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► **To cite this version:**

David Laurent, Olivier Legrand, Patrick Sebbah, Christian Vanneste, Fabrice Mortessagne. Direct observation of localized modes in an open disordered microwave cavity. 2007. hal-00133810v2

**HAL Id: hal-00133810**

**<https://hal.science/hal-00133810v2>**

Preprint submitted on 23 Mar 2007 (v2), last revised 17 Oct 2007 (v3)

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# Direct Observation of Localized Modes in an Open Disordered Microwave Cavity

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We present measurements of the spatial intensity distribution of localized modes in a two-dimensional open microwave cavity randomly filled with cylindrical dielectric scatterers. The robustness of these modes with respect to a modification of the boundaries of the scattering region is successfully established. From a comparison with numerically calculated modes, we can extract precise values of the localization length which are in agreement with the measured spectral widths. These results constitute unambiguous signatures of the existence of strongly localized electromagnetic modes in two-dimensional open random media.

PACS numbers: 42.25.Dd, 72.15.Rn

In spite of two decades of intensive research, the experimental observation of strong localization of classical waves remains a tremendous challenge [1]. One of the main difficulties lies in the fact that the signature of localization is sought after statistic measurements of transmission. Indeed, the expected exponential decrease of transmission could not be attributed unambiguously to localization rather than absorption [2]. In contrast, transient analysis, as demonstrated by Weaver in ultrasound measurements in a two-dimensional (2D) disordered medium [3], allows to discriminate between these two phenomena. A similar approach has been proposed in disordered microwave systems by Genack who demonstrated localization in the microwave regime using variance of total transmission as a genuine probe to localization even in the presence of strong absorption [4]. More recently, G. Maret's group have observed deviation from diffusion in the time of flight distribution, which cannot be explained by absorption but plead for a transition to strong localization of photons in three dimensions [5]. Such signatures of localization in transmission reflects the nature of the underlying quasimodes, which are spatially localized inside the system. The increasing contribution of such long-lived modes, as time progresses, is responsible for the observed deviations from the regime of purely diffusive transport [6]. Besides the fact that their localized nature is not affected by absorption, exhibiting the localized modes inside the random system would be a direct demonstration of localization and the key to the understanding of the mechanism underlying the transition from diffusive to localized regime.

Observations of localized modes have been mostly reported in 1D random systems [7] where localization is readily achieved. In the marginal dimension of 2, localization of bending waves in randomly loaded steel plates [8] and spatial concentration of microwave field in microwave cavities [9] have been reported where the influence of the reflecting boundaries precludes the unambiguous attribution of the localization effect to the sole presence of scatterers. The aim of this letter is to provide an unequivocal observation of localized modes in an

open 2D random system. Beyond the mere observation of such modes, we put them under a test of robustness and successfully confront them to numerical simulations and theoretical predictions.

All our results are obtained from transmission signals measured in a 2D microwave cavity operated at frequencies ranging from 500 MHz to 10 GHz. The original bare cavity is composed of two copper plates sandwiching a copper rectangular frame of thickness 5 mm. The quality of copper is OFHC to reduce ohmic losses. Due to its height of 5 mm (smaller than half the smallest wavelength used), this cavity only admits transverse magnetic two-dimensional modes of order 0. Through one of the copper plates, a few antennas are introduced, their optimal coupling being obtained by fixing their penetration length inside the cavity at 2 mm. The antennas are monopolar with SMA connectors and their positions are displayed on Fig. 1. In transmission measurements, only two antennas are used at a time, the other ones being terminated by  $50\Omega$  loads so that all antennas behave the same way regarding the losses they imply. These antennas are linked to an HP 8720 D vector analyzer through flexible cables. The details about transmission measurements are described in reference [10].

In order to mimic a truly open system, frames of microwave absorbing foams are inserted in the cavity, surrounding an empty rectangular-shaped space. Two different layers of absorbers (ECCOSORB<sup>®</sup> LS-14 and LS-16) are used with different electromagnetic impedances (see Fig. 1). Their thickness is the overall thickness of the cavity and their respective widths are determined to provide 20 dB of attenuation in the intensity reflected back to the empty space at frequencies above 500 MHz. As shown in Fig. 1 a disordered medium is introduced in the empty space. It consists of 196 cylindrical dielectric scatterers with a measured dielectric constant  $\epsilon = 37$ , a radius of 3 mm and a height of 5 mm. These scatterers have been chosen for their particular low loss (quality factor  $Q = 7000$  at 7 GHz) and their high index. In the frequency range of interest, the possible excitation of transverse electric waves inside the dielectric scatterers

can be assumed negligible.

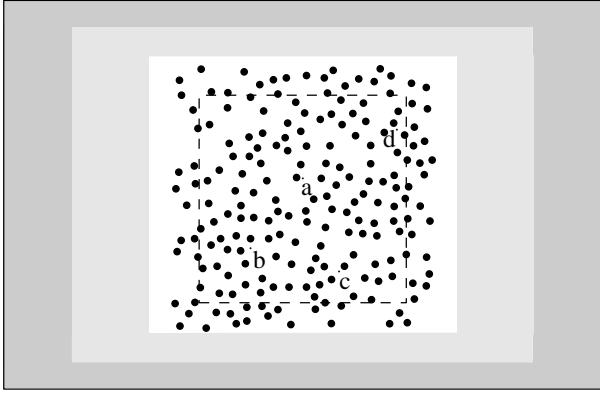


FIG. 1: Schematic view of the cavity: 196 scatterers located in a central square region. For some measurements, the scatterers outside the dashed square are withdrawn. Two different layers of microwave absorbers ensure a weak inward-reflection of the field. The antennas (a, b, c, d) are shown through small dots.

The positions of the scatterers are determined at random with a filling fraction of 8.9% and a minimal distance  $d_{\min}$  between the centers of neighboring scatterers. At the above given filling fraction, the values of  $d_{\min}$  range from 8 mm to 19 mm. The smallest value is dictated by practical constraint, the largest corresponds to the lattice parameter of a square array. We fixed  $d_{\min} = 11$  mm. The figure 1 displays a particular realization of disorder using the above parameters. In Fig. 2, a typical transmission (between antennas a and b) spectrum is shown. One can clearly identify a frequency range which is reminiscent of the bandgap found in the case of the periodic structure with the same filling fraction. This gap is strongly correlated with the Mie cross-section for a single scatterer [8, 11] (the Mie cross section is shown superimposed to the measured spectrum in Fig. 2). Using a *diffusion based theory* [12, 13, 14] we evaluated the localization length  $\xi_{theory}$  in two dimensions for our system of scatterers within the independent scattering approximation (ISA). It reads :

$$\xi_{theory} = \ell \exp(\pi \text{Re}(k_{eff}) \ell / 2)$$

where  $k_{eff}$  is the effective complex wave number and  $\ell = (2\text{Im}(k_{eff}))^{-1}$  is the mean free path, which is inversely proportional to the Mie cross section. The minimal values of this localization length are roughly located at the maxima of the Mie cross-section and are of the order of 10 mm, a value comparable to the mean free path and also to the mean distance between scatterers for the filling fraction mentioned above. We are well aware of the fact that the ISA is not a valid approximation in the regime we investigate in the present letter and that a collective effect, similar in nature to the proximity resonance

effect [15], tends to produce extra sharp resonances which are blue-shifted with respect to the Mie resonances. Note that Vanneste and Sebbah [16], in a numerical study of random lasing in the localized regime, also noticed that the frequencies of the localized modes lie between the maxima of the Mie cross-section. This is also clearly confirmed in our findings. However, in spite of these important drawbacks, as will be established later, the ISA based theory has the principal merit of yielding the right orders of magnitude for the minimal values of the localization length.

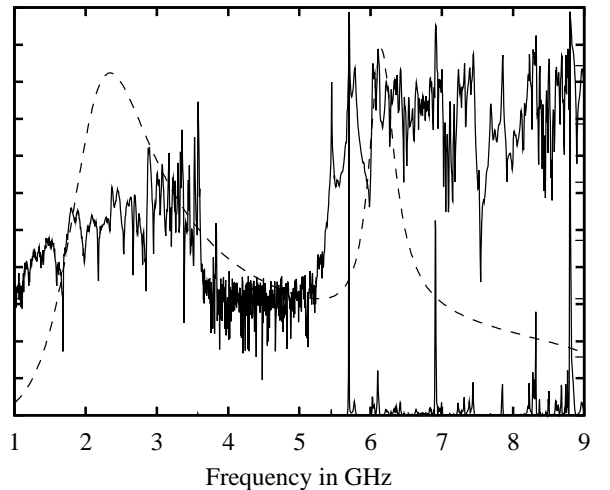


FIG. 2: Full curve : transmission signal measured for the disordered realization shown in Fig. 1 (linear and log scales are shown). Dashed curve: Mie cross-section of a single dielectric scatterer.

By using an appropriate fitting procedure [10] we extracted the spectral widths of the resonances. Only a few widths take values close to those expected when taking only the ohmic losses into account, whereas the vast majority are significantly larger. These purely ohmic widths could be associated to resonances which very likely do not reach the absorbing foam at the boundary. Localized modes, if any, should be found among these resonances. We focused our attention on three of them: one in the *gap* at 5.45 GHz, one on the edge of the *gap* at 5.66 GHz, and one at 7.80 GHz. For different couples of antennas we obtained spectra very similar to the one presented in Fig. 2, the most sensitive resonances being the sharpest, like the three selected, as they can even disappear for a given couple. This behavior pleads in favor of the observation of truly localized modes.

We obtained the spatial distribution of these resonances by using a well-known scanning perturbation technique (see ref. [17] and references therein). This technique is based on the fact that, for a given resonance, a frequency shift is caused by the presence of a small stainless steel bead placed inside the cavity. This

shift is a measure of the intensity of the local field at the bead location. More precisely, this shift can be shown to be essentially proportional to the square of the electric field [17]. Indeed, through measurements on computable modes of an empty rectangular cavity we could check that the transverse magnetic field contribution is at most of the order of 1% of the maximum contribution of the electric field. The bead can be moved on a very finely defined grid (5-mm-step) from the outside of the cavity by means of a strong magnet which is fixed on an X-Y precise translation stage.

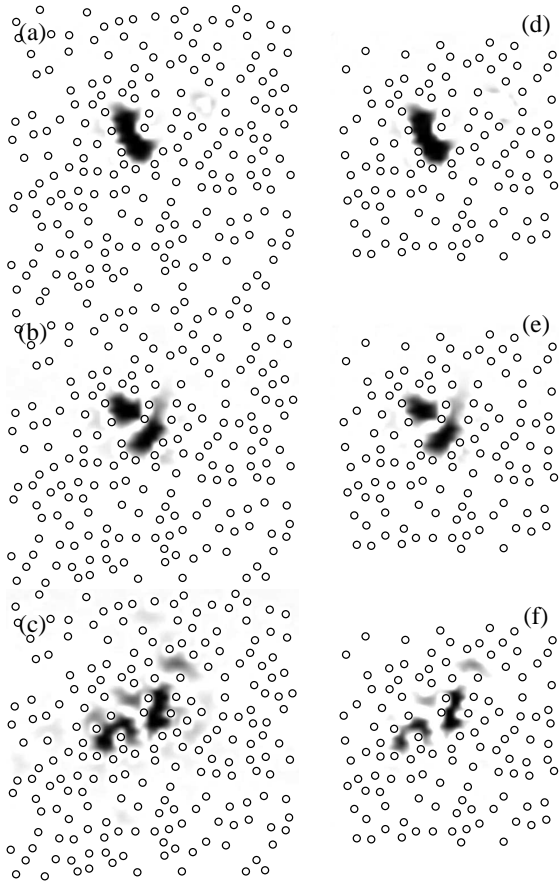


FIG. 3: Experimental maps of the amplitude of the electric field for localized modes at 5.45 GHz (a) and (d), 5.66 GHz (b) and (e), and at 7.80 GHz (c) and (f), for two different sizes of the scattering region (a  $25\text{ cm} \times 25\text{ cm}$  square and a  $20\text{ cm} \times 20\text{ cm}$  square). The 3 modes are clearly very little sensitive to the suppression of the peripheral scatterers.

In figure 3 (a,b,c), two-dimensional scans of the amplitude (square root of the intensity) of the three long-lived modes mentioned above are depicted. They display a clear spatial concentration. Whereas mode (a) seems to be trapped in a small region surrounded by scatterers, modes (b) and (c) are more extended. Following a method indicated below, we could evaluate the localization lengths for these modes: between 10 and 12 mm for

mode (a), 14-25 mm for mode (b) and 22-30 mm for mode (c) (the ranges depending on the anisotropy of the spatial pattern).

In order to test the robustness of these modes with respect to a modification of the boundaries of the scattering region, we reduced this region from a  $25\text{ cm} \times 25\text{ cm}$  square to a  $20\text{ cm} \times 20\text{ cm}$  square. This was simply achieved by taking away the outer scatterers as shown in Figure 3 (d,e,f). The frequency spectrum we obtained is globally similar to the previous one shown in Fig. 2 but certainly not superimposable. But the important point is that the peaks associated to the 3 modes shown above are still present with very little perturbed central frequencies and comparable sharpness. The two-dimensional scans of the amplitude of the corresponding modes are shown as (d,e,f) in Fig. 3. Obviously, the important modification of the boundaries of the scattering region did not affect the integrity of these modes that we can most legitimately qualify as localized.

For the sake of completeness, we also performed numerical simulations of the Maxwell equations in our system through a Finite-Difference Time-Domain (FDTD) method [18]. To mimic perfectly open boundary conditions Perfectly Matched Layer (PML) boundaries have been implemented [19]. These PML conditions are used in the same spirit as the absorbing microwave layers of the actual experiment. In the numerical simulations, losses are only due to the leakage through the boundary since no ohmic dissipation is included. Hence, spectral responses display sharper peaks than in the experiment but all the sharp resonances of the experiment are found at the corresponding frequencies. Fig. 4 shows a comparison between experimental and simulated modes. The agreement is spectacular. Note the presence of a non-vanishing field inside the scatterers, such an information being beyond the reach of our experimental measuring technique. As the calculated modes have precisely calculated exponentially vanishing tails far from their centers, we could extract values of their localization lengths  $\xi_{loc}$  by numerically evaluating the field amplitude autocorrelation. (Due to a small contribution from the squared magnetic field to the frequency shift, our experimental scanning technique is less accurate for this purpose.) The values of  $\xi_{loc}$  thus obtained are in complete agreement with the minimal values of  $\xi_{theory}$ . Moreover, the observed spectral widths  $\Gamma_{leak}$  due to leaking at the boundary, are also in fair agreement with the gross evaluation given by  $\Gamma_{leak} \simeq \exp(-2R/\xi_{loc})$  with  $R$  the distance from the center of the mode to the boundary [20]. More specifically, when reducing the scattering region, hence reducing  $R$ , we were able to ascertain the above relationship by extracting values of  $\Gamma_{leak}$  from the total experimental widths thanks to a proper evaluation of the contribution of ohmic losses to the latter [10]. All this confirms that the exponentially vanishing field of such modes at the boundary involves all the scatterers of the system



through complex multiple interference effects. In consequence, we can conclude that the modes we observe should not be simply considered as *defect-like* modes or *cavity* modes built by the mere presence of a few scatterers surrounding them. Thus, in spite of their high refraction index, all our results show that the scatterers we use should not be viewed as simple walls at the frequencies of the localized modes but as true resonators. This is numerically clearly illustrated by the presence of field maxima inside certain scatterers (see Fig. 4). To be complete, other realizations of the same type of disorder with the same scatterers have been studied and all the localized modes that were measured confirm the above results. This will be related at length in a forthcoming publication.

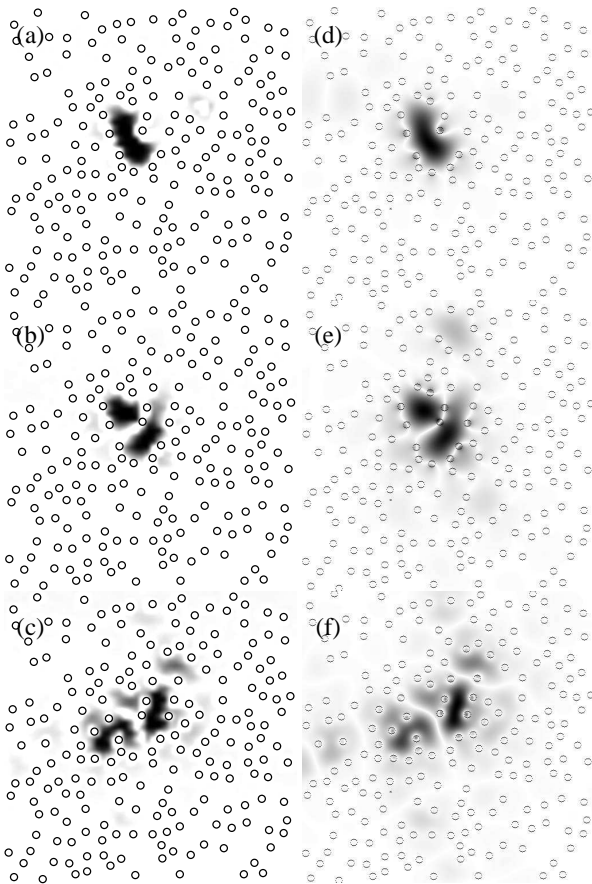


FIG. 4: Comparison between experimental results (a), (b), and (c) and numerical FDTD-based simulations (with perfectly open boundary conditions) (d), (e) and (f); The agreement is excellent.

In summary, we have provided strong evidences in favor of the first observation of truly localized modes in a two-dimensional open disordered microwave cavity. Most importantly, we have clearly shown how these localized modes are robust regarding strong modifications of the boundary of the scattering region. The fact that the

non-ohmic parts of the measured modal widths vary in inverse proportion of the exponential of the distance of these modes to the boundary also provides a strong argument in favor of strong localization in finite-size open systems and the way to probe it efficiently via measurements of decay rates.

We are very grateful to Gregory Sauder for the automation of the data acquisition. D. Laurent acknowledges financial support from DGA/CNRS Grant n. 2004487. This work is supported by the Groupement de Recherche IMCODE (CNRS n. 2253).

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