimetic games* framework [Chavalarias(2004)]. This formal framework builds on the fact that human meta-cognition and reflexivity can be introduced as features in formal models so that imitation rules have the property of being their own meta-rules. We can then build on this property to propose models based on a mimetic principle where the distribution of agents' ends in a population is endogenous. The resulting social dynamics is self-referential in the sense that driven by agents' ends it determines the distribution of ends.

In this paper, we illustrate this method of rules' distribution endogenization on the well known example of the spatial prisoner's dilemma. We then extent the method to endogenization of time constants distribution. The aim is to illustrate the epistemological shift such an approach can propose for social systems modeling. Passing, we will see how this approach renew the perspectives on the well known paradox of large scale cooperation in human societies. Before presenting this case study, we will first briefly remind the main features of metamimetic games. A more detailed description can be found in [Chavalarias(2006)].

1 Metamimetic Games

Human beings build their identity learning on their own and through social learning. One of the most important aspects of social learning, and apparently the first in ontogeny, is learning by imitation [Meltzoff and Prinz(2002)]. This learning skill is exceptionally developed in human beings compared to what exist in other animals species. Human imitation takes several forms, from automatic imitation that is present from birth and seems to persist thereafter

[1] Hayek opposes the notions of *Cosmos* which is a spontaneous order with no purpose and Taxis which is an order that relies on prior ends. *Catallaxy* is the kind of economic arrangement within a *Cosmos* whereas *Economy* is the kind of economic arrangement within a *Taxis*.

in some kind of conformism, to rational imitation where *pro* and *cons* are evaluated before ones engages in an imitation act .

Metamimetic games address this last form of imitation although it might be the case that some kinds of implicit imitation also rely on similar mechanisms. These games are only intended to account for some of the mimetic process of decision-making. They are bound to be coupled with other frameworks for decision-making processes modeling although presenting them separately helps to understand their specific contribution: providing a way to think endogenous dynamics on a set of possible ends in a population of agents.

To understand how ends are represented in the metamimetic framework let us remind a general definition of imitation rule:

Definition: Imitation rule. Given an agent A and its neighborhood Γ_A, an imitation rule is a process that:

1. Assigns a value $\nu(B, \Gamma_A) \in \Re$ to the situation of each agent B in Γ_A. ν is called a *valuation function*.
2. Selects some traits to be copied from the agent(s) in the best situation (according to the target values) and defines the copying process.

For example, in the classical payoff-biased imitation, the value assigned to each neighbor's situation is its payoffs. The agent has then to infer which of the traits of the most successful neighbor(s) are responsible for this success and try to copy these traits. The valuation function here plays a similar role to the utility function in game theory with the difference that it is subjective and evolves through an ongoing social dynamics. It will stand here for the expression of the ends of the agents (who wish to realize a given situation relatively to their relations to their social environment). Two agents can have different valuation functions and the diversity of valuation functions in a population expresses the diversity of ends[2].

To synthesize the above definition, we can say that in step 1, potential models are selected whereas step 2 determines which of these potential models are going to actually influence the agent's behavior and how.

From this definition, the general sketch for metamimetic games is the following. Agents are defined by the actions they undertake in the world plus the rules of decision-making that where used to select these actions (in our case, imitation rules). These rules of decision-making can be organized hierarchically in levels and meta-levels according to which one serves the ends of the other, which one is able to modify the other. These hierarchies of rules defines what we called *metamimetic chains* which are the equivalent of the strategies of the agents in game theory. For example on a financial market, an agent might have as first aim to maximize its profit. To achieve this end she might decide to be temporarily conformist (try to buy what the majority

[2]Several terms are used in the literature to define the principles grounding the agents' decisions and actions : ends, goals, aims, motivations, preferences, utility function, values. Although there is undoubtedly a distinction between these terms, at the level of details considered our simple example, they are subsumed under the generic notion of ends. A more accurate model would require to introduce several time scales. For example; Hayek [Hayek(1978)] distinguishes ends and values on their proper time scale. We will see in the following how these time scales can be viewed as properties of the ends considered and depend endogenously on the levels at which their appear in the hierarchical organization of ends

buys) because it is the rule for decision-making that proved to be the most efficient in the current environment. This hierarchy of strategies evolves as the agents update the different levels according to the rules of the above levels (it might happen that since the environment changed, it is more profitable to be in the minority of buyers and consequently a payoffs-maximizer agent will start to play a minority games with other agents).

The three assumption that define a metamimetic game are the following :

1. **Bounded rationality:** the number k of meta-levels in metamimetic chains is finite and bounded for each agent by its cognitive bound c_B, $(k < c_B)$.

2. **Meta-cognition:** at all levels in a metamimetic chain, imitation rules are modifiable traits. They can be changed for other rules if it is judged relevant by the application of the rule(s) of the above level.

3. **Reflexivity:** imitation rules can update reflexively changing the length of the metamimetic chain in the limit of the cognitive bound of the agents. When the cognitive bound is reached, imitation rules might update themselves.

To give an example illustrating these three assumptions, consider a payoffs-maximizer agent that has only two opportunities of action, C and D. If after reflection she concludes that a conformist behavior is the more successful in terms of material payoffs, she might decide to change her strategy accordingly which will change the length of its metamimetic chain as described in figure 2.

But if the cognitive bound of the agent does not allow her to keep in mind the two distinct ends ($C_B = 1$), she might then revise her strategy an drop her main end, as described in figure 3.

It is not the place here to discuss about the relevance of these kind of transition see [Chavalarias(2006)] for more details. Let us just mention three kinds of relations between goals and sub-goals that might be schematized by these transitions:

- The agent adopts a new end and progressively forgets for any reason its old end,

- The new end is so time-taking that although the old end is still present in mind, it is never taken into account in subsequent decisions,

- The agent enjoys more the activities associated to the new end than those associated to the old one and decides to adopt the new one as her main ends.

In all cases, the salient feature is that new ends are adopted because they are consistent with old ones at the moment of their adoption. At a given moment, the set of current ends constrains the way this set can be modified.

Now, if we consider a population of artificial metamimetic agents, with a given set of possible ends, the principles outlined above define the internal dynamics of the artificial social system (described mathematically in previous works by the Markov chain P^0). This dynamics has some stable states, *metamimetic equilibria* that are *counterfactually stable states i.e.* states such that *no agent*

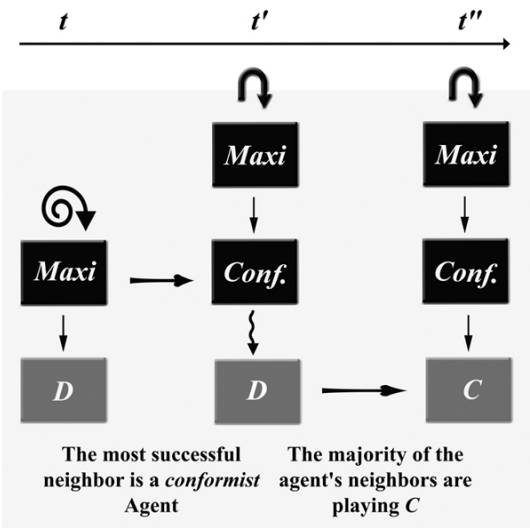

Figure 2: **Endogenous variation in the length of metamimetic chains.** At time t, a maxi-agent A has a conformist neighbor that is more successful than all agents in Γ_A. if A infers that this success is due to the conformist rule, she might adopt this rule as first level rule, and keep in mind that it is only a mean for maximizing her payoffs (meta-level). Thereafter, it might be that according to this conformist rule, the current behavior is not the best one and has to be changed.

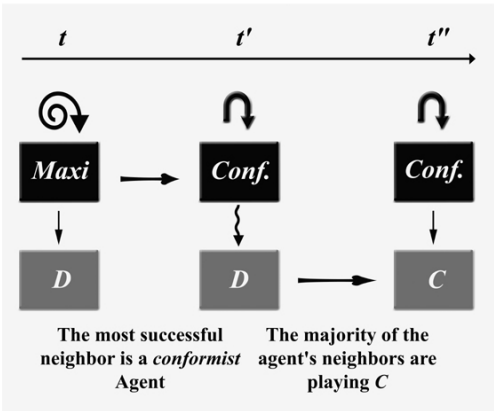

Figure 3: **Reflexive update at the limit of the cognitive bound.** At time t, a maxi-agent A has a conformist neighbor that is strictly more successful than all other neighbors. if A infers that this success is due to the conformist rule, she might adopt this rule at its first level. Since $C_B = 1$ this simply replaces the original maxi-rule. Thereafter, it might be that according to this new rule, the current behavior is not the best one, and has to be changed.

can find itself better when it imagines itself in the place of one of its neighbors. More frequently, we encounter stable sets of states, *metamimetic attractors.* Since agents do errors at the different levels of decision-making process (inference, reasoning, action implementation) and because their environment is noisy, the right object to study *in fine* is a perturbated Markov process P_ϵ^0 in the framework of stochastic evolutionary game theory [Foster and Young(1990)]. However in this paper, we will mainly focus on the internal dynamics (P^0).

After these preliminaries, we are now able to give an example of endogenization of the distribution of end in a multi-agent system with the well known example of spatial prisoner's dilemma game.

2 The Spatial metamimetic prisoner's dilemma Game

2.1 The model

Following Nowak et May [Nowak and May(1992)], we will consider a spatial model of evolution of cooperation. We choose this particular model as first example for three reasons. First, the PD dilemma and evolution of cooperation is a scientific puzzle in the consequentialist view of human decision-making. Second, rules for decision-making in the original model of Nowak and May can be interpreted as payoffs-biased mimetic rules which simplify the comparison with the model presented here. Third, the properties and limits of this original model are well known and can be summarized as follow (see for example [Hauert(2001)]):

1. Cooperation is possible in some areas of the parameters' space were there is a coexistence of zones of cooperation and defection evolving constantly with time,

2. This chaotic patterns are not robust regarding to variations in updating protocols [Huberman and Glance(1993)],

3. These areas of the parameters space are very tiny and correspond to weak social dilemmas. Consequently, cooperation is not sustainable most of the time.

The model can be described as follow. Agents are displayed at the nodes of a two dimensional toric grid. Agents play a prisoner's dilemma game (PD game) each round with each of their neighbors, choosing between two simple actions: cooperate (C) or defect (D). The actions used in the games with different neighbors are the same for a given period. Neighborhoods of players are composed by the eight adjacent cells. When two agents play together, they receive a payoff of R if both cooperate (C) and P if both defect (D). In case their strategies are different, the one who played D receives a payoff of T and the other receives S (*cf.* Table 1). The two conditions for this game to be a prisoner's dilemma are:

1. $T > R > P > S$: defection is always more advantageous from the individual point of view.

Table 1: The matrix of the prisoner's dilemma game

Player A	Player B	
	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

2. $T + S < 2.R$: mutual cooperation is the best you can do collectively.

At the end of each period, the sum of the payoffs of each agent are computed and agents update their strategies on the basis of the available information on the last period.

2.2 The set of strategies

We will consider agents with $C_B = 1$. Their strategy will thus be described by a behavior and an imitation rule for decision-making : $s = (b, r)$. Although $C_B = 1$ is not a realistic assumption[3] this will be sufficient to illustrate our purpose. For simplicity, we will also assume that the second step in an imitation process is just pure copying of the trait.
The set of rules for imitation should represent all the possible ends that agents can imagine from what they perceive. Consequently, it's important that this set is generated from what the agents can perceive and what kind of processing the agents can do on these perceptions[4]. For this reason, we define the set of imitation rules as the outcome of operation of different kinds of cognitive operators on primary perceptions: operators for the selection of a particular dimension in the perception space and operators for computation on this selection. We thus have the following scheme:

$$PERCEPTION \rightarrow PROCESSING \rightarrow IMITATION\ RULES$$

Here, we will assume that:

As for the perception, agents can perceive:

- material-payoffs of their neighbors,
- the last action (C or D) of their neighbors,
- the rule they used to choose them.

Here, perceptions are 'exact' and agents are somehow mind-readers. The issue of errors in perception and inference is addressed in [Chavalarias(2004)] but is out of the scope of this article.

As for computation, agents can:

- Compute the proportions of different behaviors and rules in their neighborhood,

[3]Neurobiological studies seems to indicate that for the 'now and here' we have something like $C_B = 2$ (Etienne Koeklin, personal communication).
[4]Note that in social systems, this is already a cultural construction.

Table 2: The four imitation rules

Computation	Perceived dimension	
	Densities	Payoffs
max	$Conformist$	$Maxi$
min	*Anti-conformists*	$Mini$

- Compare two real numbers and take the max (max of payoffs, max of proportions),
- Multiply by -1 a real number (if agents can imagine an ordering of situations, they can also imagine the reverse ordering, or to say it differently, if they can compute the max of two numbers, they can also compute the min).

Moreover, we will assume that the agents have zero memory: they can only built rule for decision making that take into account the perceptions relative to last period[5]. This generates four valuation functions and consequently four imitation rules ($cf.$ Table 2):

1. ***Maxi***: "copy the most successful agent in your neighborhood in terms of material payoffs".
2. ***Mini*** : "copy the less successful agent in your neighborhood".
3. ***Conformism*** : "copy the trait (behavior or rule) used by the majority of agents"
4. ***Anti-conformism*** : "copy the trait (behavior or rule) used by the minority of agents"

In the computational study presented here, each period proceeds with parallel updating as follow[6] (see the detailed algorithm in appendix):

1. Each agent looks at the last period's situation of its neighbors (payoffs, rules, behavior),
2. For any agent A, if according to A's valuation function there are some agents in Γ_A in a better situation than A and if all these neighbors have a valuation function different from A's, then A imitates the rule of an agent taken at random among its most successful neighbors,
3. if according to its (eventually new) valuation function, A is not among the most successful agents in Γ_A, then A chooses at random one of its neighbors with the best situation from last period and copies its behavior (C or D).
4. for each agent, the scores of the eight PD games with its neighbors are computed and the sum is the new material payoffs of the agent.

Note that an agent that does not have a neighbor with a better situation will be satisfied with its own and will not engage in an imitation process. It will just stick to its former strategy.

[5]See [Chavalarias(2004)] for a study of the influence of memory

[6]In the following we say 'it' for agents since now we are dealing with artificial agents.

3 Endogenous distribution of rules and stability of cooperation

We will now give some computational results where we will be able to se the effects of the endogenization of ends distribution we mentioned in the introduction. We will study the influence on the endogenous distribution of the strength of the social dilemma (parameter p) and of the initial disposition of the population toward cooperation (proportion of cooperators at the first period). All the simulations have for initial state a uniform distribution of the four imitation rules.

We will adopt the following exposition plan:

1. A detailed study for the 'historic' settings of the PD game [Axelrod(1984)]: $T = 5$, $R = 3$, $P = 1$ and $S = 0$, and an initial rate of cooperation of 30%.

2. Study of the robustness of these first results toward changes in the strength of the social dilemma and in the initial disposition of the population toward cooperation : stability of the qualitative properties of the attractors found in 1.

3.1 Emergence of social group and cooperation

Let's begin with a study for a particular set of parameters: $T = 5$, $R = 3$, $P = 1$ and $S = 0$, and an initial rate of cooperation of 30% (figure 4). We report here a study on 50 independent multi-agents simulations with a population of 10 000 agents each. The spatial distribution of the different kinds of rules and behaviors in the population was uniform according to the initial proportions.

The first noticeable facts are that in all simulations, the system quickly reaches an *heterogeneous* attractor while the rate of cooperation *increases*[7]. This attractor is mostly static (only a few oscillators remaining). This means that at the attractor, most agents are counterfactually stable even if all possible ends are represented in the population. The attractor is heterogeneous at both behaviors' and rules' levels. The emerging patterns make sense relatively to the ends of the immerged agents and an external observer could even guess who is who in this global picture : conformists are forming large areas where they are in majority, anti-conformists are scattered on all the territory and are locally in minority, maxi and mini agents have interlaced populations, the formers 'exploiting' the others which enjoy.

The interpretation of these emerging structure is that the structure of the attractor reflects the constraints imposed by the self-consistency of the rules, they are is the projections at the collective level of the *elementary virtualities* contained in each agent.

At the behavioral level, we found a mixed population with plain clusters of cooperators and defectors, interlaced areas and scattered exceptions. These structures can be explained only if we look above at the rule level. For example, mini agents exclusively cooperate because it is actually the best way to minimize ones payoffs. It should be emphasized that most agents changes

[7]Should we remind the reader that in the original model of Nowak and May, the rate of cooperation would have collapse down to zero with such parameters?

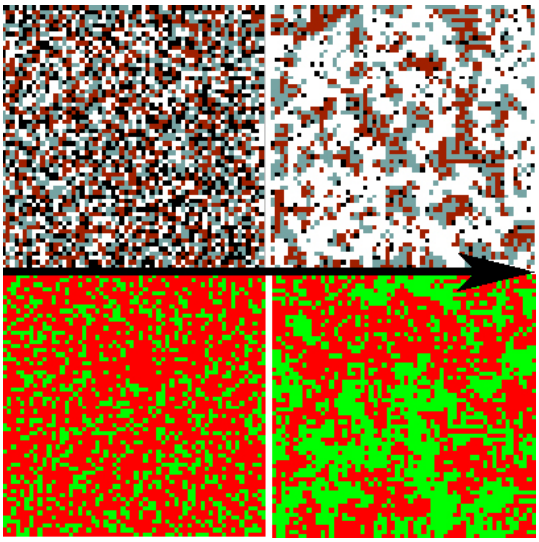

Figure 4: **Evolution of the spatial distribution of rules (upper part) and behaviors (lower part) from the initial disordered state to a structured attractor.** Each small square represents an agent. This configuration is globally stable (only few oscillators remaining at the attractor). *Legend:* Upper part - white: conformists, black: anti-conformists, light grey: mini, dark grey: maxi. Lower part - light grey: cooperators, dark grey: defectors.

both their behavior and imitation rule during their lives, sometimes several times. The original assignment of rules at first period has only week consequences on what agents will be in their social life. An agent stops changing its strategy when it finally finds a rule and a behavior such that the behavior is consistent with its social environment relatively to its rule. It is this multitude of agents looking for their identity that collectively produce a global stable order. This process is what we call *social cognition*.

Now if we run different simulations with same initial settings and look the path toward the attractors, we can see that (figure 5):

1. All populations reach very quickly their attractors,
2. these attractors are statistically and qualitatively similar as well as the trajectories to reach them (the variance on the distribution of distributions is quite small),
3. All attractors are heterogeneous and well structured.

This suggests that the kind of structure an artificial society reaches is well constrained by the internal cognitive structure of the agents (the perception and computation operators), the statistical distribution of rules and behaviors and the PD matrix. We will now study the dependence of this structure toward these last parameters.

3.2 The influence of environmental constraints

To study the influence of the initial rate of cooperators and the strength of social dilemma on the dynamics, it is more convenient to consider a matrix of

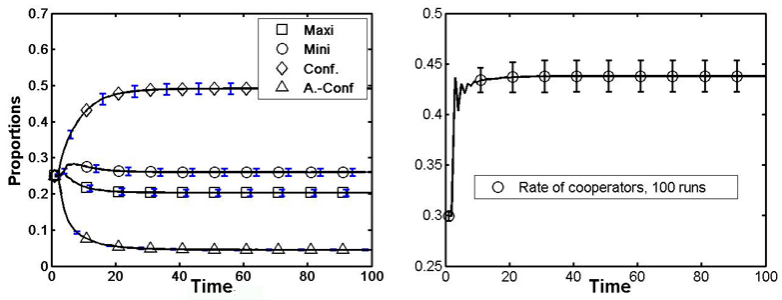

Figure 5: **The evolution of the strategies' distribution in a spatial metamimetic PD game.** *Left:* evolution of the distribution of imitation rules. Here $T = 5$, $R = 3$, $P = 1$ and $S = 0$, and the statistics have been computed on 50 runs, 10 000 agents each. The population stabilized with about 48% of of conformists, about 27% maxi and 20% mini , and 5% of anti-conformists. *Right:* Statistics on the evolution of cooperation. The rate of cooperators increases from 30% to a proportion of 44% of cooperators.

Table 3: A parametrization of the PD game matrix

Player A	Player B	
	C	D
C	$(1-p, 1-p)$	$(0, 1)$
D	$(1, 0)$	(p, p)

the game described by only one parameter. It is well known that two parameters are enough to describe the whole set of distinct games. The problem now is to select a subset of this 2D space that would nevertheless generate all the interesting dynamics. For this purpose, we will take a parametrization frequently used in the social dilemma literature: the payoffs matrix given by table 3. Here p measures the strength of the dilemma: the higher p, the stronger the dilemma.

We will assume that $0 < p < 0.5$ so that the condition $T > R > P > R$ is satisfied. The condition $T + S < 2R$ is violated (we have equality) but it doesn't have noticeable consequences on the dynamics[8].

We did a study similar to the case presented in previous section, for the initial rate of cooperation ($IniCoop$) varying between 5% and 95% and the strength p of the social dilemma varying between 0.1 and 0.4. The same qualitative properties as described in 3.1 were observed concerning the attractors (fig. 6) and stability of cooperation is robust toward change in initial conditions on p and $IniCoop$.

Behavioral level : The rate of cooperation at the attractor is plotted on figure 6. We can see that this rate is always above 9.5% and above 40% in the majority of the cases. Attractors at the behavioral level depend heavily on $IniCoop$ for low p but are almost independent of $IniCoop$ for $p > 0.2$. On the contrary, p has always a great influence on these attractors. Even if it is not the point here , it is noteworthy that the high level of cooperation for

[8]Actually, this condition is often neglected in models

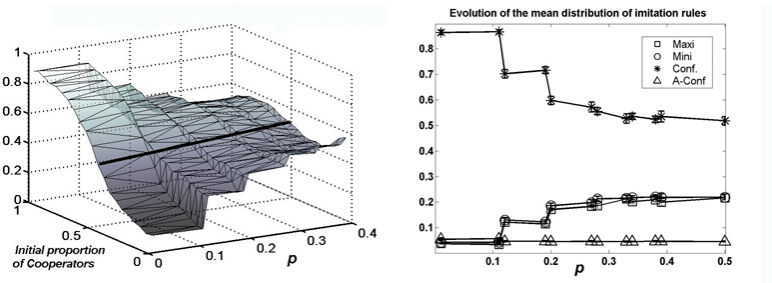

Figure 6: **Influence of initial propensity for cooperation and the strength of the social dilemma on the structure of the attractors:** *Left:* Dependence of the rate of cooperators at the attractor (100 time steps) in function of parameters p and the initial rate of cooperators $IniCoop$. Each point represents the mean rate of cooperation (on 10 independent runs) at the attractor for the couple $(p, IniCoop)$ considered. The rate of cooperation is always above 9.5%. Cooperation is sustainable under all the conditions studied. The line represents the set of simulations corresponding to the area of parameters used in the right figure. *Right:* Evolution of the distribution of metamimetic rules in function of p, for an initial rate of cooperation of 50%. Each plot represents the mean evolution of the proportions at the attractor of the corresponding imitation rule in function of p (on 10 independent runs). We can see that conformists (up triangles) are predominant under all conditions but maxi (circles) and mini (stars) agents are more represented in areas corresponding to high p than in those corresponding to low p. Error bars represent the standard deviation. We have : $0.01 < p \leq .5$ and $1\% < IniCoop < 99\%$.

most of the parameters range is a very interesting result in the perspective of the emergence of cooperation.

Meta-rules level : Here $Inicoop$ has even less influence on the meta-rules than it has on behaviors. The proportion of conformist agents decreases when p increases, while the opposite phenomenon happens with maxi and mini agents. If conformist agents are always the population with the highest density, there is a significant proportion of *maxi* and *mini* agents for $p > 0.2$ (more than 20% for each population). On the contrary, the proportion of non-conformists is not sensitive to p and is almost constant along both axes (it seems to be a function of the topology of the social network). At the attractor, most agents perform repetitive behavior without changing anything at their behavioral level or meta-level. However, few agents, at the border of clusters, keep changing one of these two traits.

The right graph of figure 6 is interesting because it predicts some qualitative trends for behaviors in the spatial prisoner's dilemma that could be verified experimentally. Given the simplicity of the model, there are much chance that such verifications would turn to refutations. However a more detailed analysis of this graph will gives some ideas about the kind of issues that can be addressed when the distribution of rule of behavior is made endogenous. The main features are the followings:

Influence of the initial rate of cooperation: The initial rate of cooperation has a positive influence on the rate of cooperation at the attractor. For a given p, this dependence has a S shape along $IniCoop$, which is the well-known signature social learning. We can thus expect that this

shape is due to some kind of conformist behavior which is confirmed by a detailed study [Chavalarias(forthcoming 2007)].

Influence of the strength of the social dilemma: As we can see, the surface of the graph becomes almost flat as p increases. This means that the higher the strength of the social dilemma, the lower the influence of the initial propensity of the population for cooperation. This leads to very counterintuitive results. For example, for a initially low proportions cooperators, the proportion of cooperators at the attractor increases when the strength of the social dilemma increases. From 1, we can also expect that the weight of conformist agents in the population will decrease as p increases if we study the dependance at the rules' level, which is also confirmed by a detailed study [Chavalarias(forthcoming 2007)].

In [Chavalarias(forthcoming 2007)] we gave more insight into the influence of these two parameters on the metamimetic prisoner's dilemma game. The important point is that such a study does not only have to explain the dependance of the behavioral level on the considered parameters, but also the dependence of the rule level. We showed in particular that with the set of rules introduced in 2.2 the proportions of rules indexed on cardinal valuation function (here *mini* and *maxi*) is influenced by two critical factors:

- The *spatial dominance* distribution of the different behaviors relatively to the valuation function of the rule considered (here D on C for maxi and mini rule). Spatial dominance of a behavior A on B can be defined as the proportion of spatial configurations on behaviors such that A and B are actions of neighbor agents and A's payoffs (according to the rule's valuation function) is higher than B's payoffs. Thus spatial dominance takes into account spatial configurations involving neighbors and second neighbors, contrary to game theory where dominance is defined in terms of outcomes with direct partners. In case of binary choice, the higher the variance on spatial dominance, the easier for an agent to find a suitable behavior. We conjecture that in case of binary choice, the proportion of a rule indexed on a cardinal valuation function increases as the variance of behaviors' spatial dominance increases relatively to the rule's valuation function.

- The *residual uncertainty of a rule*: this is related to the variance of payoffs (in terms of the rule's valuation function) associated to a given behavior along spatial configurations of neighbors and second neighbors. Even when there are some behaviors that strictly spatially dominate the others, two players with the same behavior can have different payoffs because they have different opportunities with their own neighbors. This could lead to the counterfactual instability of an agent. We conjecture that the proportion of a cardinal rule decreases as the residual uncertainty of the spatially dominant behaviors increases.

These both features are closely linked with the network topology and it should be possible to characterize the dependance of these two factors in function of social network's properties. In particular, we can expect that spatial dominance, and consequently the proportions of cardinal rules (here mini and maxi), will increase as cliquishness increases, since information on second neighbors increases.

4 An example of time constants endogenisation

We will now show that we can endogenize other variables as soon as they can be considered as part of the agent's strategy. Besides the question of robustness of our previous results toward network's topology, the other important point is robustness toward the updating protocol in the model ([Huberman and Glance(1993)]). Are the qualitative properties of our metamimetic prisoner's dilemma unchanged if strategies' updates (rules and behaviors) where asynchronous ?

A natural way to introduce asynchrony is to assume that at each period:

- Each agent updates its metarule with a probability α,
- Each agent updates its behavior if its metarule has changed. Otherwise, it updates its behavior with a probability θ.

Introducing these two variables may help to check that structures observed in section 3 are not artifacts of parallel updates. However it weakens the model because it adds supplementary degree of freedom and decreases its sensitivity to refutation in the popperian sense. But what are these time constants ? We understand that some rule of behavior can need some time to reveal efficient. On the other hand, in quickly changing environments, it may be adaptive to often check the adequation of one's strategy to one's goals. These time constants can thus be interpreted as the expression of the exploration/exploitation tradeoff of the agents, which is well known to be an important component of a strategy. But if it is part of the agent's strategy and is recognized as such by the agents, it can be imitated.

We can thus endogenize the distribution of times constants, starting with initial conditions and assuming that when an agent copies a metarule, it also copies α and θ values with some small errors. The extent of these errors would ideally be inspired by psychological experiments about time perception. To briefly give the taste of this kind of endogenization, we will take the example of 3.1 and consider that the population starts with α and θ equals to 0.5 for every agent. We will further assume that each time an agent copies one of its neighbors, it also copies values of α and θ with a relative precision $\epsilon = 5\%$, the error being determined by the following algorithm for α (and similarly but independently for θ), for each agent, at period t:

- draw a random variable ν from normal law with null average and variance ϵ
- $\frac{1}{\alpha_{t+1}} = \frac{1+\nu}{\alpha_t}$ if $\frac{1+\nu}{\alpha_t} > 1$ and $\frac{1}{\alpha_t}$ otherwise.

On figure 7, we can see that the patterns emerging at the behavioral and rule levels are similar to those observed in 3.1. We can also notice that mean frequency of rules updating is slightly lower than the one of actions updating. This is not significant here but was already observed in other simulations suggesting that the system adjusts automatically to a hierarchical distribution of time constants.

With this example, we can see that besides rule of decision making, we can naturally endogeneize some additional parameters of a model. Moreover, this enables to consider additional variables without increasing excessively the number of degrees of freedom in the model.

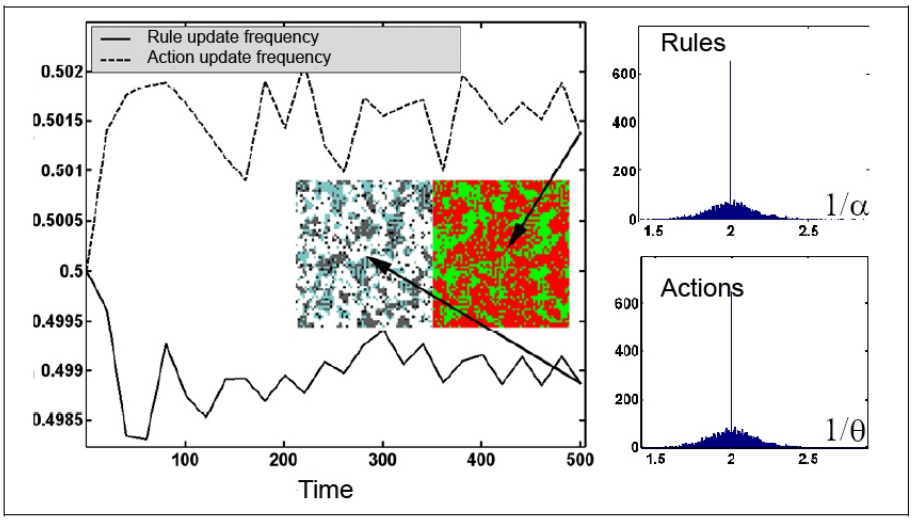

Figure 7: **Evolution of update frequencies in population of metamimetic agents playing prisoner's dilemma:** Conditions are the same as in 3.1. The right figures show the distribution of time constant in the population after 500 periods.

5 Conclusion

Multi-agents modeling when applied to social systems immediately raises the problem of individual teleologies. Except in exceptional situations where we exactly know what agents are looking for, agents' ends are loosely defined and we only have a vague idea of the set of possible ends agents might pursue. Moreover, ends evolve through social interactions and consequently, this dynamics on ends should be taken into account as soon as the period of time covered by the model are sufficiently long.

This entails that in social systems modeling making explicit the dependence of ends' distributions on environmental factors is at least as important as studying the dependance of behaviors' distribution given a distribution on ends. For example, to face global warming, the question is more how can mentalities change so that our collective way of living becomes sustainable rather than the more specific issue stating how behaviors can change under economic incentives.

The approach we suggest to take into account ends' distribution dynamics is to endogeneize this distribution through mechanisms of social learning and then study the influence of environmental factors on the dynamics of this distribution. We used for this purpose metamimetic games and conducted a short case study on emergence of cooperation.

This method to endogenize ends can also be applied to other parameters of the models like time constants. This suggests a general methods to face the problem of the choice of parameters' values in socio-economics models that explicitly integrate the fact that socio-economic systems are reflexive entities: they continuously produce themselves while functioning [Le Moigne(1988)].

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Appendix

The algorithm used for the simulations presented in this paper is the following:

Set up of the game:

- Give a value for p, $0 < p < 0.5$.
- Agent at displayed on a toric grid, their neighborhood are composed by the eight adjacent cells.

Initial Conditions:

- Give the spatial distribution of imitation rules. Here, there was four rules. For each agents, we assigned randomly one of the rules with a probability 1/4
- Give the spatial distribution of behaviors. Here, for each agents, we assigned the behavior C with a probability $IniCoop$ and D otherwise.

At each period, for each agent, with parallel update at the population level:

- For each agent, the scores of the eight PD games with its neighbors are computed and the sum is the new material payoffs of the agent.
- Each agent looks at the last period's situation of its neighbors (payoffs, rules, behavior),
- For any agent A, if according to A's valuation function there are some agents in Γ_A in a better situation than A and if all these neighbors have a valuation function different from A's, then A imitates the rule of an agent taken at random among its most successful neighbors,
- if according to its (eventually new) valuation function, A is not among the more successful agents in Γ_A, then A chooses at random one of its neighbors with the better situation from last period and copies its behavior (C or D).

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