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Crossed Andreev reflection as a probe for the pairing symmetry of Ferromagnetic Superconductors

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The coexistence of superconductivity and ferromagnetism has brought about the phenomena of ferromagnetic superconductors. The theory needed to understand the compatibility of such antagonistic phenomena cannot be built until the pairing symmetry of such superconductors is correctly identified. The proper and unambiguous identification of the pairing symmetry of such superconductors is the subject of this paper. This work shows that crossed Andreev reflection can be a very effective tool in order to identify the pairing symmetry of these superconductors.

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Introduction: Recent experiments on certain materials like $RuSr_2GdCu_2O_8$ and UGe_8 have indicated that the previously thought to be antagonistic phenomenon of ferromagnetism and superconductivity can and do coexist[1]. This suggests that the pairing symmetry of such ferromagnetic superconductors may not be of the conventional BCS $s-wave$ singlet type. Indeed experiments in many rare earth and uranium compounds provide quite distinct characteristics of triplet superconductivity[1]. This is of-course not conclusive and more so in case of organic superconductors[2]. Currently there is much debate on the symmetry of the superconducting phase in organic compounds. There is of-course lot of experimental evidence for the triplet pairing in ruthenates. It is very unlikely for singlet superconductivity to appear in the ferromagnetic state because the exchange interaction forbids the formation of cooper pairs. Antiferromagnetic correlations lead to singlet pairing (with zero spin), while ferromagnetic correlations favor triplet pairing (with one unit of spin). Another physical system where triplet pairing occurs is superfluid Helium-3. Studies[4] of ^3He suggest that unconventional superconductivity will be highly anisotropic, i.e., it will depend strongly on the energy and momentum of the electrons. In ^3He, the cooper pairs have an orbital angular momentum and spin 1. Spin 0 objects have only one projection ($S_z = 0$), spin 1 objects have three projections ($S_z = -1, 0$ or 1). Superconductivity with spin 0 cooper pairs is therefore called singlet superconductivity and superconductivity with spin 1 cooper pairs is called triplet superconductivity. The singlet state is asymmetric under exchange of spin labels while the triplet state is symmetric. A magnetic field can destroy singlet superconductivity is two ways. The first of these effects is known as the orbital effect and is simply a manifestation of the lorentz force. Since the electrons in the copper pair have opposite momenta, the lorentz force acts in opposing directions and the pair breaks up. The second phenomenon, known as the paramagnetic effect, occurs when a strong magnetic field attempts to align the spins of both the electrons along the field direction. Singlet superconductivity is destroyed by fields greater than $\sim 1.8T_c$, where T_c is the critical temperature at which the material loses its electrical resistance. Such fields, however do not wreck triplet superconductivity because the spins of both electrons may point in the same direction as the field. This means that triplet superconductivity can only be destroyed by the orbital effect.

The aim of this work is to provide a novel method of detection of the pairing symmetry of ferromagnetic superconductors. Currently the accepted and prevalent views on this subject are that spin triplet pairing is the most likely symmetry. This assertion of-course is qualified with the caveat that at weak magnetic fields spin singlet pairing or non-uniform state (FFLO, oscillating order parameter) can also be justified. This work does not consider FFLO type states but it will be shown in the course of this work that a clearer, simple and more intuitive distinction between the singlet or triplet opposite spin pairing symmetry and the triplet equal spin pairing symmetry can be easily brought about by the phenomenon of crossed Andreev reflection(CAR). In addition to the distinctive features observed in case of equal and opposite spin pairing symmetries, the phases (A_1 and A_2) seen for equal spin pairing can also be directly identified using this phenomena[2]. This problem has been dealt with in literature by taking recourse to tunnelling spectroscopy, notably in Ref.[2], where the effect of finite temperatures is also taken into account. This problem has also been dealt with through quantum pumping spectroscopy in Ref.[5]. Tunnelling spectroscopy in Ref.[2], is not as robust as the phenomenon of CAR mentioned herein, since the distinction between the A_1 and A_2 phases has not been brought out in Ref.[2]. Furthermore experiments of quantum pumping are notoriously difficult and quantum pumping has not been unambiguously detected as yet. The case of CAR is already clear cut, theoretically(Ref.[4]) as well as experimentally(Ref.[6]). This work uses the physics of CAR[3]to identify correctly and unambiguously the pairing symmetry of ferromagnetic superconductors.

To explain the phenomena of CAR[6] we take recourse

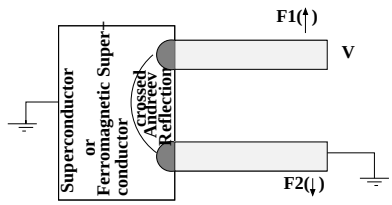

FIG. 1: Crossed Andreev reflection set-up. In the figure a bias is applied to Ferromagnetic lead F1, while Superconductor (or, Ferromagnetic Superconductor) and Ferromagnetic lead F2 are grounded.

to Fig. 1, in which two oppositely polarized ferromagnetic leads(half-metals) are coupled to a singlet (opposite spin pairing) non-magnetic superconductor. A voltage bias $eV < \Delta$ (the superconducting gap), is applied to the up-spin polarized ferromagnetic lead F1, while the down-spin polarized ferromagnetic lead F2 and the superconductor are grounded. In this case if the distance between the leads is less than the superconducting coherence length (ξ) then an up-spin electron incident in $F1$ with energy less than the superconducting gap will be completely retro-reflected as a hole in $F2$ with spin down(Ref.[4]). This is CAR. In the system as envisaged in Fig. 1 the barrier strengths at the lead-superconductor interfaces: $z_1 = z_2 = 0$. In this set-up the formula for the current in leads F1 and F2 at zero temperature are given as: $I_1 = I_0 \int_0^{eV}(1 - B_{11})dE,$ and $I_2 = I_0 \int_0^{eV} A_{12}dE$. In the preceding expressions for the currents , $I_0 = \frac{1}{2}N(0)ev_F\mathcal{A}$, with $N(0)$ being the density of states, v_F the Fermi velocity and $\mathcal{A}$ the contact area between lead and superconductor respectively. B_{11} is the probability for normal reflection and since applied bias is less than the superconducting gap and $z_1 = 0$, $B_{11} = 0$. A_{12} is the probability for CAR which in this case is 1, since there are no down-spin states in F1, the probability to be cross-reflected in F2 is unity. Thus, $I_1 = I_2 = I = I_0V$. One can also derive the current in the opposite case when a voltage bias $eV < \Delta$ (the superconducting gap) is applied to the down-spin polarized ferromagnetic lead F2, while the up-spin polarized ferromagnetic lead F1 and the superconductor are grounded. In this case too, $z_1 = z_2 = 0$. In this set-up the formula for the current in leads F1 and F2 at zero temperature are given as: $I_1 = \int_0^{eV} A_{21}dE,$ and $I_2 = I_0 \int_0^{eV}(1 - B_{22})dE$, B_{22} of-course is the probability for normal reflection and since applied bias is less than the superconducting gap and $z_2 = 0$, $B_{22} = 0$. A_{21} is the probability for CAR which in this case again is 1 because of the same reason. Thus, again $I_1 = I_2 = I = I_0V$.

Theory: As we have seen in the last paragraph of the introduction singlet s-wave superconductors show CAR. Now let us use the same set-up as envisaged in Fig. 1, but now with the superconductor replaced by a ferromagnetic superconductor. The hamiltonian of the ferromagnetic superconductor can be easily diagonalized by a Bogoliubov transformation and one can derive the Bogoliubov-deGennes equation for the ferromagnetic superconductor as written below. For the sake of brevity we only write below the Bogoliubov-de Gennes equation as in Eq. 1. The interested reader is referred to Ref.[7] for a full derivation of the following equation starting from the hamiltonian of the ferromagnetic superconductor.

$$\begin{pmatrix} \epsilon_k + h_z & 0 & -\Delta_{T,+1} & \Delta_S - \Delta_{T,0} \\ 0 & \epsilon_k - h_z & -\Delta_S - \Delta_{T,0} & -\Delta_{T,-1} \\ -\Delta^*_{T,+1} & -(\Delta^*_S + \Delta^*_{T,0}) & -(\epsilon_{-k} + h_z) & 0 \\ -(\Delta^*_S - \Delta^*_{T,0}) & -\Delta^*_{T,-1} & 0 & -\epsilon_{-k} + h_z \end{pmatrix} \begin{pmatrix} u_{\uparrow\sigma} \\ u_{\downarrow\sigma} \\ v_{\uparrow\sigma} \\ v_{\downarrow\sigma} \end{pmatrix} = E_\sigma \begin{pmatrix} u_{\uparrow\sigma} \\ u_{\downarrow\sigma} \\ v_{\uparrow\sigma} \\ v_{\downarrow\sigma} \end{pmatrix} \quad (1)$$

Herein we consider the magnetic field in the z-direction.The meaning of the various symbols in Eq. 1 are given below: $\Delta_S = \frac{1}{2}(\Delta_{\uparrow\downarrow} - \Delta_{\downarrow\uparrow})$, singlet opposite spin pairing; $\Delta_{T,0} = \frac{1}{2}(\Delta_{\uparrow\downarrow} + \Delta_{\downarrow\uparrow})$, triplet opposite spin pairing; $\Delta_{T,+1} = \Delta_{\uparrow\uparrow}$, triplet equal spin pairing; $\Delta_{T,-1} = \Delta_{\downarrow\downarrow}$, triplet equal spin pairing.

We analyze below four cases: (i) $\Delta_S \neq 0$, all others are zero (singlet opposite spin pairing); (ii) $\Delta_{T,0} \neq 0$, all others are zero (triplet opposite spin pairing); (iii) $\Delta_{T,+1} \neq 0, \Delta_{T,-1} \neq 0$, all others are zero [triplet equal spin pairing ($A2$ phase)]; and (iv) Either $\Delta_{T,+1} \neq 0$ and all others are zero or $\Delta_{T,-1} \neq 0$ and all others are zero, [triplet equal spin pairing ($A1$ phase)].

Case 1: Singlet opposite spin pairing coexisting with ferromagnetism: In this case Eq. 1, can be separated into two sets of equations:

$$\begin{pmatrix} \epsilon_k + h_z & \Delta_S \\ -\Delta^*_S & -\epsilon_{-k} + h_z \end{pmatrix} \begin{pmatrix} u_{\uparrow\uparrow} \\ v_{\downarrow\uparrow} \end{pmatrix} = E_\uparrow \begin{pmatrix} u_{\uparrow\uparrow} \\ v_{\downarrow\uparrow} \end{pmatrix}, \text{and} \quad (2)$$

$$\begin{pmatrix} \epsilon_k - h_z & -\Delta_S \\ -\Delta^*_S & -\epsilon_{-k} - h_z \end{pmatrix} \begin{pmatrix} u_{\downarrow\downarrow} \\ v_{\uparrow\downarrow} \end{pmatrix} = E_\downarrow \begin{pmatrix} u_{\downarrow\downarrow} \\ v_{\uparrow\downarrow} \end{pmatrix} \quad (3)$$

Solving Eqs. 2 and 3 for E_σ, we have: $E_\sigma = \sqrt{\epsilon_k^2 + |\Delta_S|^2} + \sigma h_z$ and the superconducting coherence factors $u_{\sigma\sigma}, v_{\sigma\sigma}$ are given by: $u^2_{\sigma\sigma} = \frac{1}{2}(1 + \frac{\epsilon_k}{E_0})$ and

$v^2_{\sigma\overline{\sigma}} = \frac{1}{2}(1 - \frac{\epsilon_k}{E_0})$. These coherence factors have the same form as in the non-magnetic superconductor. Hence, in the gap equation, exchange field enters only in the fermi function and at zero temperature the gap equation is independent of exchange field. Thus the results for the non-magnetic case are valid in this case too at zero temperature.

Case 2: Triplet opposite spin pairing coexisting with ferromagnetism: In this case Eq. 1 can be separated into two sets of equations:

$$\begin{pmatrix} \epsilon_k + h_z & -\Delta_{T,0} \\ -\Delta^*_{T,0} & -\epsilon_{-k} + h_z \end{pmatrix} \begin{pmatrix} u_{\uparrow\uparrow} \\ v_{\downarrow\uparrow} \end{pmatrix} = E_\uparrow \begin{pmatrix} u_{\uparrow\uparrow} \\ v_{\downarrow\uparrow} \end{pmatrix}, \text{and} \quad (4)$$

$$\begin{pmatrix} \epsilon_k - h_z & -\Delta_{T,0} \\ -\Delta^*_{T,0} & -\epsilon_k - h_z \end{pmatrix} \begin{pmatrix} u_{\downarrow\downarrow} \\ v_{\uparrow\downarrow} \end{pmatrix} = E_\downarrow \begin{pmatrix} u_{\downarrow\downarrow} \\ v_{\uparrow\downarrow} \end{pmatrix} \quad (5)$$

Solving Eqs. 4 and 5 for E_σ, we have: $E_\sigma = \sqrt{\epsilon_k^2 + |\Delta_{T,0}|^2} + \sigma h_z$ and the superconducting coherence factors $u_{\sigma\sigma}, v_{\sigma\overline{\sigma}}$ are given by: $u^2_{\sigma\sigma} = \frac{1}{2}(1 + \frac{\epsilon_k}{E_0})$ and $v^2_{\sigma\overline{\sigma}} = \frac{1}{2}(1 - \frac{\epsilon_k}{E_0})$. These coherence factors again have the same form as in the non-magnetic superconductor. Thus in the gap equation, exchange field enters only in the fermi function and at zero temperature the gap equation is independent of exchange field. This is exactly similar to the case of singlet opposite spin pairing coexisting with ferromagnetism. Thus the results for the non-magnetic case are valid in this case too at zero temperature. With this we go over to the final case study on triplet equal spin pairing symmetry.

Case 3: Triplet equal spin pairing coexisting with ferromagnetism: In this case Eq. 1, can be separated into two sets of equations:

$$\begin{pmatrix} \epsilon_k + h_z & -\Delta_{T,+1} \\ -\Delta^*_{T,+1} & -\epsilon_{-k} - h_z \end{pmatrix} \begin{pmatrix} u_{\uparrow\uparrow} \\ v_{\uparrow\uparrow} \end{pmatrix} = E_\uparrow \begin{pmatrix} u_{\uparrow\uparrow} \\ v_{\uparrow\uparrow} \end{pmatrix} \text{and} \quad (6)$$

$$\begin{pmatrix} \epsilon_k - h_z & -\Delta_{T,-1} \\ -\Delta^*_{T,-1} & -\epsilon_{-k} + h_z \end{pmatrix} \begin{pmatrix} u_{\downarrow\downarrow} \\ v_{\downarrow\downarrow} \end{pmatrix} = E_\downarrow \begin{pmatrix} u_{\downarrow\downarrow} \\ v_{\downarrow\downarrow} \end{pmatrix} \quad (7)$$

Solving Eqs. 6 and 7 for E_σ, we have: $E_\sigma = \sqrt{(\epsilon_k + \sigma h_z)^2 + |\Delta_{\sigma\sigma}|^2}$ and the superconducting coherence factors $u_{\sigma\sigma}, v_{\sigma\sigma}$ are given by: $u^2_{\sigma\sigma} = \frac{1}{2}(1 + \frac{\epsilon_k + \sigma h_z}{E_\sigma})$ and $v^2_{\sigma\sigma} = \frac{1}{2}(1 - \frac{\epsilon_k + \sigma h_z}{E_\sigma})$. In this case of-course as is evident from the coherence factors the exchange field does enter into the factors. Further since spin-up and spin-down are completely separated, there are two kinds of pairing state: (i) When only one kind of pairing is non-zero, i.e., either $\Delta_{\uparrow\uparrow} \neq 0$ or $\Delta_{\downarrow\downarrow} \neq 0$, which is identified as the $A1$ phase in the literature on spin triplet materials and (ii) when both kinds of pairing are non-zero which is identified as the $A2$ phase. One can as stated above further categorize the $A1$ phase. When only $\Delta_{\uparrow\uparrow} \neq 0$ and all others are zero we identify it as $A1_\uparrow$ phase and when only $\Delta_{\downarrow\downarrow} \neq 0$ and all others are zero we identify it as $A1_\downarrow$ phase. The above phases can be easily distinguished with the method of CAR as is shown below.

Case A: Triplet equal spin pairing $A1_\uparrow$ phase, with $\Delta_{\uparrow\uparrow} \neq 0$ and $\Delta_{\downarrow\downarrow} = 0$: We use the set-up as envisaged in Fig. 1. First we apply a bias $eV < \Delta_{\uparrow\uparrow}$ to lead F1 and keep the Ferromagnetic-Superconductor(FS) and lead F2 grounded. Since in this case the pairing is only between two electrons with up-spin, CAR into F2 is absent. There is only Andreev reflection (of a hole with same up-spin as the incoming electron) into the same lead, i.e., F1. Further since as before there are no interface barriers at the junction between leads and superconductor, the normal reflection at F1 is also absent. Thus, the currents in the two leads are: $I_1 = I_0 \int_0^{eV} (1 - B_{11} + A_{11}) dE$, and $I_2 = I_0 \int_0^{eV} A_{12} dE$. Since applied bias is less than the superconducting gap and $z_1 = 0, B_{11} = 0$. A_{11} the probability for normal reflection is 1. Further since there is complete absence of CAR $A_{12} = 0$. Thus, $I_1 = 2I = 2I_0V$ and $I_2 = 0$. One can also derive the current in the opposite case when a voltage bias $eV < \Delta_{\uparrow\uparrow}$ (the $A1_\uparrow$ phase superconducting gap) is applied to lead F2, while lead F1 and FS are grounded. In this case as before, $z_1 = z_2 = 0$. In this set-up, the formula for the current in leads F1 and F2 at zero temperature are given as: $I_1 = I_0 \int_0^{eV} A_{21} dE$, and $I_2 = I_0 \int_0^{eV} (1 - B_{22} + A_{22}) dE$. Since applied bias is less than the superconducting gap and $z_2 = 0, B_{22} = 0$. A_{22} is the probability for normal Andreev reflection which in this case is zero, since there are no up-spin states in F2 to get reflected into. Further the probability for CAR A_{21} is also nil, since in the $A1_\uparrow$ phase envisaged $\Delta_{\downarrow\downarrow} = 0$. This is because of the fact that F2 is completely down-spin polarized. Thus, $I_1 = 0$ and $I_2 = I = I_0V$.

Case B: Triplet equal spin pairing $A1_\downarrow$ phase, with $\Delta_{\uparrow\uparrow} = 0$ and $\Delta_{\downarrow\downarrow} \neq 0$: We use the set-up as envisaged in Fig. 1. First we apply a bias $eV < \Delta_{\downarrow\downarrow}$ to lead F1 and keep the FS and lead F2 grounded. Since in this case the pairing is only between two electron with down-spin, CAR into F2 is absent, for the same reasons Andreev reflection (of a hole with same up-spin as the incoming electron) into the same lead, i.e., F1 is also absent. Further since as before there are no interface barriers at the junction between leads and superconductor, the normal reflection at F1 is also absent. Thus, the currents in the two leads are: $I_1 = I_0 \int_0^{eV} (1 - B_{11} + A_{11}) dE$, and $I_2 = I_0 \int_0^{eV} A_{12} dE$. Since applied bias is less than the superconducting gap and $z_1 = 0$, $B_{11} = 0$. A_{11} is zero, since pairing only involves down-spin electrons. Further since there is complete absence of CAR, $A_{12} = 0$. Thus, $I_1 = I = I_0V$ and $I_2 = 0$. One can also derive the current in the opposite case when a voltage bias $eV < \Delta_{\downarrow\downarrow}$ (the $A1_\downarrow$ phase superconducting gap) is applied to lead F2, while lead F1 and the FS are grounded. In this case as before, $z_1 = z_2 = 0$. For this set-up the current in leads F1 and F2 at zero temperature are given as: $I_1 = I_0 \int_0^{eV} A_{21} dE$, and $I_2 = I_0 \int_0^{eV} (1 - B_{22} + A_{22}) dE$. Since applied bias is less than the superconducting gap and $z_2 = 0$, $B_{22} = 0$. A_{22} is 1. Further the probability for CAR A_{21} is nil, since the $A1_\downarrow$ phase envisages, $\Delta_{\uparrow\uparrow} = 0$.

This is of-course due to the fact that F2 is completely down-spin polarized, and there are no up-spin states in F2. Thus, $I_1 = 0$ and $I_2 = 2I = 2I_0V$. At the end we analyze the case wherein both $\Delta_{\uparrow\uparrow} \neq 0$ and $\Delta_{\downarrow\downarrow} \neq 0$, i.e., the $A2$ phase.

Case C: Triplet equal spin pairing $A2$ phase, with $\Delta_{\uparrow\uparrow} \neq 0$ and $\Delta_{\downarrow\downarrow} \neq 0$: We use the set-up as envisaged in Fig. 1. First we apply a bias $eV < \Delta_{\downarrow\downarrow}$(and $\Delta_{\uparrow\uparrow}$) to lead F1 and keep FS and lead F2 grounded. It should be noted that the applied bias should be less than both $\Delta_{\uparrow\uparrow}$ and $\Delta_{\downarrow\downarrow}$. Since in this case the pairing is both between two electrons with down-spin as well as between two electrons with up-spin, Andreev reflection (of a hole with same up-spin as the incoming electron) into the same lead, i.e., F1 is present. Further since as before there are no interface barriers at the junction between leads and superconductor, the normal reflection at F1 is absent. As far as CAR A_{12} into F2 is concerned, since F2 is completely down spin polarized and in this case incident electron with up-spin is incident in F1, $A_{12} = 0$. Thus, the currents in the two leads are: $I_1 = I_0 \int_0^{eV}(1 - B_{11} + A_{11})dE$, and $I_2 = I_0 \int_0^{eV} A_{12}dE$. Since applied bias is less than the super-conducting gap and $z_1 = 0$, $B_{11} = 0$. A_{11} in this case is 1, while as discussed above $A_{12} = 0$. Thus, $I_1 = 2I = 2I_0V$ and $I_2 = 0$. One can also derive the current in the opposite case when a voltage bias $eV < \Delta_{\downarrow\downarrow}$(and $\Delta_{\uparrow\uparrow}$) (the super-conducting gap) is applied to lead F2, while F1 and FS are grounded. In this case too, $z_1 = z_2 = 0$. In this set-up the formula for the current in leads F1 and F2 at zero temperature are thus given as: $I_1 = I_0 \int_0^{eV} A_{21}dE$, and $I_2 = I_0 \int_0^{eV}(1 - B_{22} + A_{22})dE$. Since applied bias is less than the super-conducting gap and $z_2 = 0$, $B_{22} = 0$. A_{22} in this case is 1. Further the probability for CAR A_{21} is nil, since F1 is purely up-spin polarized and there are no down-spin states. Hence, $I_1 = 0$, and $I_2 = 2I = 2I_0V$. Thus, the important message we get from the above analysis of singlet opposite spin pairing case and of the triplet opposite spin pairing case is not whether the pairing symmetry is triplet or singlet but whether it is opposite spin pairing or equal spin pairing which really matters.

Experimental Realization: The phenomenon of CAR has been demonstrated in two recent experiments[8]. To adapt it so that it can reveal the pairing symmetry of FS one has to replace the normal superconductor used in the aforementioned experiments with the FS. After this the two oppositely polarized Ferromagnetic leads are half metals. A proper choice for the purely up-spin polarized Ferromagnetic lead F1 could be Chromium Oxide CrO_2, while for the purely down-spin polarized Ferromagnetic lead F2 could be the Strontium alloy Sr_2FeMoO_6.

Conclusions: Finally we juxtapose all the results obtained in this work in Table 1. We see that the currents for all the cases bear the characteristic of the pairing symmetry considered. Especially one can clearly mark out the characteristic differences between $A1$ and $A2$ phases. Furthermore one can also make out which pairing (up/down) is non-zero in the $A1$ phase. This makes the phenomena of CAR ideal for the detection of the pairing symmetry of Ferromagnetic Superconductors. We also calculate the differential conductance $dG = dI/dV$ for the different symmetry classes. The differential conductance when bias is applied to F1 is just current in F2 divided by the voltage, and when bias is applied to F2 is just current in F1 divided by the voltage which can be inferred from table 1 by replacing I in the I_1/I_2 currents (depending upon to which lead bias is applied) by I_0. Two interesting quantities labelled in Ref.[3] as $I_T = I_1 - I_2$ and $I_G = I_1 + I_2$, the differential conductance from these quantities are $dG_T = dI_T/dV$ and $dG_G = dI_G/dV$, these quantities can also reveal the differences between the different symmetry classes as again pointed out in table 1.

TABLE I: Distinguishing opposite spin-pairing and equal-spin pairing in Ferromagnetic Superconductors

	Opposite Spin-Pairing (Singlet/Triplet)	*Equal Spin-Pairing (Triplet)*		
		A1 Phase		A2 Phase
		$A1_\uparrow(\Delta_{\uparrow\uparrow} \neq 0)$	$A1_\downarrow(\Delta_{\downarrow\downarrow} \neq 0)$	
Bias applied to F1	$I_1 = I$ $I_2 = I$ $dG_T = 0$ $dG_G = 2I_0$	$I_1 = 2I$ $I_2 = 0$ $dG_T = I_0$ $dG_G = 2I_0$	$I_1 = I$ $I_2 = 0$ $dG_T = I_0/2$ $dG_G = I_0$	$I_1 = 2I$ $I_2 = 0$ $dG_T = I_0$ $dG_T = 2I_0$
Bias applied to F2	$I_1 = I$ $I_2 = I$ $dG_T = 0$ $dG_G = 2I_0$	$I_1 = 0$ $I_2 = I$ $dG_T = -I_0/2$ $dG_G = I_0$	$I_1 = 0$ $I_2 = 2I$ $dG_T = -I_0$ $dG_G = 2I_0$	$I_1 = 0$ $I_2 = 2I$ $dG_T = -I_0$ $dG_G = 2I_0$

A perspective on the application is called for. First, if ferromagnets are not half-metals, then the situation changes depending on the polarization strength of the ferromagnets, see Ref.[9]. The effects of impurities have been dealt with in Ref.[10], where they showed that novel features like reflectionless tunnelling appear. This suggests CAR will be contaminated by elastic co-tunneling when ferromagnets are not half-metals. For half-metals impurities will not change anything qualitatively, i.e., when currents are zero they remain zero. Lastly temperature also wont change anything qualitatively, since temperature only appears in the fermi functions. To conclude, one can say for half-metals, impurities or finite temperature will not affect the currents qualitatively.

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[1] J. Flouquet and A. Buzdin, Physics World January 2002.
[2] C. J. Bolech, and T. Giamarchi, Phys. Rev. B **71**, 024517 (2005); Phys. Rev. Lett. **92**, 127001 (2004).
[3] J. M. Byers and M. E. Flatte, Phys. Rev. Lett. **74**, 306 (1995).
[4] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975); Rev. Mod. Phys. 76, 999 (2004).
[5] A. Brataas and Y. Tserkovnyak, Phys. Rev. Lett. **93**, 087201 (2004).
[6] G. Deutscher and D. Feinberg, Appl. Phys. Lett. **76**, 487 (2000).
[7] B. J. Powell, J. F. Annett and B. L. Gyorffy, J. Phys. A **36**, 9289 (2003); B. J. Powell, Ph. D Thesis, Bristol University (U.K.) (2002).
[8] D. Beckmann, H.B. Weber and H. v. Löhneysen, Phys. Rev. Lett. **93**, 197003 (2004); S. Russo, et. al., Phys. Rev. Lett. **95**, 027002 (2005).
[9] T. Yamashita, S. Takahashi and S. Maekawa, Phys. Rev. B **68**, 174504 (2003).
[10] S. Duhot and R. Melin, cond-mat/0601442.