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► **To cite this version:**

Patrick Le Meur. Topological invariants of piecewise hereditary algebras. Transactions of the American Mathematical Society, 2011, 363 (4), pp.2143–2170. 10.1090/S0002-9947-2010-05185-2 . hal-00131235v4

**HAL Id: hal-00131235**

**<https://hal.science/hal-00131235v4>**

Submitted on 26 Feb 2009

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# Topological invariants of piecewise hereditary algebras

Patrick Le Meur

February 26, 2009

## Abstract

We investigate the Galois coverings of piecewise algebras and more particularly their behaviour under derived equivalences. Under a technical assumption which is satisfied if the algebra is derived equivalent to a hereditary algebra, we prove that there exists a universal Galois covering whose group of automorphisms is free and depends only on the derived category of the algebra. As a corollary, we prove that the algebra is simply connected if and only if its first Hochschild cohomology vanishes.

## Introduction

Let  $k$  be an algebraically closed field and  $A$  a basic finite dimensional  $k$ -algebra (or, simply, an algebra). The representation theory studies the category  $\text{mod } A$  of finite dimensional (right)  $A$ -modules and also its bounded derived category  $\mathcal{D}^b(\text{mod } A)$ . From this point of view, some classes of algebras play an important rôle: The *hereditary* algebras, that is, path algebras  $kQ$  of finite quivers  $Q$  with no oriented cycle; the *tilted* algebras, that is, of the form  $\text{End}_{kQ}(T)$ , where  $T$  is a tilting  $kQ$ -module; and, more generally, the *piecewise hereditary* algebras, that is, the algebras  $A$  such that  $\mathcal{D}^b(\text{mod } A)$  is triangle equivalent to  $\mathcal{D}^b(\mathcal{H})$  where  $\mathcal{H}$  is a Hom-finite hereditary abelian category with split idempotents (if  $\mathcal{H} = \text{mod } kQ$  then  $A$  is called *piecewise hereditary of type  $Q$* ). These algebras are particularly well understood (see [2, 14, 26, 27], for instance).

The piecewise hereditary algebras arise in many areas of representation theory. For example, the cluster category  $C_A$  of a piecewise hereditary algebra was introduced in [5, 7] as a tool to study conjectures related to cluster algebras ([12]). Another example is the study of self-injective algebras, that is, algebras  $A$  such that  $A \simeq DA$  as right  $A$ -modules. Indeed, to any algebra  $A$  is associated the repetitive category  $\hat{A}$ , which is a Galois covering with group  $\mathbb{Z}$  of the trivial extension  $A \rtimes DA$  (see [18]). Assume that some group  $G$  acts freely on  $\hat{A}$  thus defining a Galois covering  $\hat{A} \rightarrow \hat{A}/G$  with group  $G$ . If  $\hat{A}/G$  is a finite dimensional algebra, that is, if it has finitely many objects as a category, then it is self-injective and called *of type  $Q$*  if  $A$  is tilted of type  $Q$ . It is proved in [28] that any self-injective algebra of polynomial growth and admitting a Galois covering by a strongly simply connected category is of the form  $\hat{A}/G$  for some tilted algebra  $A$  and some infinite cyclic group  $G$ . The class of self-injective algebras of type  $Q$  has been the object of many studies recently (see [30, 31, 32]).

In this text we investigate the Galois coverings of piecewise hereditary algebras. The Galois coverings of algebras and, more generally, of  $k$ -categories, were introduced in [13, 24] for the classification of representation-finite algebras. Consider  $A$  as a locally bounded  $k$ -category: If  $1 = e_1 + \dots + e_n$  is a decomposition of the unity into primitive orthogonal idempotents, then  $\text{ob}(A) = \{e_1, \dots, e_n\}$  and the space of morphisms from  $e_i$  to  $e_j$  is  $e_j A e_i$ . Then a *Galois covering* of the  $k$ -category  $A$  is a  $k$ -linear functor  $F: \mathcal{C} \rightarrow A$  where  $\mathcal{C}$  is a  $k$ -category endowed with a free action of  $G$ , that is,  $G$  acts freely on  $\text{ob}(\mathcal{C})$ , such that  $F \circ g = F$  for every  $g \in G$  and the induced functor  $\mathcal{C}/G \rightarrow A$  is an isomorphism ([13]). In such a situation,  $\text{mod } \mathcal{C}$  and  $\text{mod } A$  are related by the so-called push-down functor  $F_\lambda: \text{mod } \mathcal{C} \rightarrow \text{mod } A$ , that is, the extension-of-scalars functor. Often,  $F_\lambda$  allows nice comparisons between  $\text{mod } \mathcal{C}$  and  $\text{mod } A$ . For example: The action of  $G$  on  $\mathcal{C}$  naturally defines an action  $(g, X) \mapsto {}^g X$  of  $G$  on  $\mathcal{C}$ -modules. When this action is free on indecomposable  $\mathcal{C}$ -modules,  $F_\lambda$  defines an isomorphism of translation quivers between  $\Gamma(\text{mod } \mathcal{C})/G$  and a union of some components of the Auslander-Reiten quiver  $\Gamma(\text{mod } A)$  of  $A$  (see [11, 13]).

The comparisons allowed by the covering techniques raise naturally the following questions: Given an algebra  $A$ , is it possible to describe all the Galois coverings of  $A$  (in particular, does  $A$  admit a universal Galois covering, as happens in topology)? Is it possible to characterise the simple connectedness of  $A$  (that is, the fact that  $A$  has no proper Galois covering by a connected and locally bounded  $k$ -category)? In view of the above discussion on self-injective algebras, these questions are particularly relevant when  $A$  is piecewise hereditary of type  $Q$ . In case  $A = kQ$ , the answers are well-known: The Galois coverings of  $kQ$  correspond to the ones of the underlying graph of  $Q$ ; and  $kQ$  is simply connected if and only if  $Q$  is a tree, which is also equivalent to the vanishing of the first Hochschild cohomology group  $\text{HH}^1(kQ)$  ([8]). Keeping in mind the general objective of representation theory, one can wonder if the data of the Galois coverings of  $A$  and the simple connectedness of  $A$  depend only on the bounded derived category  $\mathcal{D}^b(\text{mod } A)$ . Again, it is natural to treat this problem for piecewise hereditary algebras. Up to now, there are no general solutions to the above problems. The question of the description of the Galois coverings and the one of the characterisation of simple connectedness have found a satisfactory answer in the case of standard representation-finite algebras (see [6, 13]). This is mainly due to the fact that the Auslander-Reiten quiver is connected and completely describes the module category in this case. However,

the infinite-representation case seems to be more complicated. As an example, there exist string algebras which admit no universal Galois covering ([22]). In the present text, we study the above problems when  $A$  is piecewise hereditary. As a main result, we prove the following theorem.

**Theorem A.** *Let  $A$  be a connected algebra derived equivalent to a hereditary abelian category  $\mathcal{H}$  whose oriented graph  $\vec{\mathcal{K}}_{\mathcal{H}}$  of tilting objects is connected. Then  $A$  admits a universal Galois covering  $\widehat{\mathcal{C}} \rightarrow A$  with group a free group  $\pi_1(A)$  uniquely determined by  $\mathcal{D}^b(\text{mod } A)$ . This means that  $\widehat{\mathcal{C}}$  is connected and locally bounded and for any Galois covering  $\mathcal{C} \rightarrow A$  with group  $G$  where  $\mathcal{C}$  is connected and locally bounded there exists a commutative diagram:*

$$\begin{array}{ccc} \widehat{\mathcal{C}} & & \mathcal{C} \\ \downarrow & \searrow & \downarrow \\ A & \xrightarrow{\sim} & A \end{array}$$

where the bottom horizontal arrow is an isomorphism extending the identity map on  $\text{ob}(A)$ . Moreover,  $\widehat{\mathcal{C}} \rightarrow \mathcal{C}$  is Galois with group  $N$  such that there is an exact sequence of groups  $1 \rightarrow N \rightarrow \pi_1(A) \rightarrow G \rightarrow 1$ .

Finally, if  $A$  is hereditary of type  $Q$  then  $\pi_1(A)$  is the fundamental group  $\pi_1(Q)$  of the underlying graph of  $Q$  and, otherwise, the rank of  $\pi_1(A)$  equals  $\dim_k \text{HH}^1(A)$  (which is 0 or 1).

We refer the reader to the next section for the definition of  $\vec{\mathcal{K}}_{\mathcal{H}}$ . Recall ([17]) that the assumption on  $A$  is satisfied if  $A$  is piecewise hereditary of type  $Q$ .

The above theorem implies that the Galois coverings of a piecewise hereditary algebra are determined by the factor groups of  $\pi_1(Q)$ . Also it shows that the data of the Galois coverings is an invariant of the derived category. Therefore so does the simple connectedness. Using the fact that the Hochschild cohomology is invariant under derived equivalences (see [19]), we deduce the following corollary of our main result.

**Corollary B.** *Let  $A$  be as in Theorem A. The following are equivalent:*

- (a)  $A$  is simply connected.
- (b)  $\text{HH}^1(A) = 0$ .

If  $A$  is piecewise hereditary of type  $Q$ , then (a) and (b) are also equivalent to:

- (c)  $Q$  is a tree.

This corollary generalises some of the results of [1, 3] which studied the same characterisation for tilted algebras of euclidean type and for tame tilted algebras. Also, it gives a new class of algebras for which the following question of Skowroński ([29, Pb. 1]) has a positive answer: *Is  $A$  simply connected if and only if  $\text{HH}^1(A) = 0$ ?* Originally, this question was asked for tame triangular algebras.

The methods we use to prove Theorem A allow us to prove the last main result of this text. It shows that the Galois coverings have a nice behaviour for piecewise hereditary algebras.

**Theorem C.** *Let  $A$  be piecewise hereditary of type  $Q$  and  $F: \mathcal{C} \rightarrow A$  be a Galois covering with group  $G$  where  $\mathcal{C}$  is connected and locally bounded. Then  $\mathcal{C}$  is piecewise hereditary of type a quiver  $Q'$  such that there exists a Galois covering of quivers  $Q' \rightarrow Q$  with group  $G$ .*

We now give some explanations on the proof of Theorem A. For unexplained notions, we refer the reader to the next section. Assume that  $A$  is piecewise hereditary. It is known from [16, Thm. 2.6] that there exists an algebra  $B$  such that  $A \simeq \text{End}_{\mathcal{D}^b(\text{mod } B)}(X)$  for some tilting complex  $X \in \mathcal{D}^b(\text{mod } B)$  and such that  $B$  has one of the following forms:

1.  $B = kQ$ , with  $Q$  a finite quiver with no oriented cycle.
2.  $B$  is a squid algebra.

It is easy to check that Theorem A holds true for path algebras of quivers and for squid algebras. Therefore we are reduced to proving that Theorem A holds true for  $A$  and only if it holds true for  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  for any tilting complex  $T \in \mathcal{D}^b(\text{mod } A)$ . Roughly speaking, we need a correspondence between the Galois coverings of  $A$  and those of  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$ . Therefore we use a construction introduced in [21] for tilting modules: Given a Galois covering  $F: \mathcal{C} \rightarrow A$  with group  $G$ , the push-down functor  $F_{\lambda}: \text{mod } \mathcal{C} \rightarrow \text{mod } A$  is exact and therefore induces an exact functor  $F_{\lambda}: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$ . Also, the  $G$ -action on modules extends to a  $G$ -action on  $\mathcal{D}^b(\text{mod } \mathcal{C})$  by triangle automorphisms. Now, let  $T \in \mathcal{D}^b(\text{mod } A)$  be a tilting complex and  $T = T_1 \oplus \dots \oplus T_n$  be an indecomposable decomposition. Assume that the following conditions hold true for every  $i \in \{1, \dots, n\}$ :

- (H<sub>1</sub>) There exists an indecomposable  $\mathcal{C}$ -module  $\widetilde{T}_i$  such that  $F_{\lambda}\widetilde{T}_i = T_i$ .
- (H<sub>2</sub>) The stabiliser  $\{g \in G \mid {}^g\widetilde{T}_i \simeq \widetilde{T}_i\}$  is the trivial group.

Under these assumptions, the complexes  ${}^g\widetilde{T}_i$  (for  $g \in G$  and  $i \in \{1, \dots, n\}$ ) form a full subcategory of  $\mathcal{D}^b(\text{mod } \mathcal{C})$  which we denote by  $\text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})}(\widetilde{T})$ . Then  $F_{\lambda}: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$  induces a Galois covering with group  $G$ :

$$\begin{array}{ccc} \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})}(\widetilde{T}) & \rightarrow & \text{End}_{\mathcal{D}^b(\text{mod } A)}(T) \\ {}^g\widetilde{T}_i & \mapsto & T_i \\ {}^g\widetilde{T}_i \xrightarrow{u} {}^h\widetilde{T}_j & \mapsto & T_i \xrightarrow{F_{\lambda}(u)} T_j \end{array} .$$

Hence  $(H_1)$  and  $(H_2)$  are technical conditions which allow one to associate a Galois covering of  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  to a Galois covering of  $A$ . In particular, if  $A$  admits a universal Galois covering, then the associated Galois covering of  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  is a good candidate for being a universal Galois covering. This is indeed the case provided that the following technical condition is satisfied:

$(H_3)$  If  $\psi: A \xrightarrow{\sim} A$  is an automorphism such that  $\psi(x) = x$  for every  $x \in \text{ob}(A)$ , then  $\psi_\lambda T_i \simeq T_i$ , for every  $i$ .

We therefore need to prove the assertions  $(H_1)$ ,  $(H_2)$  and  $(H_3)$  for every Galois covering  $F: \mathcal{C} \rightarrow A$  and every tilting complex  $T \in \mathcal{D}^b(\text{mod } A)$ .

The text is therefore organised as follows. In Section 1, we recall some useful definitions and fix some notations. In Section 2, we define the exact functor  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$  associated to a Galois covering  $F: \mathcal{C} \rightarrow A$ . In Section 3, we introduce elementary transformations on tilting complexes using approximations. The main result of the section asserts that for  $A$  piecewise hereditary of type  $Q$  and for any tilting complexes  $T, T'$ , there exists a sequence of elementary transformations relating  $T$  and  $T'$ . We prove the assertions  $(H_1)$ ,  $(H_2)$  and  $(H_3)$  in Section 4 using the elementary transformations. We prove Theorem C as an application of these results. Then, in Section 5, we establish a correspondence between the Galois coverings of  $A$  and those of  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  for every tilting complex  $T$ . Finally, we prove Theorem A and Corollary B in Section 6.

## 1 Definitions and notations

### Modules over $k$ -categories

We refer the reader to [4] for the definition of  $k$ -categories and locally bounded  $k$ -categories. All locally bounded  $k$ -categories are assumed to be small and all functors between  $k$ -categories are assumed to be  $k$ -linear (our module categories and derived categories will be skeletally small). Let  $\mathcal{C}$  be a  $k$ -category. Following [4], a (right)  $\mathcal{C}$ -module is a  $k$ -linear functor  $M: \mathcal{C}^{op} \rightarrow \text{MOD } k$  where  $\text{MOD } k$  is the category of  $k$ -vector spaces. The category of  $\mathcal{C}$ -modules is denoted by  $\text{MOD } \mathcal{C}$ . A module  $M \in \text{MOD } \mathcal{C}$  is called *finite dimensional* if  $\sum_{x \in \text{ob}(\mathcal{C})} \dim_k M(x) < \infty$ .

The category of finite dimensional  $\mathcal{C}$ -modules is denoted by  $\text{mod } \mathcal{C}$ . Note that the indecomposable projective  $\mathcal{C}$ -module associated to  $x \in \text{ob}(\mathcal{C})$  is the representable functor  $\mathcal{C}(-, x)$ . The projective dimension of a  $\mathcal{C}$ -module  $X$  is denoted by  $\text{pd}_{\mathcal{C}}(X)$ . If  $X \in \text{mod } \mathcal{C}$ , then  $\text{add}(X)$  denotes the smallest full subcategory of  $\text{mod } \mathcal{C}$  closed under direct summands and direct sums. We refer the reader to [2] for notions on tilting theory. If  $A$  is an algebra, an  $A$ -module  $T$  is called *tilting* if: (a)  $T$  is multiplicity-free; (b)  $\text{pd}_A(T) \leq 1$ ; (c)  $\text{Ext}_A^1(T, T) = 0$ ; (d) for every indecomposable projective  $A$ -module  $P$  there is an exact sequence  $0 \rightarrow P \rightarrow X \rightarrow Y \rightarrow 0$  in  $\text{mod } A$  where  $X, Y \in \text{add}(T)$ . Let  $\mathcal{H}$  be a hereditary abelian category. An object  $T \in \mathcal{H}$  is called *tilting* (see [16]) if: (a)  $T$  is multiplicity-free; (b)  $\text{Ext}_{\mathcal{H}}^1(T, T) = 0$ ; (c) whenever  $\text{Hom}_{\mathcal{H}}(T, X) = \text{Ext}_{\mathcal{H}}^1(T, X) = 0$  for  $X \in \mathcal{H}$ , then  $X = 0$ . The set of isomorphism classes of tilting objects in  $\mathcal{H}$  has a partial order such that  $T \leq T'$  if and only if  $\text{Fac } T \subseteq \text{Fac } T'$  where  $\text{Fac } T$  is the class of epimorphic images of direct sums of copies of  $T$ . The Hasse diagram of this poset is denoted by  $\overrightarrow{\mathcal{K}}_{\mathcal{H}}$  and called the *oriented graph of tilting objects in  $\mathcal{H}$*  (see [17] for more details).

If  $\mathcal{A}$  is an additive category, then  $\text{ind } \mathcal{A}$  denotes the full subcategory of all indecomposable objects of  $\mathcal{A}$ .

### Galois coverings of $k$ -categories

Let  $F: \mathcal{E} \rightarrow \mathcal{B}$  be a Galois covering with group  $G$  between  $k$ -categories (see the introduction). It is called *connected* if both  $\mathcal{C}$  and  $\mathcal{B}$  are connected and locally bounded. Let  $A$  be a connected and locally bounded  $k$ -category and  $x_0 \in \text{ob}(A)$ . A *pointed Galois covering*  $F: (\mathcal{C}, x) \rightarrow (A, x_0)$  is a connected Galois covering  $F: \mathcal{C} \rightarrow A$  endowed with  $x \in \text{ob}(\mathcal{C})$  such that  $F(x) = x_0$ . A *morphism of pointed Galois coverings*  $F \xrightarrow{u} F'$  from  $F: (\mathcal{C}, x) \rightarrow (A, x_0)$  to  $F': (\mathcal{C}', x') \rightarrow (A, x_0)$  is a functor  $u: \mathcal{C} \rightarrow \mathcal{C}'$  such that  $F' \circ u = F$  and  $u(x) = x'$ . Note that, given  $F$  and  $F'$ , there is at most one such morphism (see [20, Lem. 3.1]). This defines the category  $\text{Gal}(A, x_0)$  of pointed Galois coverings. If  $F \in \text{Gal}(A, x_0)$ , then we let  $F^{\rightarrow}$  be the full subcategory of  $\text{Gal}(A, x_0)$  with objects those  $F'$  such that there exists a morphism  $F \rightarrow F'$ .

### Covering properties on module categories (see [4, 24])

Let  $F: \mathcal{E} \rightarrow \mathcal{B}$  be a Galois covering with group  $G$ . The  $G$ -action on  $\mathcal{E}$  defines a  $G$ -action on  $\text{MOD } \mathcal{E}$ : If  $M \in \text{MOD } \mathcal{E}$  and  $g \in G$ , then  ${}^g M := F \circ g^{-1} \in \text{MOD } \mathcal{E}$ . If  $X \in \text{MOD } \mathcal{E}$ , the *stabiliser* of  $X$  is the subgroup  $G_X := \{g \in G \mid {}^g X \simeq X\}$  of  $G$ . The Galois covering  $F$  defines two exact functors: The extension-of-scalars functor  $F_\lambda: \text{MOD } \mathcal{E} \rightarrow \text{MOD } \mathcal{B}$  which is called the *push-down functor* and the restriction-of-scalars functor  $F: \text{MOD } \mathcal{B} \rightarrow \text{MOD } \mathcal{E}$  which is called the *pull-up functor*. They form an adjoint pair  $(F_\lambda, F)$  and  $F_\lambda$  is  $G$ -invariant, that is,  $F_\lambda \circ g = F_\lambda$  for every  $g \in G$ . We refer the reader to [4] for details on  $F_\lambda$  and  $F$ . For any  $M, N \in \text{mod } \mathcal{E}$ , the following maps induced by  $F_\lambda$  are bijective:

$$\bigoplus_{g \in G} \text{Hom}_{\mathcal{E}}({}^g M, N) \rightarrow \text{Hom}_{\mathcal{B}}(F_\lambda M, F_\lambda N) \text{ and } \bigoplus_{g \in G} \text{Hom}_{\mathcal{E}}(M, {}^g N) \rightarrow \text{Hom}_{\mathcal{B}}(F_\lambda M, F_\lambda N).$$

An indecomposable module  $X \in \text{mod } \mathcal{B}$  is called *of the first kind with respect to  $F$*  if and only if  $F_\lambda \tilde{X} \simeq X$  for some  $\tilde{X} \in \text{mod } \mathcal{E}$  (necessarily indecomposable). In such a case, one may choose  $\tilde{X}$  such that  $F_\lambda \tilde{X} = X$ . More generally,  $X \in \text{mod } \mathcal{B}$  is called *of the first kind with respect to  $F$*  if and only if it is the direct sum of indecomposable  $\mathcal{B}$ -modules of the first kind with respect to  $F$ .

## 2 Covering techniques on the bounded derived category

Let  $F: \mathcal{C} \rightarrow \mathcal{A}$  be a Galois covering with group  $G$  and with  $\mathcal{C}$  and  $\mathcal{A}$  locally bounded categories of finite global dimension. The  $G$ -action on  $\text{mod } \mathcal{C}$  naturally defines a  $G$ -action on  $\mathcal{D}^b(\text{mod } \mathcal{C})$ , still denoted by  $(g, M) \mapsto {}^g M$ , by triangle automorphisms. We introduce an exact functor  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  induced by  $F_\lambda: \text{mod } \mathcal{C} \rightarrow \text{mod } \mathcal{A}$ .

**Proposition 2.1.** *There exists an exact functor  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  such that the following diagram commutes:*

$$\begin{array}{ccc} \text{mod } \mathcal{C} & \longrightarrow & \mathcal{D}^b(\text{mod } \mathcal{C}) \\ F_\lambda \downarrow & & \downarrow F_\lambda \\ \text{mod } \mathcal{A} & \longrightarrow & \mathcal{D}^b(\text{mod } \mathcal{A}) \end{array} .$$

The functor  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  has the covering property, that is, it is  $G$ -invariant and the two following maps are linear bijections for every  $M, N \in \mathcal{D}^b(\text{mod } \mathcal{C})$ :

$$\begin{aligned} \bigoplus_{g \in G} \mathcal{D}^b(\text{mod } \mathcal{C})({}^g M, N) &\xrightarrow{F_\lambda} \mathcal{D}^b(\text{mod } \mathcal{A})(F_\lambda M, F_\lambda N), \\ \text{and } \bigoplus_{g \in G} \mathcal{D}^b(\text{mod } \mathcal{C})(M, {}^g N) &\xrightarrow{F_\lambda} \mathcal{D}^b(\text{mod } \mathcal{A})(F_\lambda M, F_\lambda N). \end{aligned}$$

**Proof:** The existence and exactness of  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  follow from the exactness of  $F_\lambda: \text{mod } \mathcal{C} \rightarrow \text{mod } \mathcal{A}$ . On the other hand,  $F_\lambda$  induces an additive functor  $F_\lambda: \mathcal{K}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{K}^b(\text{mod } \mathcal{A})$  between bounded homotopy categories of complexes. It easily checked that it has the covering property in the sense of the proposition. Since  $\mathcal{A}$  and  $\mathcal{C}$  have finite global dimension, we deduce that  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  has the covering property.  $\square$

*Remark 2.2.* It follows from the preceding proposition that  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  is faithful.

We are mainly interested in indecomposable objects  $X \in \mathcal{D}^b(\text{mod } \mathcal{A})$  which are of the form  $F_\lambda \tilde{X}$  for some  $\tilde{X} \in \mathcal{D}^b(\text{mod } \mathcal{C})$ . The following shows that the possible objects  $\tilde{X}$  lie in the same  $G$ -orbit for a given  $X$ .

**Lemma 2.3.** *Let  $X, Y \in \mathcal{D}^b(\text{mod } \mathcal{C})$  be such that  $F_\lambda X$  and  $F_\lambda Y$  are indecomposable and isomorphic in  $\mathcal{D}^b(\text{mod } \mathcal{A})$ . Then  $X \simeq {}^g Y$  for some  $g \in G$ .*

**Proof:** Let  $u: F_\lambda X \rightarrow F_\lambda Y$  be an isomorphism in  $\mathcal{D}^b(\text{mod } \mathcal{A})$ . By 2.1, there exists  $(u_g)_g \in \bigoplus_{g \in G} \mathcal{D}^b(\text{mod } \mathcal{C})(X, {}^g Y)$  such that  $u = \sum_{g \in G} F_\lambda(u_g)$ . Since  $F_\lambda X$  and  $F_\lambda Y$  are indecomposable, there exists  $g_0 \in G$  such that  $F_\lambda(u_{g_0}): F_\lambda X \rightarrow F_\lambda Y$  is an isomorphism. Since  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{A})$  is exact and faithful,  $u_{g_0}: X \rightarrow {}^{g_0} Y$  is an isomorphism in  $\mathcal{D}^b(\text{mod } \mathcal{C})$ .  $\square$

## 3 Transforming tilting complexes into tilting modules

Let  $\mathcal{H}$  be a hereditary abelian category over  $k$  with finite dimensional Hom-spaces, split idempotents and tilting objects. Let  $n$  the rank of its Grothendieck group. For short, we set  $\text{Hom} := \text{Hom}_{\mathcal{D}^b(\mathcal{H})}$  and  $\text{Ext}^i(X, Y) := \text{Hom}_{\mathcal{D}^b(\mathcal{H})}(X, Y[i])$ . We write  $\mathcal{T}$  for the class of objects  $T \in \mathcal{D}^b(\mathcal{H})$  such that:

- (a)  $T$  is multiplicity-free and has  $n$  indecomposable summands.
- (b)  $\text{Ext}^i(T, T) = 0$  for every  $i \geq 1$ .

We identify an object in  $\mathcal{T}$  with its isomorphism class. A complex  $T$  lies in  $\mathcal{T}$  if and only if  $T[1] \in \mathcal{T}$ . Also, all tilting complexes in  $\mathcal{D}^b(\mathcal{H})$  and, therefore, all tilting objects in  $\mathcal{H}$ , lie on  $\mathcal{T}$ . Given  $T \in \mathcal{D}^b(\mathcal{H})$ , we let  $\langle T \rangle$  be the smallest full subcategory of  $\mathcal{D}^b(\mathcal{H})$  containing  $T$  and stable under direct sums, direct summands and shifts in either direction. The aim of this section is to define elementary transformations on objects in  $\mathcal{T}$  which, by repetition, allow one to relate any two objects in  $\mathcal{T}$ . For this purpose, we introduce some notation. Given  $T \in \mathcal{T}$ , we have a unique decomposition  $T = Z_0[i_0] \oplus Z_1[i_0 + 1] \oplus \dots \oplus Z_l[i_0 + l]$  where  $Z_0, \dots, Z_l \in \mathcal{H}$  and  $Z_0, Z_l \neq 0$ . Here,  $Z_i$  needs not be indecomposable. We let  $r(T)$  be the number of indecomposable summands of  $Z_1[i_0 + 1] \oplus \dots \oplus Z_l[i_0 + l]$ . Note that:  $r(T) \in \{0, \dots, n - 1\}$ ;  $r(T) = 0$  if and only if  $T[-i_0]$  is a tilting object in  $\mathcal{H}$ ; and  $r(T) = r(T[1])$ . We are interested in transformations which map an object  $T \in \mathcal{T}$  to  $T'$  such that  $r(T') < r(T)$ . Hence, by repeating the process, we may end up with a tilting object in  $\mathcal{H}$  (up to a shift).

### Transformations of the first kind

Our first elementary transformation is given by the following lemma.

**Lemma 3.1.** *Let  $T \in \mathcal{T}$ . There exists  $T' \in \mathcal{T}$  such that  $T' \in \langle T \rangle$ ,  $r(T') \leq r(T)$  and  $T' = Z'_0 \oplus Z'_1[1] \oplus \dots \oplus Z'_l[l']$  where:*

- (a)  $Z'_0, \dots, Z'_{l'} \in \mathcal{H}$  and  $Z'_0, Z'_{l'} \neq 0$ .  
(b)  $\text{Hom}(Z'_0, Z'_1[1]) \neq 0$  if  $l' \neq 0$ .

**Proof:** Given  $T' \in \langle T \rangle$ , we have the unique decomposition  $T' = Z'_0[i'_0] \oplus Z'_1[i'_0 + 1] \oplus \dots \oplus Z'_{l'}[i'_0 + l']$  as explained at the beginning of the section. We choose  $T' \in \langle T \rangle \cap \mathcal{T}$  such that  $r(T') \leq r(T)$  and such that the pair  $(l', r(T'))$  is minimal for the lexicographical order. We may assume that  $i_0 = 0$ . We prove that  $T'$  satisfies (a) and (b). If  $l' = 0$ , there is nothing to prove. So we assume that  $l' > 0$ . Assume first that  $Z'_1 = 0$ . Then we let  $T''$  be as follows:

$$T'' := Z'_0 \oplus Z'_2[1] \oplus Z'_3[2] \oplus \dots \oplus Z'_{l'}[l' - 1].$$

Then  $T'' \in \langle T' \rangle = \langle T \rangle$ . Also,  $\text{Ext}^i(T'', T'') = 0$  for every  $i \geq 1$  because  $T' \in \mathcal{T}$  and  $\mathcal{H}$  is hereditary. Finally,  $T''$  is the direct sum of  $n$  pairwise non isomorphic indecomposable objects. Thus,  $T'' \in \langle T \rangle \cap \mathcal{T}$  and  $(l' - 1, r(T'')) < (l', r(T'))$  which contradicts the minimality of  $(l', r(T'))$ . So  $Z'_1 \neq 0$ . Now, assume that  $\text{Hom}(Z'_0, Z'_1[1]) = 0$ . We let  $T''$  be the following object:

$$T'' := (Z'_0 \oplus Z'_1) \oplus Z'_2[2] \oplus Z'_3[3] \oplus \dots \oplus Z'_{l'}[l'].$$

As above, we have  $T'' \in \langle T \rangle \cap \mathcal{T}$  and  $(l', r(T'')) < (l', r(T'))$  which contradicts the minimality of  $(l', r(T'))$ . So  $\text{Hom}(Z'_0, Z'_1[1]) \neq 0$ .  $\square$

With the notations of 3.1, we say that  $T$  and  $T'$  are related by a *transformation of the first kind*.

### Transformations of the second kind

We now turn to the second elementary transformation. It is inspired by the characterisation of the quiver of tilting objects in hereditary categories (see [17] and also [5] for the corresponding construction in cluster categories). Let  $T, T' \in \mathcal{T}$  be such that  $T = X \oplus \overline{T}$  with  $X$  indecomposable,  $T' = Y \oplus \overline{T}$  with  $Y$  indecomposable and there exists a triangle  $X \xrightarrow{u} M \xrightarrow{v} Y \rightarrow X[1]$  such that  $u$  is a left minimal  $\text{add}(\overline{T})$ -approximation or  $v$  is a right minimal  $\text{add}(\overline{T})$ -approximation. In such a situation, we say that  $T$  and  $T'$  are related by a *transformation of the second kind*.

*Remark 3.2.* Following [17], if  $T \rightarrow T'$  is an arrow in  $\overrightarrow{\mathcal{K}}_{\mathcal{H}}$  then  $T$  and  $T'$  are related by a transformation of the second kind.

Note that, with the previous notations, both  $u$  and  $v$  are minimal  $\text{add}(\overline{T})$ -approximations, as shows the following lemma.

**Lemma 3.3.** *Let  $T \in \mathcal{T}$ . Assume that  $T = X \oplus \overline{T}$  with  $X$  indecomposable.*

- (a) *Let  $X \xrightarrow{u} M \xrightarrow{v} Y \rightarrow X[1]$  be a triangle where  $u$  is a left minimal  $\text{add}(\overline{T})$ -approximation. Then  $v$  is a right minimal  $\text{add}(\overline{T})$ -approximation.*  
(b) *Let  $Y \xrightarrow{u} M \xrightarrow{v} X \rightarrow Y[1]$  be a triangle where  $v$  is a right minimal  $\text{add}(\overline{T})$ -approximation. Then  $u$  is a left minimal  $\text{add}(\overline{T})$ -approximation.*

**Proof:** We only prove (a) because the proof of (b) is similar. Every morphism  $\overline{T} \rightarrow Y$  factorises through  $v$  because  $\text{Hom}(\overline{T}, X[1]) = 0$ . So  $v$  is a right  $\text{add}(\overline{T})$ -approximation. Let  $\alpha: M \rightarrow M$  be a morphism such that  $v\alpha = v$ . So there exists  $\lambda: M \rightarrow X$  such that  $u\lambda = \alpha - \text{Id}_M$ . Note that  $u$  is not a section because  $T$  is multiplicity-free. So  $u\lambda$  is nilpotent and  $\alpha = \text{Id}_M + u\lambda$  is an isomorphism. Therefore  $v$  is right minimal.  $\square$

It is not true that any two objects  $T, T' \in \mathcal{T}$  can be related by a sequence of transformations of second kind (whereas this is the case, for example, for tilting objects in a cluster category, see [5]). However, we have the following result from [17].

**Proposition 3.4.** *Assume that at least one of the two following assertions is true:*

- (a)  $\mathcal{H} = \text{mod } kQ$  where  $Q$  is a finite connected quiver without oriented cycles and of Dynkin type.  
(b)  $\mathcal{H}$  has no non-zero projective object and  $\mathcal{D}^b(\mathcal{H})$  is triangle equivalent to  $\mathcal{D}^b(\text{mod } kQ)$  with  $Q$  a connected finite quiver without oriented cycles.

*Then  $\overrightarrow{\mathcal{K}}_{\mathcal{H}}$  is connected. In particular (see 3.2) for every tilting objects  $T, T' \in \mathcal{H}$ . There exists a sequence  $T = T_0, \dots, T_l = T'$  of tilting objects in  $\mathcal{H}$  such that  $T_i$  and  $T_{i+1}$  are related by a transformation of the second kind for every  $i$ .*

We are going to prove that any  $T \in \mathcal{T}$  can be related to some tilting object in  $\mathcal{H}$  by a sequence of transformations of the first or of the second kind. Let  $T \in \mathcal{T}$ . With the notations established at the beginning of the section, assume that  $\text{Hom}(Z_0, Z_1[1]) \neq 0$ . Since the ordinary quiver of  $\text{End}(T)$  has no oriented cycle, there exists  $M \in \text{add}(Z_1[i_0 + 1])$  indecomposable such that:

1.  $\text{Hom}(Z_0[i_0], M) \neq 0$ .
2.  $\text{Hom}(Z, M) = 0$  for any indecomposable direct summand  $Z$  of  $\bigoplus_{t=1}^l Z_t[i_0 + t]$  not isomorphic to  $M$ .

Let  $\overline{T}$  be such that  $T = \overline{T} \oplus M$ . Let  $B \rightarrow M$  be a right minimal  $\text{add}(\overline{T})$ -approximation of  $M$ . Complete it into a triangle in  $\mathcal{D}^b(\mathcal{H})$ :

$$M^* \rightarrow B \rightarrow M \rightarrow M^*[1]. \quad (\Delta)$$

**Lemma 3.5.** *With the above setting, let  $T' = \overline{T} \oplus M^*$ . Then  $T' \in \mathcal{T}$  and  $T, T'$  are related by a transformation of the second kind. Moreover,  $r(T') < r(T)$ .*

**Proof:** We only need to prove that  $T' \in \mathcal{T}$ . We may assume that  $i_0 = 0$ . By hypothesis on  $M$ , we have  $B \in \text{add}(Z_0) \subseteq \mathcal{H}$ . Since  $M \in \mathcal{H}[1]$ , the triangle  $\Delta$  does not split. We now list some properties on  $T'$ . In most cases, the proof is due to arguments taken from [5, § 6]. Although these arguments were originally given in the setting of cluster categories (that is, triangulated categories which are Calabi–Yau of dimension 2), it is easily verified that they still work in our situation (that is, the Calabi–Yau property is unnecessary):

1.  $\text{Ext}^1(\overline{T}, M^*) = 0$  (see [5, Lem. 6.3]).
2.  $\text{Ext}^i(\overline{T}, M^*) = 0$  for every  $i \geq 2$ . Indeed, applying  $\text{Hom}(\overline{T}, -)$  to  $\Delta$  gives the exact sequence

$$0 = \text{Ext}^{i-1}(\overline{T}, M) \rightarrow \text{Ext}^i(\overline{T}, M^*) \rightarrow \text{Ext}^i(\overline{T}, B) = 0.$$

3.  $\text{Ext}^i(M^*, \overline{T}) = 0$  for every  $i \geq 1$ . Indeed, applying  $\text{Hom}(-, \overline{T})$  to  $\Delta$  gives the exact sequence

$$0 = \text{Ext}^i(B, \overline{T}) \rightarrow \text{Ext}^i(M^*, \overline{T}) \rightarrow \text{Ext}^{i+1}(M, \overline{T}) = 0.$$

4. The map  $M^* \rightarrow B$  is a left minimal  $\text{add}(\overline{T})$ -approximation (see [5, Lem. 6.4]).
5.  $M^*$  is indecomposable and does not lie on  $\text{add}(\overline{T})$  (see [5, Lems. 6.5, 6.6]). Therefore  $T'$  is the direct sum of  $n$  pairwise indecomposable objects.
6.  $M^* \in \mathcal{H}$ . Indeed,  $M$  is indecomposable and there are two non-zero maps  $M[-1] \rightarrow M^*$  and  $M^* \rightarrow B$  with  $M[-1], B \in \mathcal{H}$ .
7.  $\text{Ext}^1(M^*, M^*) = 0$  (see [5, Lem. 6.7]).
8.  $\text{Ext}^i(M^*, M^*) = 0$  for every  $i \geq 2$  because  $M^*$  is indecomposable and  $\mathcal{H}$  is hereditary.

The facts 1. – 8. express that  $T' \in \mathcal{T}$ . Moreover,  $r(T') < r(T)$  because  $M^* \in \mathcal{H}$  and  $M \in \mathcal{H}[1]$ .  $\square$

**Lemma 3.6.** *Let  $T \in \mathcal{T}$ . Let  $\mathcal{A}$  be the smallest subclass of  $\mathcal{T}$  containing  $T$  and stable under transformations of the first or of the second kind. Then  $\mathcal{A}$  contains a tilting object in  $\mathcal{H}$ .*

**Proof:** Let  $T' \in \mathcal{A}$  be such that  $r(T')$  is minimal for this property. Assume that  $r(T') > 0$ . By 3.1 and 3.5, there exists  $T'' \in \mathcal{A}$  such that  $r(T'') < r(T')$ . This contradicts the minimality of  $r(T')$ . Hence  $r(T') = 0$  and there exists an integer  $i_0$  such that  $T'[-i_0]$  is a tilting object in  $\mathcal{H}$  and lies in  $\mathcal{A}$ .  $\square$

The following result is a direct consequence of 3.4 and 3.6.

**Proposition 3.7.** *Assume that  $\overrightarrow{\mathcal{K}}_{\mathcal{H}}$  is connected. Let  $T \in \mathcal{T}$ . Then  $\mathcal{T}$  is the smallest subset of  $\mathcal{T}$  containing  $T$  and stable under transformations of the first or the second kind.*

*Remark 3.8.* (a) A tilting object in  $\mathcal{H}$  generates  $\mathcal{D}^b(\mathcal{H})$ . By definition of the two kinds of transformations, 3.7 implies, under the same hypotheses, that any  $T \in \mathcal{T}$  generates  $\mathcal{D}^b(\mathcal{H})$ .

- (b) Let  $A$  be an algebra derived equivalent to a hereditary algebra. Then 3.4 implies that the conclusion of 3.7 holds true if one replaces  $\mathcal{H}$  by  $\text{mod } A$ .

## 4 Tilting complexes of the first kind

Throughout this section, we assume that  $A$  is an algebra derived equivalent to a hereditary abelian category  $\mathcal{H}$  such that  $\overrightarrow{\mathcal{K}}_{\mathcal{H}}$  is connected. We denote by  $n$  the rank of its Grothendieck group and  $\Theta: \mathcal{D}^b(\mathcal{H}) \rightarrow \mathcal{D}^b(\text{mod } A)$  a triangle equivalence. We fix a Galois covering  $F: \mathcal{C} \rightarrow A$  with group  $G$  and with  $\mathcal{C}$  locally bounded. We use 2.1 without reference. The aim of this section is to prove that the following facts hold true for any tilting complex  $T \in \mathcal{D}^b(\text{mod } A)$ :

- (H<sub>1</sub>) For every indecomposable direct summand  $X$  of  $T$ , there exists  $\tilde{X} \in \mathcal{D}^b(\text{mod } \mathcal{C})$  such that  $F_{\lambda} \tilde{X} \simeq X$  in  $\mathcal{D}^b(\text{mod } A)$ .
- (H<sub>2</sub>)  $\tilde{X} \not\cong {}^g \tilde{X}$  for every indecomposable direct summand  $X$  of  $T$  and  $g \in G \setminus \{1\}$ .
- (H<sub>3</sub>) If  $\psi: A \xrightarrow{\sim} A$  is an automorphism such that  $\psi(x) = x$  for every  $x \in \text{ob}(A)$ , then  $\psi_{\lambda} X \simeq X$  in  $\mathcal{D}^b(\text{mod } A)$  for every indecomposable direct summand  $X$  of  $T$ .

Some results presented in this section have been proved in [21, § 3] in the case of tilting modules.

### Proof of assertion ( $H_1$ )

In this paragraph, we prove the following.

**Proposition 4.1.** *Let  $A$  be as above. Let  $T \in \mathcal{D}^b(\text{mod } A)$  be a tilting complex. Then:*

- (a) *For every indecomposable direct summand  $X$  of  $T$  there exists  $\tilde{X} \in \mathcal{D}^b(\text{mod } \mathcal{C})$  (necessarily indecomposable) such that  $F_\lambda \tilde{X} \simeq X$ .*

Moreover, the class  $\{\tilde{X} \in \mathcal{D}^b(\text{mod } \mathcal{C}) \mid F_\lambda \tilde{X} \text{ is an indecomposable direct summand of } T\}$  satisfies the following:

- (b) *It generates the triangulated category  $\mathcal{D}^b(\text{mod } \mathcal{C})$ .*  
(c) *It is stable under the action of  $G$ .*  
(d)  *$\mathcal{D}^b(\text{mod } \mathcal{C})(\tilde{X}, {}^g \tilde{Y}[i]) = 0$  for every  $\tilde{X}, \tilde{Y}$  in this class,  $i \neq 0$  and  $g \in G$ .*

We need the two following dual lemmas.

**Lemma 4.2.** *Let  $\Delta : X \xrightarrow{u} M \rightarrow Y \rightarrow X[1]$  be triangle in  $\mathcal{D}^b(\text{mod } A)$  such that:*

- (a) *There exists  $\tilde{X} \in \mathcal{D}^b(\text{mod } \mathcal{C})$  satisfying  $X = F_\lambda \tilde{X}$ .*  
(b)  *$M = M_1 \oplus \dots \oplus M_t$  where  $M_1, \dots, M_t$  are indecomposables such that there exist indecomposable objects  $\tilde{M}_1, \dots, \tilde{M}_t$  satisfying  $F_\lambda \tilde{M}_i = M_i$  for every  $i$ .*  
(c)  *$\mathcal{D}^b(\text{mod } A)(Y, M[1]) = 0$ .*

Then  $\Delta$  is isomorphic to a triangle in  $\mathcal{D}^b(\text{mod } A)$ :

$$X \xrightarrow{[F_\lambda u'_1 \quad \dots \quad F_\lambda u'_t]^t} M_1 \oplus \dots \oplus M_t \rightarrow Y \rightarrow X[1]$$

where  $u'_i \in \mathcal{D}^b(\text{mod } \mathcal{C})(\tilde{X}, {}^{g_i} \tilde{M}_i)$  for some  $g_i \in G$  for every  $i$ .

**Lemma 4.3.** *Let  $\Delta : X \rightarrow M \xrightarrow{v} Y \rightarrow X[1]$  be triangle in  $\mathcal{D}^b(\text{mod } A)$  such that:*

- (a) *There exists  $\tilde{Y} \in \mathcal{D}^b(\text{mod } \mathcal{C})$  satisfying  $Y = F_\lambda \tilde{Y}$ .*  
(b)  *$M = M_1 \oplus \dots \oplus M_t$  where  $M_1, \dots, M_t$  are indecomposables such that there exist indecomposable objects  $\tilde{M}_1, \dots, \tilde{M}_t$  satisfying  $F_\lambda \tilde{M}_i = M_i$  for every  $i$ .*  
(c)  *$\mathcal{D}^b(\text{mod } A)(M, X[1]) = 0$ .*

Then  $\Delta$  is isomorphic to a triangle in  $\mathcal{D}^b(\text{mod } A)$ :

$$X \rightarrow M_1 \oplus \dots \oplus M_t \xrightarrow{[F_\lambda v'_1 \quad \dots \quad F_\lambda v'_t]} Y \rightarrow X[1]$$

where  $v'_i \in \mathcal{D}^b(\text{mod } \mathcal{C})({}^{g_i} \tilde{M}_i, \tilde{Y})$  for some  $g_i \in G$  for every  $i$ .

**Proof of 4.2:** We say that a morphism  $u \in \mathcal{D}^b(\text{mod } A)(X, M_i)$  is homogeneous of degree  $g \in G$  if and only if there exists  $u' \in \mathcal{D}^b(\text{mod } \mathcal{C})(\tilde{X}, {}^g \tilde{M}_i)$  such that  $u = F_\lambda(u')$ . Since  $F_\lambda : \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$  has the covering property, any morphism  $X \rightarrow M_i$  is (uniquely) the sum of  $d$  non zero homogeneous morphisms of pairwise different degrees (with  $d \geq 0$ ). Let  $u = [u_1 \quad \dots \quad u_t]^t$  with  $u_i : X \rightarrow M_i$  for each  $i$ . We may assume that  $u_1 : X \rightarrow M_1$  is not homogeneous. Thus  $u_1 = h_1 + \dots + h_d$ , where  $d \geq 2$  and  $h_1, \dots, h_d : X \rightarrow M_1$  are non-zero homogeneous morphisms of pairwise different degrees. In order to prove the lemma, it suffices to prove that  $\Delta$  is isomorphic to a triangle  $X \xrightarrow{u'} M \rightarrow Y \rightarrow X[1]$  with  $u' = [u'_1 \quad u_2 \quad \dots \quad u_t]^t$  such that  $u'_1$  is equal to the sum of at most  $d-1$  non-zero homogeneous morphisms  $X \rightarrow M_1$  of pairwise different degrees. For simplicity we adopt the following notations:

1.  $\overline{M} = M_2 \oplus \dots \oplus M_t$  (so  $M = M_1 \oplus \overline{M}$ ).
2.  $\overline{u} = [u_2 \quad \dots \quad u_t]^t : X \rightarrow \overline{M}$  (so  $u = [u_1 \quad \overline{u}]^t : X \rightarrow M_1 \oplus \overline{M}$ ).
3.  $\overline{h} = h_2 + \dots + h_d : X \rightarrow M_1$  (so  $u_1 = h_1 + \overline{h}$ ).

Applying the functor  $\mathcal{D}^b(\text{mod } A)(-, M_1)$  to  $\Delta$  gives the exact sequence:

$$\mathcal{D}^b(\text{mod } A)(M_1 \oplus \overline{M}, M_1) \xrightarrow{\text{Hom}(u, M_1)} \mathcal{D}^b(\text{mod } A)(X, M_1) \rightarrow \mathcal{D}^b(\text{mod } A)(Y, M_1[1]) = 0.$$

So there exists  $[\lambda, \mu] : M_1 \oplus \overline{M} \rightarrow M_1$  such that  $h_1 = [\lambda, \mu]u$ . Hence:

$$h_1 = \lambda u_1 + \mu \overline{u} = \lambda h_1 + \lambda \overline{h} + \mu \overline{u}. \quad (i)$$

We distinguish two cases according to whether  $\lambda \in \text{End}_{\mathcal{D}^b(\text{mod } A)}(M_1, M_1)$  is invertible or nilpotent. If  $\lambda$  is invertible, then the following is an isomorphism in  $\mathcal{D}^b(\text{mod } A)$ :

$$\theta := \begin{bmatrix} \lambda & \mu \\ 0 & \text{Id}_{\overline{M}} \end{bmatrix} : M_1 \oplus \overline{M} \rightarrow M_1 \oplus \overline{M}.$$

Using (i) we deduce an isomorphism of triangles:

$$\begin{array}{ccccc} X & \xrightarrow{[u_1 \ \bar{u}]^t} & M_1 \oplus \bar{M} & \longrightarrow & Y & \longrightarrow & X[1] & \Delta \\ \parallel & & \downarrow \theta & & \downarrow \sim & & & \\ X & \xrightarrow{[h_1 \ \bar{u}]^t} & M_1 \oplus \bar{M} & \longrightarrow & Y & \longrightarrow & X[1] & \Delta' \end{array}$$

Since  $h_1: X \rightarrow M_1$  is homogeneous,  $\Delta'$  satisfies our requirements. If  $\lambda$  is nilpotent, let  $p \geq 0$  be such that  $\lambda^p = 0$ . Using (i) we get the following equalities:

$$\begin{aligned} h_1 &= \lambda^2 h_1 + (\lambda^2 + \lambda) \bar{h} + (\lambda + \text{Id}_{M_1}) \mu \bar{u} \\ &\vdots \\ h_1 &= \lambda^t h_1 + (\lambda^t + \lambda^{t-1} + \dots + \lambda) \bar{h} + (\lambda^{t-1} + \dots + \lambda + \text{Id}_{M_1}) \mu \bar{u} \\ &\vdots \\ h_1 &= \lambda^p h_1 + (\lambda^p + \lambda^{p-1} + \dots + \lambda) \bar{h} + (\lambda^{p-1} + \dots + \lambda + \text{Id}_{M_1}) \mu \bar{u} \end{aligned}$$

Since  $\lambda^p = 0$  and  $u_1 = h_1 + \bar{h}$  we infer that:

$$u_1 = \lambda' \bar{h} + \lambda' \mu \bar{u},$$

where  $\lambda'$  is the invertible morphism  $\text{Id}_{M_1} + \lambda + \dots + \lambda^{p-1} \in \text{End}_{\mathcal{D}^b(\text{mod } A)}(M_1, M_1)$ . So we have an isomorphism in  $\mathcal{D}^b(\text{mod } A)$ :

$$\theta := \begin{bmatrix} \lambda' & \lambda' \mu \\ 0 & \text{Id}_{\bar{M}} \end{bmatrix} : M_1 \oplus \bar{M} \rightarrow M_1 \oplus \bar{M}.$$

Consequently we have an isomorphism of triangles:

$$\begin{array}{ccccc} X & \xrightarrow{[\bar{h} \ \bar{u}]^t} & M_1 \oplus \bar{M} & \longrightarrow & Y & \longrightarrow & X[1] & \Delta' \\ \parallel & & \downarrow \theta & & \downarrow \sim & & & \\ X & \xrightarrow{[u_1 \ \bar{u}]^t} & M_1 \oplus \bar{M} & \longrightarrow & Y & \longrightarrow & X[1] & \Delta \end{array}$$

where  $\bar{h} = h_2 + \dots + h_p$  is the sum of  $p-1$  non zero homogeneous morphisms of pairwise different degrees. So  $\Delta'$  satisfies our requirements.  $\square$

The proof of 4.3 is the dual of the one of 4.2 so we omit it. Now we can prove 4.1.

**Proof of 4.1:** If (a) holds true, then so does (c) because  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$  is  $G$ -invariant. Recall that  $\Theta: \mathcal{D}^b(\mathcal{H}) \rightarrow \mathcal{D}^b(\text{mod } A)$  is a triangle equivalence. As in Section 3, we write  $\mathcal{T}$  (or  $\mathcal{T}'$ ) for the set of isomorphism classes of objects  $T \in \mathcal{D}^b(\mathcal{H})$  (or  $T \in \mathcal{D}^b(\text{mod } A)$ ) such that  $T$  is the direct sum of  $n$  pairwise non isomorphic indecomposable objects and  $\mathcal{D}^b(\mathcal{H})(T, T[i]) = 0$  (or  $\mathcal{D}^b(\text{mod } A)(T, T[i]) = 0$ , respectively) for every  $i \geq 1$ . Therefore:

(i)  $\Theta$  defines a bijection  $\Theta: \mathcal{T} \rightarrow \mathcal{T}'$ . Under this bijection, tilting complexes in  $\mathcal{D}^b(\mathcal{H})$  correspond to tilting complexes in  $\mathcal{D}^b(\text{mod } A)$ .

We prove that (a) and (b) hold true for any  $T \in \mathcal{T}'$  (and therefore for any tilting object in  $\mathcal{D}^b(\text{mod } A)$ ). For this purpose, we use the results of Section 3. First of all, remark that:

(ii) The assertions (a) and (b) hold true for  $T = A$ . In this case,  $F_\lambda \tilde{X}$  is an indecomposable summand of  $A$  if and only if  $\tilde{X}$  is an indecomposable projective  $\mathcal{C}$ -module.

If  $\tilde{X} \in \mathcal{D}^b(\mathcal{H})$ , then  $F_\lambda(\tilde{X}[1]) = (F_\lambda \tilde{X})[1]$ . Therefore:

(iii) Let  $T, T' \in \mathcal{T}'$  be such that  $\Theta^{-1}(T')$  is obtained from  $\Theta^{-1}(T)$  by a transformation of the first kind. Then (a) and (b) hold true for  $T$  if and only if they do so for  $T'$ .

Now assume that  $T, T' \in \mathcal{T}'$  are such that  $\Theta^{-1}(T')$  is obtained from  $\Theta^{-1}(T)$  by a transformation of the second kind. We prove that (a) and (b) hold true for  $T$  if and only if they do so for  $T'$ . In such a situation there exist  $X, Y \in \mathcal{D}^b(\text{mod } A)$  indecomposables and  $\bar{T} \in \mathcal{D}^b(\text{mod } A)$  such that  $T = X \oplus \bar{T}$  and  $T' = Y \oplus \bar{T}$ . Also, there exists a triangle in  $\mathcal{D}^b(\text{mod } A)$  of one the two following forms:

1.  $X \rightarrow M \rightarrow Y \rightarrow X[1]$  where  $M \in \text{add}(\bar{T})$ .
2.  $Y \rightarrow M \rightarrow X \rightarrow Y[1]$  where  $M \in \text{add}(\bar{T})$ .

Assume that (a) and (b) hold true for  $T$  and that there is a triangle  $X \rightarrow M \rightarrow Y \rightarrow X[1]$  (the other cases are dealt with using similar arguments). In order to prove that (a) and (b) hold true for  $T'$  we prove that  $Y \simeq F_\lambda \tilde{Y}$  for some  $\tilde{Y} \in \mathcal{D}^b(\text{mod } \mathcal{C})$ . Fix an indecomposable decomposition  $M = M_1 \oplus \dots \oplus M_t$ . By assumption on  $T$ , there exist indecomposable objects  $\tilde{X}, \tilde{M}_1, \dots, \tilde{M}_t \in \mathcal{D}^b(\text{mod } \mathcal{C})$  such that  $F_\lambda \tilde{X} \simeq X, F_\lambda \tilde{M}_1 \simeq M_1, \dots, F_\lambda \tilde{M}_t \simeq M_t$ . Using

these isomorphisms, we identify  $F_\lambda \widetilde{X}$  and  $F_\lambda \widetilde{M}_i$  to  $X$  and  $M_i$ , respectively. By 4.2, there exist  $g_1, \dots, g_t \in G$  and morphisms  $u_i \in \mathcal{D}^b(\text{mod } \mathcal{C})(\widetilde{X}, {}^{g_i} \widetilde{M}_i)$  (for  $i \in \{1, \dots, t\}$ ) such that the triangle  $X \rightarrow M \rightarrow Y \rightarrow X[1]$  is isomorphic to a triangle of the form:

$$X \xrightarrow{[F_\lambda(u_1) \quad \dots \quad F_\lambda(u_t)]^t} M \rightarrow Y \rightarrow X[1].$$

Set  $u = [u_1 \quad \dots \quad u_t]^t : \widetilde{X} \rightarrow \widetilde{M}_1 \oplus \dots \oplus \widetilde{M}_t$ . We complete  $u$  into a triangle  $\widetilde{X} \xrightarrow{u} \widetilde{M}_1 \oplus \dots \oplus \widetilde{M}_t \xrightarrow{v} \widetilde{X}[1]$  in  $\mathcal{D}^b(\text{mod } \mathcal{C})$ . So we have a triangle  $X \xrightarrow{F_\lambda(u)} M \xrightarrow{F_\lambda(v)} F_\lambda \widetilde{Y} \rightarrow X[1]$  in  $\mathcal{D}^b(\text{mod } A)$ . Therefore  $Y \simeq F_\lambda \widetilde{Y}$ . So (a) holds for  $T'$  and the class  $\{\widetilde{Z} \mid F_\lambda \widetilde{Z} \text{ is an indecomposable direct summand of } T'\}$  coincides with the class  $\{{}^g \widetilde{Y} \mid g \in G\} \cup \{\widetilde{Z} \mid F_\lambda \widetilde{Z} \text{ is an indecomposable direct summand of } \widetilde{T}\}$  (see 2.3). Because (b) holds true for  $T$  and because of the triangle  $\widetilde{X} \rightarrow \widetilde{M}_1 \oplus \dots \oplus \widetilde{M}_t \rightarrow \widetilde{Y} \rightarrow \widetilde{X}[1]$ , we deduce that (b) holds for  $T'$ . So we have proved the following:

(iv) Let  $T, T' \in \mathcal{T}'$  be such that  $\Theta^{-1}(T')$  is obtained from  $\Theta^{-1}(T)$  by a transformation of the second kind. Then (a) and (b) hold true for  $T$  if and only if they do so for  $T'$ .

By 3.7 and (i – iv), the assertions (a), (b) and (c) are satisfied for any  $T \in \mathcal{T}$ . Finally, if  $T$  is a tilting complex, then (d) follows from the fact that  $\mathcal{D}^b(\text{mod } A)(T, T[i]) = 0$  for every  $i \neq 0$  and from 2.1.  $\square$

It is interesting to note that the transformations of the second kind in  $\mathcal{D}^b(\text{mod } A)$  give rise to transformations of the second kind in  $\mathcal{D}^b(\text{mod } \mathcal{C})$ . Indeed, let  $T, T'$  be in  $\mathcal{T}'$  where  $T'$  is as in the proof of 4.1. Assume that  $T = M \oplus \widetilde{T}$  with  $M$  indecomposable,  $T = M^* \oplus \widetilde{T}$  with  $M^*$  indecomposable and there exists a triangle  $\Delta : M \rightarrow B \rightarrow M^* \rightarrow M[1]$  in  $\mathcal{D}^b(\text{mod } A)$  where  $M \rightarrow B$  (or  $B \rightarrow M^*$ ) is a left minimal  $\text{add}(\widetilde{T})$ -approximation of  $M$  (or a right minimal  $\text{add}(\widetilde{T})$ -approximation of  $M^*$ , respectively). Then the following holds.

**Lemma 4.4.** *Keep the above setting. Let  $B = \bigoplus_{i=1}^t B_i$  be an indecomposable decomposition (maybe with multiplicities). Then there exists a triangle  $\widetilde{\Delta} : \widetilde{M} \xrightarrow{u} \bigoplus_{i=1}^t {}^{g_i} \widetilde{B}_i \xrightarrow{v} {}^{g_0} \widetilde{M}^* \rightarrow M[1]$  in  $\mathcal{D}^b(\text{mod } \mathcal{C})$  whose image under  $F_\lambda$  is isomorphic to  $\Delta$ . Moreover, if  $\mathcal{X}$  (or  $\mathcal{X}'$ ) denotes the additive full subcategory of  $\mathcal{D}^b(\text{mod } \mathcal{C})$  generated by the indecomposables  $X \in \mathcal{D}^b(\text{mod } \mathcal{C})$  not isomorphic to  $\widetilde{M}$  (or to  $\widetilde{M}^*$ ) and such that  $F_\lambda X$  is an indecomposable summand of  $T$  (or of  $T'$ , respectively), then:*

(a)  $u$  is a left minimal  $\mathcal{X}$ -approximation.

(b)  $v$  is a right minimal  $\mathcal{X}'$ -approximation.

**Proof:** The existence of  $\widetilde{\Delta}$  follows from the proof of 4.1. So  $F_\lambda(u)$  is a left minimal  $\text{add}(\widetilde{T})$ -approximation. This and the exactness of  $F_\lambda$  imply that  $u$  is left minimal. Let  $f : \widetilde{M} \rightarrow {}^g \widetilde{Y}$  be a non-zero morphism where  ${}^g \widetilde{Y} \in \mathcal{X}$  and  $Y \in \text{add}(T)$ . The linear map  $\bigoplus_{h \in G} \mathcal{D}^b(\text{mod } \mathcal{C})(\widetilde{M}, {}^h \widetilde{M}) \rightarrow \text{End}_{\mathcal{D}^b(\text{mod } A)}(M, M)$  induced by  $F_\lambda$  is bijective.

Also  $\dim_k \text{End}_{\mathcal{D}^b(\text{mod } A)}(M, M) = 1$  because  $M$  is an indecomposable and  $\mathcal{D}^b(\text{mod } A)(M, M[i]) = 0$  for every  $i > 0$ . So  ${}^g \widetilde{Y} \not\cong {}^h \widetilde{M}$  for every  $h \neq 1$ . Hence  $Y \in \text{add}(\widetilde{T})$  and, therefore,  $F_\lambda(f)$  factorises through  $F_\lambda(u)$ :

$$\begin{array}{ccc} M & \xrightarrow{F_\lambda(u)} & \bigoplus_{i=1}^t B_i \\ & \searrow F_\lambda(f) & \downarrow f' \\ & & Y \end{array} .$$

There exists  $(f'_h)_h \in \bigoplus_{h \in G} \mathcal{D}^b(\text{mod } \mathcal{C})(\bigoplus_{i=1}^t {}^{g_i} \widetilde{B}_i, {}^h \widetilde{Y})$  such that  $f' = \sum_{h \in G} F_\lambda(f'_g)$  because of the covering property of  $F_\lambda$ . So  $F_\lambda(f) = \sum_{h \in G} F_\lambda(f'_h u)$  and, therefore,  $f = f'_g u$  for the same reason. Thus  $u$  is a left minimal  $\mathcal{X}$ -approximation. Similarly,  $v$  is a right minimal  $\mathcal{X}'$ -approximation.  $\square$

Since tilting  $A$ -modules are particular cases of tilting complexes, we get the following result.

**Corollary 4.5.** *Let  $A$  be an algebra derived equivalent to a hereditary abelian category  $\mathcal{H}$  such that  $\vec{\mathcal{K}}_{\mathcal{H}}$  is connected. Let  $F : \mathcal{C} \rightarrow A$  be a Galois covering with group  $G$  where  $\mathcal{C}$  is locally bounded,  $T$  a tilting  $A$ -module and  $X \in \text{mod } A$  an indecomposable summand of  $T$ . Then there exists  $\widetilde{X} \in \text{mod } \mathcal{C}$  such that  $F_\lambda \widetilde{X} \simeq X$ .*

**Proof:** By 4.1, such an  $\widetilde{X}$  exists in  $\mathcal{D}^b(\text{mod } \mathcal{C})$ . We prove that  $\widetilde{X}$  is isomorphic to a  $\mathcal{C}$ -module. Let  $P \in \text{mod } \mathcal{C}$  be projective and  $i \in \mathbb{Z} \setminus \{0\}$ . Then  $F_\lambda P \in \text{mod } A$  is projective and  $\mathcal{D}^b(\text{mod } A)(F_\lambda P, X[i]) = 0$  because  $X$  is an  $A$ -module. On the other hand, the spaces  $\mathcal{D}^b(\text{mod } A)(F_\lambda P, X[i])$  and  $\bigoplus_{g \in G} \mathcal{D}^b(\text{mod } \mathcal{C})({}^g P, \widetilde{X}[i])$  are isomorphic.

So  $\mathcal{D}^b(\text{mod } \mathcal{C})(P, \widetilde{X}[i]) = 0$  for every  $i \neq 0$ . Thus,  $\widetilde{X} \simeq H^0(\widetilde{X}) \in \text{mod } \mathcal{C}$ .  $\square$

### Proof of assertion ( $H_2$ )

**Proposition 4.6.** *Let  $A$  be as in 4.5,  $F: \mathcal{C} \rightarrow A$  a Galois covering with group  $G$  where  $\mathcal{C}$  is locally bounded and  $X \in \text{ind } A$  a direct summand of a tilting complex in  $\mathcal{D}^b(\text{mod } A)$ . Assume that  $F_\lambda \tilde{X} \simeq X$  for some  $\tilde{X} \in \mathcal{D}^b(\text{mod } \mathcal{C})$ . Then  ${}^g \tilde{X} \not\cong \tilde{X}$  for every  $g \in G \setminus \{1\}$ .*

**Proof:** We have  $\dim_k \text{End}_{\mathcal{D}^b(\text{mod } A)}(X, X) = 1$  because  $X$  is indecomposable and  $\mathcal{D}^b(\text{mod } A)(X, X[i]) = 0$  for every  $i \neq 0$ . On the other hand, the spaces  $\bigoplus_{g \in G} \mathcal{D}^b(\text{mod } \mathcal{C})({}^g \tilde{X}, \tilde{X})$  and  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(X, X)$  are isomorphic. So  $\mathcal{D}^b(\text{mod } \mathcal{C})({}^g \tilde{X}, \tilde{X}) = 0$  and, therefore,  ${}^g \tilde{X} \not\cong \tilde{X}$  if  $g \neq 1$ .  $\square$

### Proof of assertion ( $H_3$ )

If  $\psi: A \rightarrow A$  is an automorphism (and therefore a Galois covering with trivial group), then  $\psi_\lambda: \text{mod } A \rightarrow \text{mod } A$  is an equivalence. It thus induces a triangle equivalence  $\psi_\lambda: \mathcal{D}^b(\text{mod } A) \rightarrow \mathcal{D}^b(\text{mod } A)$ .

**Proposition 4.7.** *Let  $A$  be as in 4.5,  $\psi: A \xrightarrow{\sim} A$  an automorphism such that  $\psi(x) = x$  for every  $x \in \text{ob}(A)$  and  $T \in \mathcal{D}^b(\text{mod } A)$  a tilting complex. Then  $\psi_\lambda X \simeq X$  in  $\mathcal{D}^b(\text{mod } A)$  for every indecomposable summand  $X$  of  $T$ .*

**Proof:** Since  $\psi(x) = x$  for every  $x \in \text{ob}(A)$ , we have the following fact:

(i) The conclusion of the proposition holds true if  $X$  is an indecomposable projective  $A$ -module.

Recall that  $\Theta: \mathcal{D}^b(\mathcal{H}) \rightarrow \mathcal{D}^b(\text{mod } A)$  is a triangle equivalence. We keep the notations  $\mathcal{T}$  and  $\mathcal{T}'$  introduced in the proof of 4.1. We prove the proposition for any  $T \in \mathcal{T}'$ . By construction of  $\Theta$ , we have:

(ii)  $\Theta$  induces a bijection  $\Theta: \mathcal{T} \rightarrow \mathcal{T}'$ . Under this bijection, tilting complexes in  $\mathcal{D}^b(\mathcal{H})$  correspond to tilting complexes in  $\mathcal{D}^b(\text{mod } A)$ .

Since  $\psi_\lambda: \mathcal{D}^b(\text{mod } A) \rightarrow \mathcal{D}^b(\text{mod } A)$  is an equivalence, we also have:

(iii) Let  $T, T' \in \mathcal{T}$  be such that  $T'$  is obtained from  $T$  by a transformation of the first kind. Then the proposition holds true for  $T$  if and only if it does for  $T'$ .

Now assume that  $T, T' \in \mathcal{T}'$  are such that  $\Theta^{-1}(T')$  is obtained from  $\Theta^{-1}(T)$  by a transformation of the second kind. We prove that the proposition holds true for  $T$  if and only if it does for  $T'$ . There exist  $X, Y \in \mathcal{D}^b(\text{mod } A)$  indecomposables and  $\bar{T} \in \mathcal{D}^b(\text{mod } A)$  such that  $T = X \oplus \bar{T}$  and  $T' = Y \oplus \bar{T}$ . Also, there exists a triangle in  $\mathcal{D}^b(\text{mod } A)$  of one the two following forms:

1.  $X \rightarrow M \rightarrow Y \rightarrow X[1]$  where  $X \rightarrow M$  is a left minimal  $\text{add}(\bar{T})$ -approximation and  $M \rightarrow Y$  is a right minimal  $\text{add}(\bar{T})$ -approximation.
2.  $Y \rightarrow M \rightarrow X \rightarrow Y[1]$  where  $Y \rightarrow M$  is a left minimal  $\text{add}(\bar{T})$ -approximation and  $M \rightarrow X$  is a right minimal  $\text{add}(\bar{T})$ -approximation.

Assume that the proposition holds for  $T$  and that there is a triangle  $X \rightarrow M \rightarrow Y \rightarrow X[1]$  (the other cases are dealt with using similar arguments). We only need to prove that  $\psi_\lambda Y \simeq Y$ . Apply  $\psi_\lambda$  to the triangle  $X \rightarrow M \rightarrow Y \rightarrow X[1]$ . Since  $\psi_\lambda$  is an equivalence and the proposition holds true for  $T$ , there exists a triangle  $X \rightarrow M \rightarrow \psi_\lambda Y \rightarrow X[1]$  in  $\mathcal{D}^b(\text{mod } A)$  where  $X \rightarrow M$  is a left minimal  $\text{add}(\bar{T})$ -approximation. Therefore  $\psi_\lambda Y \simeq Y$  in  $\mathcal{D}^b(\text{mod } A)$ . So we proved that:

(iv) If  $T, T' \in \mathcal{T}'$  are such that  $\Theta^{-1}(T')$  is obtained from  $\Theta^{-1}(T)$  by a transformation of the second kind, then the proposition holds true for  $T$  if and only if it does for  $T'$ .

As in the proof of 4.1, the conclusion follows from (i), (ii), (iii), (iv) and 3.7.  $\square$

### Application: proof of Theorem C

As an application of the preceding results of the section, we prove Theorem C. We need the following lemma. If  $T = T_1 \oplus \dots \oplus T_n \in \text{mod } A$  is an indecomposable decomposition of a multiplicity-free module  $T$ , then  $\text{End}_A(T)$  is naturally a  $k$ -category, equal to the full subcategory of  $\text{mod } A$  with objects  $T_1, \dots, T_n$ .

**Lemma 4.8.** *Let  $A$  be a piecewise hereditary algebra of type  $Q$ . Let  $F: \mathcal{C} \rightarrow A$  be a connected Galois covering with group  $G$ . Let  $T \in \mathcal{D}^b(\text{mod } A)$  be a tilting complex,  $B = \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  and  $T = T_1 \oplus \dots \oplus T_n$  an indecomposable decomposition. Let  $\lambda_i: F_\lambda \tilde{T}_i \rightarrow T_i$  be an isomorphism where  $\tilde{T}_i \in \mathcal{D}^b(\text{mod } \mathcal{C})$  is indecomposable for every  $i$ . This defines the bounded complex of (not necessarily finite dimensional)  $\mathcal{C}$ -modules  $\tilde{T} := \bigoplus_{i,g} {}^g \tilde{T}_i$ ,*

*where the sum runs over  $g \in G$  and  $i \in \{1, \dots, n\}$ . Let  $\mathcal{C}'$  be the full subcategory of  $\mathcal{D}^b(\text{mod } \mathcal{C})$  with objects the complexes  ${}^g \tilde{T}_i$  (for  $g \in G, i \in \{1, \dots, n\}$ ). Then the triangle functor  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$  induces a connected Galois covering with group  $G$ :*

$$F_{\tilde{T}, \lambda}: \begin{array}{ccc} \mathcal{C}' & \rightarrow & B \\ {}^g \tilde{T}_i & \mapsto & T_i \\ {}^g \tilde{T}_i \xrightarrow{u} {}^h \tilde{T}_j & \mapsto & T_i \xrightarrow{\lambda_j F_\lambda(u) \lambda_i^{-1}} T_j \end{array} .$$

The complex  $\tilde{T}$  is naturally a bounded complex of  $\mathcal{C}' - \mathcal{C}$ -bimodules: As a functor from  $\text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})}(\tilde{T}) \times \mathcal{C}^{op}$ , it assigns the vector space  ${}^g\tilde{T}_i(x)$  to the pair of objects  $({}^g\tilde{T}_i, x)$ . The total derived functor:

$$- \underset{\mathcal{C}'}{\overset{\mathbb{L}}{\otimes}} \tilde{T}: \mathcal{D}^b(\text{mod } \mathcal{C}') \rightarrow \mathcal{D}^b(\text{mod } \mathcal{C})$$

is a  $G$ -equivariant triangle equivalence. Finally, if  $T$  is a tilting  $A$ -module and all the objects  $\tilde{T}_i$  are  $\mathcal{C}$ -modules (see 4.5), then:

- (a)  $\text{Ext}_{\mathcal{C}}^1({}^g\tilde{T}_i, {}^h\tilde{T}_j) = 0$  for every  $i, j \in \{1, \dots, n\}$  and  $g, h \in G$ .
- (b)  $\text{pd}_{\mathcal{C}}({}^g\tilde{T}_i) \leq 1$  for every  $i, g$ .
- (c) If  $P \in \text{mod } \mathcal{C}$  is an indecomposable projective  $\mathcal{C}$ -module, then there exists an exact sequence  $0 \rightarrow P \rightarrow T^{(1)} \rightarrow T^{(2)} \rightarrow 0$  in  $\text{mod } \mathcal{C}$  where  $T^{(1)}, T^{(2)} \in \text{add}(\{{}^g\tilde{T}_i \mid i \in \{1, \dots, n\}, g \in G\})$ .

**Proof:** By 2.1, the functor  $F_{\tilde{T}, \lambda}$  is a well-defined Galois covering. By 4.6, we know that that  $\mathcal{C}'$  is a locally bounded  $k$ -category (see [21, 2.1], for more details on the construction of  $F_{\tilde{T}, \lambda}$ ). We prove that  $\mathcal{C}'$  is connected.

By definition of  $\tilde{T}$ , we have  ${}^g\tilde{T} = \tilde{T}$  for every  $g \in G$ . Hence the functor  $- \underset{\mathcal{C}'}{\overset{\mathbb{L}}{\otimes}} \tilde{T}$  is  $G$ -equivariant. On the other hand,  $- \underset{\mathcal{C}'}{\overset{\mathbb{L}}{\otimes}} \tilde{T}$  is a triangle equivalence. Indeed, by 4.1, (d), and by classical arguments on derived equivalences (see [14, III.2], for instance), this functor is full and faithful. Moreover its image contains the complexes  ${}^g\tilde{T}_i$  (for  $g \in G$  and  $i \in \{1, \dots, n\}$ ). So 4.1, (b), implies that this functor is dense and, therefore, a triangle equivalence  $\mathcal{D}^b(\text{mod } \mathcal{C}') \rightarrow \mathcal{D}^b(\text{mod } \mathcal{C})$ . In particular,  $\mathcal{C}'$  is connected.

Now we assume that  $T$  is a tilting  $A$ -module. Assertion (a) follows from 4.1, (d). Assertion (b) follows from the fact that  $\text{pd}_A(T) \leq 1$  and  $F_\lambda: \text{mod } \mathcal{C} \rightarrow \text{mod } A$  is exact. We prove assertion (c). Let  $P \in \text{mod } \mathcal{C}$  be indecomposable projective. Since  $F_\lambda P$  is projective, there exists an exact sequence  $0 \rightarrow F_\lambda P \rightarrow X \rightarrow Y \rightarrow 0$  in  $\text{mod } A$  with  $X, Y \in \text{add}(T)$ . By 4.2, the triangle  $F_\lambda P \rightarrow X \rightarrow Y \rightarrow F_\lambda P[1]$  is isomorphic to the image under  $F_\lambda$  of a triangle  $P \rightarrow X' \rightarrow Y' \rightarrow P[1]$  where  $X', Y' \in \text{add}(\{{}^g\tilde{T}_i \mid g \in G, i \in \{1, \dots, n\}\})$ . Since  $F_\lambda$  is exact, the sequence  $0 \rightarrow P \rightarrow X' \rightarrow Y' \rightarrow 0$  is an exact sequence in  $\text{mod } \mathcal{C}$ .  $\square$

*Remark 4.9.* Keep the hypotheses and notations of the preceding lemma. If  $G$  is finite and if  $T$  is a tilting  $A$ -module, then the lemma expresses that  $\bigoplus_{g,i} {}^g\tilde{T}_i$  is a tilting  $\mathcal{C}$ -module.

Now we can prove Theorem C which was stated in the introduction.

**Proof of Theorem C:** By [14, Cor. 5.5], there exists a sequence of algebras:

$$A_0 = kQ, A_1 = \text{End}_{A_0}(T^{(0)}), \dots, A_l = \text{End}_{A_{l-1}}(T^{(l-1)}) = A$$

such that  $T^{(i)} \in \text{mod } A_{l-1}$  is tilting for every  $i$ . We prove the theorem by induction on  $l$ . If  $l = 0$ , then  $A = kQ$ . For any connected Galois covering  $\mathcal{C} \rightarrow A$  with group  $G$  there exists a Galois covering of quivers  $Q' \rightarrow Q$  with group  $G$  such that  $\mathcal{C} \simeq kQ'$  (see [20, Prop. 4.4]). Assume that  $l > 0$  and the conclusion of the theorem holds true for  $A_{l-1}$ . Let  $\mathcal{C} \rightarrow A$  be a connected Galois covering with group  $G$ . Note that  $T^{(l-1)}$  is a tilting  $A^{op}$ -module. So the preceding lemma yields a connected Galois covering  $\mathcal{C}' \rightarrow \text{End}_{A^{op}}(T^{(l-1)})$  with group  $G$  such that  $\mathcal{D}^b(\text{mod } \mathcal{C}^{op})$  and  $\mathcal{D}^b(\text{mod } \mathcal{C}')$  are triangle equivalent. On the other hand,  $A_{l-1} \simeq \text{End}_{A^{op}}(T^{(l-1)})^{op}$ . Therefore the induction hypothesis implies that  $\mathcal{D}^b(\text{mod } \mathcal{C}'^{op})$  is triangle equivalent to  $\mathcal{D}^b(\text{mod } kQ')$  where  $Q'$  is a quiver such that there exists a Galois covering of quivers  $Q' \rightarrow Q$  with group  $G$ .  $\square$

*Remark 4.10.* Let  $A$  be a finite dimensional algebra endowed with a (non necessarily free)  $G$ -action. Then:

- (a) If the  $G$ -action on  $A$  is free, then the quotient algebra  $A/G$  is well-defined. The proof of Theorem C shows that if  $A/G$  is tilted (or, more generally, piecewise hereditary), then so is  $A$ .
- (b) It is proved in [10, Thm. 3] that if the order of  $G$  is invertible in  $k$  and if  $A$  is piecewise hereditary, then so is the skew-group algebra  $A[G]$ . Recall that if  $G$  acts freely on  $A$ , then the algebras  $A[G]$  and  $A/G$  are Morita equivalent (see [9, Thm.2.8]).

## 5 Correspondence between Galois coverings

We still assume that  $A$  is derived equivalent to a hereditary abelian category  $\mathcal{H}$  such that  $\vec{\mathcal{K}}_{\mathcal{H}}$  is connected. Let  $T \in \mathcal{D}^b(\text{mod } A)$  be a tilting complex and  $B = \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$ . In this section, we construct a correspondence between the Galois coverings of  $A$  and those of  $B$ . This work has been done in [21] in the particular case where  $T$  is a tilting  $A$ -module. In order to compare the Galois coverings of  $A$  and those of  $B$ , it is convenient to use the notion of equivalent Galois covering. Given two Galois coverings  $F: \mathcal{C} \rightarrow A$  and  $F': \mathcal{C}' \rightarrow A$ , we say that  $F$  and  $F'$  are equivalent if there exists a commutative diagram:

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow{\sim} & \mathcal{C}' \\ F \downarrow & & \downarrow F' \\ A & \xrightarrow{\sim} & A \end{array}$$

where the horizontal arrows are isomorphisms and  $\varphi: A \rightarrow A$  is an automorphism such that  $\varphi(x) = x$  for every  $x \in \text{ob}(A)$ .

### Equivalence classes of Galois coverings of $A$ associated to equivalence classes of Galois coverings of $B$

In 4.8, we have associated a Galois covering  $F_{\tilde{T}, \lambda}$  of  $B$  to any Galois covering of  $A$  and to any data consisting of isomorphisms  $(\lambda_i: F_\lambda \tilde{T}_i \rightarrow T_i)_{i=1, \dots, n}$  in  $\mathcal{D}^b(\text{mod } \mathcal{C})$ . The following lemma shows that different choices for these data give rise to equivalent Galois coverings.

**Lemma 5.1.** [21, § 2] *Let  $F: \mathcal{C} \rightarrow A$  be a connected Galois covering with group  $G$ . Let  $T \in \mathcal{D}^b(\text{mod } A)$  be a tilting complex and  $T = T_1 \oplus \dots \oplus T_n$  be an indecomposable decomposition.*

- (a) *Let  $(\lambda_i: F_\lambda \tilde{T}_i \rightarrow T_i)_{i=1, \dots, n}$  and  $(\mu_i: F_\lambda \hat{T}_i \rightarrow T_i)_{i=1, \dots, n}$  be isomorphisms in  $\mathcal{D}^b(\text{mod } A)$  defining the Galois coverings  $F_{\tilde{T}, \lambda}: \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})}(\tilde{T}) \rightarrow \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  and  $F_{\hat{T}, \mu}: \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})}(\hat{T}) \rightarrow \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$ , respectively. Then  $F_{\tilde{T}, \lambda}$  and  $F_{\hat{T}, \mu}$  are equivalent. We write  $[F]_T$  for the corresponding equivalence class of Galois coverings of  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$ .*
- (b) *Let  $F': \mathcal{C}' \rightarrow A$  be a connected Galois covering with group  $G$  and equivalent to  $F$ . Then the equivalence classes  $[F]_T$  and  $[F']_T$  coincide.*

**Proof:** In the case of tilting modules, (a) and (b) were proved in [21, Lem. 2.4] and [21, Lem. 2.5], respectively. Using 2.3 and 4.7, it is easily checked that the same arguments apply to prove (a) and (b) in the present situation.  $\square$

In the sequel, we keep the notation  $[F]_T$  introduced in 5.1.

### Galois coverings of $A$ induced by Galois coverings of $B$

We now express any Galois covering of  $A$  as induced by a Galois covering of  $B$  as in 4.8. The tilting complex  $T$  is naturally a complex of  $B - A$ -bimodules. Also, it defines a triangle equivalence:

$$-\overset{\mathbb{L}}{\otimes}_B X: \mathcal{D}^b(\text{mod } B) \rightarrow \mathcal{D}^b(\text{mod } A).$$

Fix a connected Galois covering  $F: \mathcal{C} \rightarrow A$  with group  $G$ , an indecomposable decomposition  $T = T_1 \oplus \dots \oplus T_n$  and isomorphisms  $(\mu_i: F_\lambda \tilde{T}_i \xrightarrow{\sim} T_i)_{i=1, \dots, n}$ . According to 4.8, these data define the Galois covering  $F_{\tilde{T}, \mu}: \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})}(\tilde{T}) \rightarrow B$  which we denote by  $F': \mathcal{C}' \rightarrow B$  for simplicity.

**Lemma 5.2.** *The following diagram commutes up to an isomorphism of functors.*

$$\begin{array}{ccc} \mathcal{D}^b(\text{mod } \mathcal{C}') & \xrightarrow{-\overset{\mathbb{L}}{\otimes}_{\mathcal{C}'} \tilde{T}} & \mathcal{D}^b(\text{mod } \mathcal{C}) \\ F'_\lambda \downarrow & & \downarrow F_\lambda \\ \mathcal{D}^b(\text{mod } B) & \xrightarrow{-\overset{\mathbb{L}}{\otimes}_B T} & \mathcal{D}^b(\text{mod } A) \end{array},$$

**Proof:** Recall that  $F_\lambda: \text{mod } \mathcal{C} \rightarrow \text{mod } A$  (or  $F'_\lambda: \text{mod } \mathcal{C}' \rightarrow \text{mod } B$ ) is equal to  $-\overset{\mathbb{L}}{\otimes}_{\mathcal{C}} A$  (or to  $-\overset{\mathbb{L}}{\otimes}_{\mathcal{C}'} B$ , respectively).

Since these two functors are exact and map projective modules to projective modules and the horizontal arrows of the diagram are triangle equivalences (see 5.1), we deduce that:

1. The composition  $\mathcal{D}^b(\text{mod } \mathcal{C}') \xrightarrow{F'_\lambda} \mathcal{D}^b(B) \xrightarrow{-\overset{\mathbb{L}}{\otimes}_B T} \mathcal{D}^b(\text{mod } A)$  is isomorphic to  $-\overset{\mathbb{L}}{\otimes}_{\mathcal{C}'} \left( B \overset{\mathbb{L}}{\otimes}_B T \right)$ .
2. The composition  $\mathcal{D}^b(\text{mod } \mathcal{C}') \xrightarrow{-\overset{\mathbb{L}}{\otimes}_{\mathcal{C}'} \tilde{T}} \mathcal{D}^b(\text{mod } \mathcal{C}) \xrightarrow{F_\lambda} \mathcal{D}^b(\text{mod } A)$  is isomorphic to  $-\overset{\mathbb{L}}{\otimes}_{\mathcal{C}'} \left( \tilde{T} \overset{\mathbb{L}}{\otimes}_{\mathcal{C}} A \right)$ .

On the other hand, the isomorphisms  $\mu_i: F_\lambda \tilde{T}_i \xrightarrow{\sim} T_i$  (for  $i \in \{1, \dots, n\}$ ) define an isomorphism  $B \overset{\mathbb{L}}{\otimes}_B T \xrightarrow{\sim} \tilde{T} \overset{\mathbb{L}}{\otimes}_{\mathcal{C}} A$  of  $\mathcal{C}' - A$ -bimodules. This proves that the diagram commutes up to an isomorphism of functors.  $\square$

Since  $-\overset{\mathbb{L}}{\otimes}_B T$  is an equivalence, there exists an isomorphism  $\varphi_x: X_x \overset{\mathbb{L}}{\otimes}_B T \rightarrow A(-, x)$  in  $\mathcal{D}^b(\text{mod } A)$  with  $X_x \in \mathcal{D}^b(\text{mod } B)$  for every  $x \in \text{ob}(A)$ . In particular,  $\bigoplus_{x \in \text{ob}(A)} X_x$  is an indecomposable decomposition of a

tilting complex in  $\mathcal{D}^b(\text{mod } B)$ . Then by the preceding section, there exists an isomorphism  $\nu_x: F'_\lambda \tilde{X}_x \xrightarrow{\sim} X_x$  in  $\mathcal{D}^b(\text{mod } B)$  with  $\tilde{X}_x \in \mathcal{D}^b(\text{mod } \mathcal{C}')$  for every  $x \in \text{ob}(A)$ . By 4.8, the datum  $(\nu_x)_{x \in \text{ob}(A)}$  defines a connected Galois covering with group  $G$ :

$$\begin{array}{ccc} F'_{\tilde{X}, \nu}: \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C}')}(\tilde{X}) & \rightarrow & \text{End}_{\mathcal{D}^b(\text{mod } B)}(X) \\ & \xrightarrow{g} & \tilde{X}_x \mapsto X_x \\ & \xrightarrow{g} & \tilde{X}_x \xrightarrow{u} h \tilde{X}_y \mapsto X_x \xrightarrow{\nu_y F'_\lambda(u) \nu_x^{-1}} X_y \end{array}.$$

On the other hand, the isomorphisms  $\varphi_x$  (for  $x \in \text{ob}(A)$ ) define the following isomorphism of  $k$ -categories:

$$\begin{aligned} \rho_{X,\varphi}: \quad \text{End}_{\mathcal{D}^b(\text{mod } B)}(X) &\rightarrow A \\ X_x &\mapsto x \\ X_x \xrightarrow{u} X_y &\mapsto (\varphi_y \circ (u \otimes T) \circ \varphi_x^{-1})(\text{Id}_x) \in A(x, y) \quad . \end{aligned}$$

Thus, we have a connected Galois covering  $\rho_{X,\varphi} \circ F'_{\tilde{X},\nu}: \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C}')}(\tilde{X}) \rightarrow A$  with group  $G$  which we denote by  $F''$ . The following lemma relates  $F$  and  $F''$ .

**Lemma 5.3.** *The Galois coverings  $F$  and  $F''$  are equivalent.*

**Proof:** We need to construct a commutative diagram:

$$\begin{array}{ccc} \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C}')}(\tilde{X}) & \xrightarrow{\sim} & \mathcal{C} \\ F'' \downarrow & & \downarrow F \\ A & \xrightarrow{\sim} & A \end{array} \quad (D)$$

where the horizontal arrows are isomorphisms and the bottom horizontal isomorphism extends the identity map on objects. For this purpose, we proceed in two steps.

**Step 1:** We express  $F$  as a functor between subcategories of  $\mathcal{D}^b(\text{mod } \mathcal{C})$  and  $\mathcal{D}^b(\text{mod } A)$ . Given  $x \in \text{ob}(\mathcal{C})$ , the  $A$ -module  $F_\lambda(\mathcal{C}(-, x))$  does depend only on  $F(x)$  (and not on  $x$ ) because  $F_\lambda$  is  $G$ -invariant. Besides, there is a canonical isomorphism  $\iota_x: F_\lambda(\mathcal{C}(-, x)) \xrightarrow{\sim} A(-, F(x))$  of  $A$ -modules induced by  $F$ : If  $y \in \text{ob}(A)$ , then  $(F_\lambda(\mathcal{C}(-, x)))(y) = \bigoplus_{Fy=F(y)} \mathcal{C}(y', x)$  and an element  $(u_{y'})_{y'}$  of this vector space is mapped by  $\iota_x$  to  $\sum_{y'} F(u_{y'}) \in A(F(y), F(x))$ . Clearly, this isomorphism does depend only on  $F(x)$  (and not on  $x$ ) whence the notation  $\iota_x$ . Now, let  $\mathcal{P}_A$  and  $\mathcal{P}_\mathcal{C}$  be the full subcategories of  $\mathcal{D}^b(\text{mod } A)$  and  $\mathcal{D}^b(\text{mod } \mathcal{C})$  with object sets  $\{A(-, x) \mid x \in \text{ob}(A)\}$  and  $\{\mathcal{C}(-, x) \mid x \in \text{ob}(\mathcal{C})\}$ , respectively. Hence we have a commutative diagram:

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow{\sim} & \mathcal{P}_\mathcal{C} \\ F \downarrow & & \downarrow \\ A & \xrightarrow{\sim} & \mathcal{P}_A \end{array} \quad (D_1)$$

where the unlabelled functors are as follows:

1. The functor  $\mathcal{C} \rightarrow \mathcal{P}_\mathcal{C}$  is the following isomorphism:

$$\begin{aligned} \mathcal{C} &\rightarrow \mathcal{P}_\mathcal{C} \\ x \in \text{ob}(\mathcal{C}) &\mapsto \mathcal{C}(-, x) \\ u \in \mathcal{C}(x, y) &\mapsto \mathcal{C}(-, u): \mathcal{C}(-, x) \rightarrow \mathcal{C}(-, y) \quad . \end{aligned}$$

2. The functor  $A \rightarrow \mathcal{P}_A$  is the following isomorphism:

$$\begin{aligned} A &\rightarrow \mathcal{P}_A \\ x \in \text{ob}(A) &\mapsto A(-, x) \\ u \in A(x, y) &\mapsto A(-, u): A(-, x) \rightarrow A(-, y) \quad . \end{aligned}$$

3. The functor  $\mathcal{P}_\mathcal{C} \rightarrow \mathcal{P}_A$  is as follows:

$$\begin{aligned} \mathcal{P}_\mathcal{C} &\rightarrow \mathcal{P}_A \\ \mathcal{C}(-, x) &\mapsto A(-, F(x)) \\ \mathcal{C}(-, x) \xrightarrow{u} \mathcal{C}(-, y) &\mapsto A(-, F(x)) \xrightarrow{\iota_y \circ F_\lambda(u) \circ \iota_x^{-1}} A(-, F(y)) \quad . \end{aligned} \quad (i)$$

In particular,  $\mathcal{P}_\mathcal{C} \rightarrow \mathcal{P}_A$  is a Galois covering with group  $G$ .

**Step 2:** We now relate  $F''$  to the Galois covering  $\mathcal{P}_\mathcal{C} \rightarrow \mathcal{P}_A$ . We first construct an isomorphism  $\text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C}')}(\tilde{X}) \xrightarrow{\sim} \mathcal{P}_\mathcal{C}$ . Let  $\Theta: F'_\lambda(-) \otimes_B T \xrightarrow{\sim} F_\lambda(-) \otimes_{\mathcal{C}'} \tilde{T}$  be an isomorphism of functors (see 5.2). Let  $x \in \text{ob}(A)$ . So we have a composition of isomorphisms in  $\mathcal{D}^b(\text{mod } A)$ :

$$F_\lambda \left( \tilde{X}_x \otimes_{\mathcal{C}'} \tilde{T} \right) \xrightarrow{\Theta_{\tilde{X}_x}^{-1}} F'_\lambda \tilde{X}_x \otimes_B T \xrightarrow{\nu_x \otimes T} X_x \otimes_B T \xrightarrow{\varphi_x} A(-, x) \quad .$$

Therefore, by 2.3, there exists an isomorphism  $\psi_x: \tilde{X}_x \otimes_{\mathcal{C}'} \tilde{T} \xrightarrow{\sim} \mathcal{C}(-, L(x))$  in  $\mathcal{D}^b(\text{mod } \mathcal{C})$  with  $L(x) \in F^{-1}(x)$ .

We deduce that the following is an isomorphism of  $k$ -categories because  $-\otimes_{\mathcal{C}'} \tilde{T}$  is a  $G$ -equivariant functor (see 4.8):

$$\begin{aligned} \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C}')}(\tilde{X}) &\rightarrow \mathcal{P}_\mathcal{C} \\ g \tilde{X}_x &\mapsto \mathcal{C}(-, gL(x)) \\ g \tilde{X}_x \xrightarrow{u} h \tilde{X}_y &\mapsto \mathcal{C}(-, gL(x)) \xrightarrow{h \psi_y \circ (u \otimes \tilde{T}) \circ (g \psi_x)^{-1}} \mathcal{C}(-, hL(y)) \quad . \end{aligned} \quad (ii)$$

We now construct another isomorphism between  $A$  and  $\mathcal{P}_A$ . We have the following composition of isomorphisms in  $\mathcal{D}^b(\text{mod } A)$  which we denote by  $\alpha_x$ :

$$\alpha_x : A(-, x) \xrightarrow{\varphi_x^{-1}} X_x \otimes_B^{\mathbb{L}} T \xrightarrow{(\nu_x \otimes T)^{-1}} F'_\lambda \tilde{X}_x \otimes_B^{\mathbb{L}} T \xrightarrow{\Theta_{\tilde{X}_x}} F_\lambda \left( \tilde{X}_x \otimes_{C'}^{\mathbb{L}} \tilde{T} \right) \xrightarrow{F_\lambda(\psi_x)} F_\lambda(\mathcal{C}(-, L(x))) \xrightarrow{\iota_x} A(-, x).$$

Note that  $\alpha_x : A(-, x) \xrightarrow{\sim} A(-, x)$  is necessarily equal to the multiplication by a scalar in  $k^*$  because  $A(-, x)$  is an indecomposable projective  $A$ -module and  $A$  is piecewise hereditary. Therefore we have an isomorphism of categories:

$$\begin{aligned} A &\rightarrow \mathcal{P}_A \\ x &\mapsto A(-, x) \\ u \in A(x, y) &\mapsto \alpha_y \circ A(-, u) \circ \alpha_x^{-1} \end{aligned} \quad (iii)$$

Hence the horizontal arrows of the following diagram are isomorphisms:

$$\begin{array}{ccc} \text{End}_{\mathcal{D}^b(\text{mod } C')}(\tilde{X}) & \xrightarrow{(ii)} & \mathcal{P}_C \\ F'' \downarrow & & \downarrow (i) \\ A & \xrightarrow{(iii)} & \mathcal{P}_A \end{array} \quad (D_2)$$

We claim that this diagram commutes. The commutativity is clearly satisfied on objects. Let  $u : {}^g \tilde{X}_x \rightarrow {}^h \tilde{X}_y$  be a morphism in  $\text{End}_{\mathcal{D}^b(\text{mod } C')}(\tilde{X})$ . Denote by  $u_1 : A(-, x) \rightarrow A(-, y)$  the image of  $u$  under the composition of (i) and (ii). Then:

$$\begin{aligned} u_1 &= \iota_y \circ F_\lambda \left( {}^h \psi_y \circ (u \otimes \tilde{T}) \circ ({}^g \psi_x)^{-1} \right) \circ \iota_x^{-1} \\ &= \iota_y \circ F_\lambda(\psi_y) \circ F_\lambda \left( u \otimes \tilde{T} \right) \circ (F_\lambda(\psi_x))^{-1} \circ \iota_x^{-1} && \text{because } F_\lambda \text{ is } G\text{-invariant,} \\ &= \iota_y \circ F_\lambda(\psi_y) \circ \Theta_{\tilde{X}_y} \circ (F'_\lambda(u) \otimes T) \circ \Theta_{\tilde{X}_x}^{-1} \circ (F_\lambda(\psi_x))^{-1} \circ \iota_x^{-1} && \text{by definition of } \Theta, \\ &= \alpha_y \circ \varphi_y \circ (\nu_y \otimes T) \circ (F'_\lambda(u) \otimes T) \circ (\nu_x \otimes T)^{-1} \circ \varphi_x^{-1} \circ \alpha_x^{-1} && \text{by definition of } \alpha_x \text{ and } \alpha_y, \\ &= \alpha_y \circ \varphi_y \circ ((\nu_y \circ F'_\lambda(u) \circ \nu_x^{-1}) \otimes T) \circ \varphi_x^{-1} \circ \alpha_x^{-1} \end{aligned}$$

Now, let  $u_2 \in A(x, y)$  be the image of  $u$  under  $F''$ , that is  $u_2 = (\varphi_y \circ ((\nu_y \circ F'_\lambda(u) \circ \nu_x^{-1}) \otimes T) \circ \varphi_x^{-1}) (\text{Id}_x)$ . Therefore  $A(-, u_2)$  is equal to the morphism  $\varphi_y \circ ((\nu_y \circ F'_\lambda(u) \circ \nu_x^{-1}) \otimes T) \circ \varphi_x^{-1} : A(-, x) \rightarrow A(-, y)$  of  $\mathcal{P}_A$ . In particular, the image of  $u_2$  under (iii) coincides with  $u_1$ . Therefore (D<sub>2</sub>) is commutative. Since (D<sub>1</sub>) also commutes, we deduce that so does (D). Thus,  $F$  and  $F''$  are equivalent.  $\square$

## Correspondence between the Galois coverings of $A$ and those of $B$

**Proposition 5.4.** *Let  $A$  be an algebra derived equivalent to a hereditary abelian category  $\mathcal{H}$  such that  $\vec{\mathcal{K}}_{\mathcal{H}}$  is connected. Let  $T \in \mathcal{D}^b(\text{mod } A)$  be a tilting complex,  $B = \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  and  $G$  a group. With the notations of 5.1, the map  $[F] \mapsto [F]_T$  is a well-defined bijection from the set of equivalence classes of connected Galois coverings with group  $G$  of  $A$  to the set of equivalence classes of Galois coverings with group  $G$  of  $B$ .*

**Proof:** Let  $\text{Gal}_A(G)$  be the set of equivalence classes of connected Galois coverings with group  $G$  of  $A$ . By 5.1, there is a well-defined map:

$$\begin{aligned} \gamma_A : \text{Gal}_A(G) &\rightarrow \text{Gal}_B(G) \\ [F] &\mapsto [F]_T \end{aligned}$$

We keep the notations  $X_x, \varphi_x$  (for  $x \in \text{ob}(A)$ ) introduced after the proof of 5.2. Then we also have a well-defined map:

$$\begin{aligned} \gamma_B : \text{Gal}_B(G) &\rightarrow \text{Gal}_{\text{End}_{\mathcal{D}^b(\text{mod } B)}(X)}(G) \\ [F] &\mapsto [F]_X \end{aligned}$$

By 5.3, we know that  $\gamma_A$  is injective and  $\gamma_B$  is surjective. Therefore  $\gamma_A$  is bijective because  $A, T$  and  $B, X$  play symmetric rôles.  $\square$

By 5.4, we have some information on the existence of a universal cover. Indeed, we have the following result.

**Proposition 5.5.** *Let  $A$  be as in 5.4 and  $T \in \mathcal{D}^b(\text{mod } A)$  a tilting complex. Assume that  $A$  admits a universal cover  $\tilde{F} : \tilde{\mathcal{C}} \rightarrow A$ . Then  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  admits a universal cover with group isomorphic to the one of  $\tilde{F}$ .*

**Proof:** Fix an indecomposable decomposition  $T = T_1 \oplus \dots \oplus T_n$ . Let  $B = \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$ . So  $B$  is the full subcategory of  $\mathcal{D}^b(\text{mod } A)$  with objects  $T_1, \dots, T_n$ . Let  $x_0 \in \text{ob}(A)$  be a base-point for the category  $\text{Gal}(A, x_0)$  of pointed Galois coverings of  $A$ . We construct a (full and faithful) functor  $\tilde{F}^{\rightarrow} \rightarrow \text{Gal}(B, T_1)$ . Recall that  $\tilde{F}^{\rightarrow}$  was defined in Section 1 and there is at most one morphism between two pointed Galois coverings. We need the following data:

1. For every  $i \in \{1, \dots, n\}$ , let  $\tilde{T}_i \in \text{mod } \tilde{\mathcal{C}}$  be such that  $\tilde{F}_\lambda \tilde{T}_i \simeq T_i$ . Therefore the  $k$ -categories  $B = \text{End}_{\mathcal{D}^b(\text{mod } A)}(T)$  and  $\text{End}_{\mathcal{D}^b(\text{mod } A)}(\bigoplus_{i=1}^n F_\lambda \tilde{T}_i)$  are isomorphic. For simplicity, we assume that  $\tilde{F}_\lambda \tilde{T}_i = T_i$  for every  $i$ .
2. If  $F \in \tilde{F}^\rightarrow$ , there exists a unique morphism  $p: \tilde{F} \rightarrow F$  in  $\text{Gal}(A, x_0)$ . Since  $p$  is a Galois covering (see [20, Prop. 3.4]), we set  $T_i^F = p_\lambda \tilde{T}_i$  for every  $i$ .

Then:

- (i) We have  $T_i = F_\lambda(T_i^F)$  for every  $i \in \{1, \dots, n\}$  and  $F \in \tilde{F}^\rightarrow$ . Indeed, there exists a unique morphism  $p: \tilde{F} \rightarrow F$ , so that  $\tilde{F}_\lambda = F_\lambda \circ p_\lambda$ .
- (ii) Let  $u: F \rightarrow F'$  be a morphism in  $\tilde{F}^\rightarrow$ . Let  $G$  be the group of  $F$  and  $G'$  the group of  $F'$ . Then  $u$  is a Galois covering (see [20, Prop. 3.4]). Let  $p: \tilde{F} \rightarrow F$  and  $p': \tilde{F} \rightarrow F'$  be the unique morphisms in  $\text{Gal}(A, x_0)$  from  $\tilde{F}$  to  $F$  and from  $\tilde{F}$  to  $F'$ , respectively. Then  $p' = u \circ p$  because of [20, Lem. 3.1] and because we are dealing with pointed Galois coverings. Therefore  $u_\lambda(T_i^F) = \sigma_{u(g)} T_i^{F'}$  for every  $i \in \{1, \dots, n\}$  and every  $g \in G$ . Here,  $\sigma_u: G \rightarrow G'$  is the unique (surjective) morphism of groups such that  $u \circ g = \sigma_u(g) \circ u$  for every  $g \in G$  (see [20, Prop. 3.4]).

Now we can construct a functor  $\tilde{F}^\rightarrow \rightarrow \text{Gal}(B, T_1)$ . Let  $F: (\mathcal{C}, x) \rightarrow (A, x_0)$  be in  $\tilde{F}^\rightarrow$ . Let  $G$  be the group of  $F$ . By (i) and 4.8, we have a pointed Galois covering with group  $G$  induced by  $F_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } A)$ :

$$F_T: \left( \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})} \left( \bigoplus_{g,i} {}^g T_i^F \right), T_1^F \right) \rightarrow (B, T_1)$$

$$\begin{array}{ccc} {}^g T_i^F & \mapsto & T_i \\ {}^g T_i^F \xrightarrow{f} {}^h T_j^F & \mapsto & T_i \xrightarrow{F_\lambda(f)} T_j \end{array} .$$

So  $[F_T] = [F]_T$ . Thus, we have associated a pointed Galois covering with group  $G$  of  $B$  to any pointed Galois covering with group  $G$  of  $A$ . We now associate a morphism of pointed Galois coverings of  $B$  to any morphism of pointed Galois coverings of  $A$ . Let  $u: F \rightarrow F'$  be a morphism in  $\tilde{F}^\rightarrow$  where  $F: (\mathcal{C}, x) \rightarrow (A, x_0)$  and  $F': (\mathcal{C}', x') \rightarrow (A, x_0)$  have groups  $G$  and  $G'$ , respectively. By (ii), we have a well-defined  $k$ -linear functor induced by  $u_\lambda: \mathcal{D}^b(\text{mod } \mathcal{C}) \rightarrow \mathcal{D}^b(\text{mod } \mathcal{C}')$ :

$$u_T: \left( \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C})} \left( \bigoplus_{g,i} {}^g T_i^F \right), T_1^F \right) \rightarrow \left( \text{End}_{\mathcal{D}^b(\text{mod } \mathcal{C}')} \left( \bigoplus_{g',i} {}^{g'} T_i^{F'} \right), T_1^{F'} \right)$$

$$\begin{array}{ccc} {}^g T_i^F & \mapsto & u_\lambda({}^g T_i^F) = \sigma_{u(g)} T_i^{F'} \\ {}^g T_i^F \xrightarrow{f} {}^h T_j^F & \mapsto & \sigma_{u(g)} T_i^{F'} \xrightarrow{u_\lambda(f)} \sigma_{u(h)} T_j^{F'} \end{array} .$$

The equality  $u_\lambda({}^g T_i^F) = \sigma_{u(g)} T_i^{F'}$  follows from the equality  $u \circ g = \sigma_{u(g)} \circ u$ . Also,  $u_\lambda(T_1^F) = T_1^{F'}$ . Since  $F' \circ u = F$  and  $F_T, F'_T$  and  $u_T$  are defined as restrictions of  $F_\lambda, F'_\lambda$  and  $u_\lambda$  respectively,  $u_T: F_T \rightarrow F'_T$  is a morphism in  $\text{Gal}(B, T_1)$ . Thus, to any morphism in  $\tilde{F}^\rightarrow$ , we have associated a morphism in  $\text{Gal}(B, T_1)$ . We let the reader check that the following is a functor:

$$\Psi: \begin{array}{ccc} \tilde{F}^\rightarrow & \rightarrow & \text{Gal}(B, T_1) \\ F & \mapsto & F_T \\ F \xrightarrow{u} F' & \mapsto & F_T \xrightarrow{u_T} F'_T \end{array}$$

Also, it is not difficult to prove that  $\Psi$  is full and faithful, although we shall not use this fact in the sequel. Remark that the Galois covering  $F_T$  lies in  $\Psi(\tilde{F}^\rightarrow)$  for every  $F \in \tilde{F}^\rightarrow$ .

Now we can prove that  $\Psi(\tilde{F}^\rightarrow)$  is a universal cover for  $B$ . Let  $F$  be a connected Galois covering of  $B$ . By 5.4, there exists a connected Galois covering  $F'$  of  $A$  such that  $[F] = [F']_T$ . Since  $\tilde{F}$  is a universal cover of  $A$ , the Galois covering  $F'$  of  $A$  is equivalent to some  $F'' \in \tilde{F}^\rightarrow$ , that is  $[F'] = [F'']$ . As noticed above, we have  $[F'_T] = [F''_T]$ . Therefore  $[F] = [F']_T = [F''_T]_T = [F''_T]$ , that is,  $F$  is equivalent to a Galois covering of  $B$  lying in  $\Psi(\tilde{F}^\rightarrow)$ . So  $\Psi(\tilde{F}^\rightarrow)$  is a universal Galois covering of  $B$ .  $\square$

## 6 The main theorem and its corollary

In this section, we prove Theorem A and Corollary B. We assume that  $A$  is a connected algebra derived equivalent to a hereditary abelian category  $\mathcal{H}$  such that  $\overrightarrow{\mathcal{K}}_{\mathcal{H}}$  is connected.

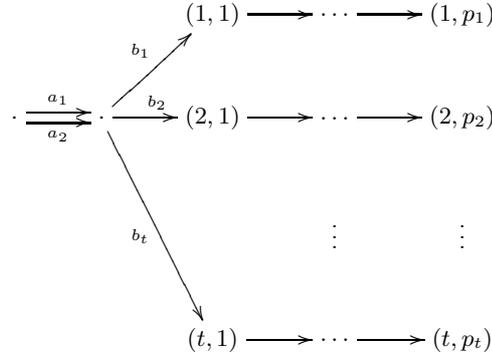
### Two particular cases: paths algebras and squid algebras

We first check that our main results hold for paths algebras and for squid algebras.

**Lemma 6.1.** *Assume that  $A = kQ$  where  $Q$  is a finite connected quiver with no oriented cycle. Then Theorem A and Corollary B hold true for  $A$ .*

**Proof:** Let  $\tilde{Q} \rightarrow Q$  be the universal Galois covering of quivers (see [23]). It follows from [20, Prop. 4.4] that the induced Galois covering  $k\tilde{Q} \rightarrow kQ$  with group  $\pi_1(Q)$  is a universal cover of  $A$ . Whence Theorem C. On the other hand,  $\mathrm{HH}^1(kQ) = 0$  if and only if  $Q$  is a tree (see [8]). Whence Corollary B.  $\square$

We now turn to the case of squid algebras. We refer the reader to [25] for more details on squid algebras. A squid algebra over an algebraically closed field  $k$  is defined by the following data: An integer  $t \geq 2$ , a sequence  $p = (p_1, \dots, p_t)$  of non negative integers and a sequence  $\tau = (\tau_3, \dots, \tau_t)$  of pairwise distinct non-zero elements of  $k$ . With this data, the squid algebra  $S(t, p, \tau)$  is the  $k$ -algebra  $kQ/I$  where  $Q$  is the following quiver:



and  $I$  is the ideal generated by the following relations:

$$b_1 a_1 = b_2 a_2 = 0, \quad b_i a_2 = \tau_i b_i a_1 \quad \text{for } i = 3, \dots, t.$$

Using Happel's long exact sequence ([15]), one can compute  $\mathrm{HH}^1(S(t, p, \tau))$ :

$$\dim_k \mathrm{HH}^1(S(t, p, \tau)) = \begin{cases} 1 & \text{if } t = 2 \\ 0 & \text{if } t \geq 3. \end{cases}$$

Following [23], the bound quiver  $(Q, I)$  defines a Galois covering  $k\tilde{Q}/\tilde{I} \rightarrow kQ/I$  with group isomorphic to  $\mathbb{Z}$  if  $t = 2$  and with trivial group otherwise. One can easily check that this Galois covering is universal in the sense of Theorem C. The above considerations give the following.

**Lemma 6.2.** *Let  $A$  be a squid algebra. Then Theorem A and Corollary B hold true for  $A$ .*

### The general case

Using 5.5, 6.1 and 6.2 we can prove the two main results of this text.

**Proof of Theorem A and Corollary B:** Assume first that  $A$  is piecewise hereditary of type  $Q$  where  $Q$  is a finite connected quiver with no oriented cycle. So there exists a tilting complex  $T \in \mathcal{D}^b(\mathrm{mod} kQ)$  such that  $A \simeq \mathrm{End}_{\mathcal{D}^b(\mathrm{mod} kQ)}(T)$ . By 5.5 and 6.1, the algebra  $A$  admits a universal Galois covering with group isomorphic to the fundamental group of  $Q$ . In particular,  $A$  is simply connected if and only if  $Q$  is a tree. On the other hand,  $Q$  is tree if and only if  $\mathrm{HH}^1(kQ) = 0$  (by [8]) and  $\mathrm{HH}^1(kQ) \simeq \mathrm{HH}^1(A)$  (by [19]). Therefore  $A$  is simply connected if and only if  $\mathrm{HH}^1(A) = 0$ , or, if and only if  $Q$  is a tree.

Assume now that  $A$  is not derived equivalent to  $\mathcal{D}^b(\mathrm{mod} kQ)$  for any finite quiver  $Q$ . Then [16, Prop. 2.1, Thm. 2.6] implies that there exists a squid algebra  $S = S(t, p, \tau)$  and a tilting complex  $T \in \mathcal{D}^b(\mathrm{mod} S)$  such that  $A \simeq \mathrm{End}_{\mathcal{D}^b(\mathrm{mod} S)}(T)$ . By 5.5 and 6.2, the algebra  $A$  has a universal cover with group isomorphic to the trivial group or to  $\mathbb{Z}$  according to whether  $t \geq 3$  or  $t = 2$ . In particular,  $A$  is simply connected if and only if  $t = 2$ , that is, if and only if  $\mathrm{HH}^1(S) = 0$  (see 6.2). Since  $\mathrm{HH}^1(S) \simeq \mathrm{HH}^1(A)$  (by [19]), we deduce that  $A$  is simply connected if and only if  $\mathrm{HH}^1(A) = 0$ .  $\square$

## References

- [1] I. Assem, E. N. Marcos, and J. A. de La Peña. The simple connectedness of a tame tilted algebra. *J. Algebra*, 237(2):647–656, 2001.
- [2] I. Assem, D. Simson, and A. Skowroński. *Elements of the representation theory of associative algebras. Vol. 1: Techniques of representation theory*. London Mathematical Society Student Texts 65. Cambridge: Cambridge University Press. ix, 458 p., 2006.
- [3] I. Assem and A. Skowroński. On some classes of simply connected algebras. *Proc. London Math. Soc.*, 56(3):417–450, 1988.
- [4] K. Bongartz and P. Gabriel. Covering spaces in representation theory. *Invent. Math.*, 65:331–378, 1982.
- [5] A. B. Buan, R. Marsh, M. Reineke, I. Reiten, and G. Todorov. Tilting theory and cluster combinatorics. *Adv. Math.*, 204(2):572–618, 2006.

- [6] R.-O. Buchweitz and S. Liu. Hochschild cohomology and representation-finite algebras. *Proc. London Math. Soc., Ser. III*, 88(2):355–380, 2004.
- [7] P. Caldero, F. Chapoton, and R. Schiffler. Quivers with relations arising from clusters ( $A_n$  case). *Trans. Amer. Math. Soc.*, 2006.
- [8] C. Cibils. On the Hochschild cohomology of finite dimensional algebras. *Comm. Algebra*, 16(3):645–649, 1988.
- [9] C. Cibils and E. N. Marcos. Skew categories, Galois coverings and smash-product of a  $k$ -category. *Proc. Amer. Math. Soc.*, 134(1):39–50, 2006.
- [10] J. Dionne, M. Lanzilotta, and D. Smith. Skew group algebras of piecewise hereditary algebras are piecewise hereditary. *J. Pure Appl. Algebra*, 213(2):241–249, 2009.
- [11] P. Dowbor and A. Skowroński. Galois coverings of representation infinite algebras. *Comment. Math. Helv.*, 62:311–337, 1987.
- [12] S. Fomin and A. Zelevinsky. Cluster algebras I: Foundations. *Trans. Amer. Math. Soc.*, 353:497–529, 2002.
- [13] P. Gabriel. The universal cover of a representation-finite algebra. Representations of algebras, Proc. 3rd int. Conf., Puebla/Mex. 1980, Lecture Notes in Mathematics 903, 68–105 (1981)., 1981.
- [14] D. Happel. *Triangulated categories in the representation theory of finite dimensional algebras*, volume 119 of *London Mathematical Society Lecture Notes Series*. Cambridge University Press, Cambridge, 1988.
- [15] D. Happel. Hochschild cohomology of finite dimensional algebras. *Séminaire d'Algèbre Paul Dubreuil, Marie-Paule Malliavin, Lecture Notes in Mathematics*, 1404:108–126, 1989.
- [16] D. Happel and I. Reiten. Hereditary abelian categories with tilting object over arbitrary base fields. *J. Algebra*, 256(2):414–432, 2002.
- [17] D. Happel and L. Unger. On the set of tilting objects in hereditary categories. In *Representations of algebras and related topics*, volume 45 of *Fields Inst. Commun.*, pages 141–159. Amer. Math. Soc., 2005.
- [18] D. Hugues and J. Waschbüsch. Trivial extensions of tilted algebras. *Proc. London Math. Soc.*, 46:347–364, 1983.
- [19] B. Keller. Hochschild cohomology and derived Picard groups. *J. Pure Appl. Algebra*, 190:177–196, 2004.
- [20] P. Le Meur. The universal cover of an algebra without double bypass. *J. Algebra*, 312(1):330–353, 2007.
- [21] P. Le Meur. On Galois coverings and tilting modules. *J. Algebra*, 319(12):4961–4999, 2008.
- [22] P. Le Meur. The universal cover of a monomial algebra without multiple arrows. *J. Algebra Appl.*, 7(4):443–469, 2008.
- [23] R. Martínez-Villa and J. A. de la Peña. The universal cover of a quiver with relations. *J. Pure Appl. Algebra*, 30:277–292, 1983.
- [24] C. Riedtmann. Algebren, Darstellungsköcher, Ueberlagerungen und zurück. *Comment. Math. Helv.*, 55:199–224, 1980.
- [25] C. M. Ringel. The canonical algebras (with an appendix by W. Crawley-Boevey). *Banach Center Publications*, 26:407–432, 1990.
- [26] D. Simson and A. Skowroński. *Elements of the representation theory of associative algebras: 2*, volume 71 of *London Mathematical Society Student Texts*. Cambridge University Press, 2007.
- [27] D. Simson and A. Skowroński. *Elements of the representation theory of associative algebras: 3*, volume 72 of *London Mathematical Society Student Texts*. Cambridge University Press, 2007.
- [28] A. Skowroński. Selfinjective algebras of polynomial growth. *Math. Ann.*, 285:177–199, 1989.
- [29] A. Skowroński. Simply connected algebras and Hochschild cohomologies. *Can. Math. Soc. Conf. Proc.*, 14:431–447, 1993.
- [30] A. Skowroński and K. Yamagata. Stable equivalence of selfinjective algebras of tilted type. *Arch. Math.*, 70:341–350, 1998.
- [31] A. Skowroński and K. Yamagata. On invariability of self-injective algebras of tilted type under stable equivalences. *Proc. Amer. Math. Soc.*, 132(3):659–667, 2003.
- [32] A. Skowroński and K. Yamagata. Stable equivalence of selfinjective algebras of Dynkin type. *Algebr. Represent. Theory*, 9:33–45, 2006.

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