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Polynomial energy decay rate and strong stability of Kirchhoff plates with non-compact resolvent

Bopeng RAO[1] and Ali WEHBE[2]

Abstract. *Using a multiplier method, we establish the polynomial energy decay rate for the smooth solutions of Kirchhoff plates equations. Consequently, we obtain the strong stability in the absence of compactness of resolvent of the infinitesimal operator.*

Key words. Kirchhoff plates, polynomial decay rate, strong stability, non-compactness.

AMS subject classifications. 93D20, 93B05, 35B35, 35B37.

1 Introduction and main results

In this paper we study the stability of a thin elastic plate with dynamical boundary feedback controls. Let $\Omega \subset I\!R^2$ be a bounded open set with smooth boundary Γ consisting of a clamped part Γ_0, and a rimmed part Γ_1 such that $\overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \{\emptyset\}$. Following the theory of linear elasticity, the vibration of a thin elastic plate is gouverned by the linear system :

$$\begin{cases} y_{tt} - \gamma \Delta y_{tt} + \Delta^2 y = 0 & \text{in } \Omega \times I\!R^+, \\ y = \dfrac{\partial y}{\partial \nu} = 0 & \text{on } \Gamma_0 \times I\!R^+, \\ \Delta y + (1-\mu) B_1 y + \eta = 0 & \text{on } \Gamma_1 \times I\!R^+, \\ \dfrac{\partial \Delta y}{\partial \nu} + (1-\mu)\dfrac{\partial B_2 y}{\partial \tau} - \gamma \dfrac{\partial y_{tt}}{\partial \nu} + \dfrac{\partial^2 y_t}{\partial \tau^2} - \xi = 0 & \text{on } \Gamma_1 \times I\!R^+ \end{cases} \tag{1.1}$$

where $\nu = (\nu_1, \nu_2)$ denotes the unit normal vector, $\tau = (\tau_1, \tau_2)$ the unit tangent vector, and where $\gamma > 0$ and $0 < \mu < 1/2$ are physical constants. The boundary

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operators B_1, B_2 are defined by :

$$\begin{cases} B_1 y = 2\nu_1\nu_2 \dfrac{\partial^2 y}{\partial x_1 \partial x_2} - \nu_1^2 \dfrac{\partial^2 y}{\partial x_2^2} - \nu_2^2 \dfrac{\partial^2 y}{\partial x_1^2}, \\ B_2 y = (\nu_1^2 - \nu_2^2) \dfrac{\partial^2 y}{\partial x_1 \partial x_2} + \nu_1\nu_2 \left(\dfrac{\partial^2 y}{\partial x_2^2} - \dfrac{\partial^2 y}{\partial x_1^2} \right). \end{cases} \tag{1.2}$$

In the case of static feedbacks : $\eta = \partial_\nu y_t$, $\xi = y_t$, the stability of system (1.1)-(1.2) was well studied by different approaches (see [7], [9], [10], [11], [18], [23] and the references wherein). In this work, we consider the case of dynamical boundary controls $\eta(t)$, $\xi(t)$ which are given by the following integral system :

$$\begin{cases} \eta_t - \dfrac{\partial y_t}{\partial \nu} + \eta = 0 & \text{on } \Gamma_1 \times I\!R^+, \\ \xi_t - y_t + \xi = 0 & \text{on } \Gamma_1 \times I\!R^+. \end{cases} \tag{1.3}$$

The concept of dynamical control has been introduced by the automaticians in the finite dimensional case (see Francis [6]). In the infinite dimensional case, the dynamical controls form part of indirect damping mechanisms proposed by Russell [21].

Now let y be a smooth solution of system (1.1)-(1.3). We define the associated energy $E(t)$ by

$$E(t) = \frac{1}{2} \left\{ \int_\Omega (y_t^2 + \gamma \mid \nabla y_t \mid^2) dx + \int_\Omega a(y,y) dx + \int_{\Gamma_1} \left(|\eta|^2 + |\xi|^2 \right) d\Gamma \right\} \tag{1.4}$$

where for smooth functions y, z, we have put :

$$a(y,z) = \frac{\partial^2 y}{\partial x_1^2} \frac{\partial^2 z}{\partial x_1^2} + \frac{\partial^2 y}{\partial x_2^2} \frac{\partial^2 z}{\partial x_2^2} + \mu \left(\frac{\partial^2 y}{\partial x_1^2} \frac{\partial^2 z}{\partial x_2^2} + \frac{\partial^2 y}{\partial x_2^2} \frac{\partial^2 z}{\partial x_1^2} \right) + 2(1-\mu) \frac{\partial^2 y}{\partial x_1 \partial x_2} \frac{\partial^2 z}{\partial x_1 \partial x_2}.$$

Then a straightforward computation gives (see [10] and [18]) :

$$\frac{dE(t)}{dt} = - \int_{\Gamma_1} \left(|\eta|^2 + |\xi|^2 + \left| \frac{\partial y_t}{\partial \tau} \right|^2 \right) d\Gamma \leq 0. \tag{1.5}$$

Therefore system (1.1)-(1.3) is dissipative in the sense that the energy is nonincreasing. Moreover, denoting by $u = (y, y_t, \eta, \xi)$ the state of system (1.1)-(1.3), we can formulate the problem as an evolutionary equation :

$$u_t + \mathcal{A} u = 0, \qquad u(0) = u_0 \in \mathcal{H} \tag{1.6}$$

where $\mathcal{A}$ is a maximal monotone operator in an appropriate Hilbert energy space $\mathcal{H}$. In the one dimensional case [24], we have shown that equation (1.6) has non uniform energy decay rate. So we turn to establish the polynomial energy decay rate for the smooth initial data. There are several approaches about the polynomial energy decay rate. We refer to [12] for wave equation with local intern damping, [15] for abstract systems by semigroup approaches, [19] for hybrids systems and [2]

for partially damped systems by multipliers, [16] and [14] for Riesz basis approach. In this work, we will adapt a nonlinear method introduced in [19] to the linear equation (1.6) and establish the energy decay rate as $\frac{1}{t}$ for the smooth initial data in $D(\mathcal{A})$. The method is direct and gives the optimal decay rate in the underlying context.

We first establish the following polynomial energy decay rate :

Theorem 1.1 *Assume that*
$(H1)$ *:* $\Gamma_0 \neq \emptyset$ *and* $\overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \emptyset$*;*
$(H2)$ *: There exists* $\delta > 0$ *and* $x_0 \in I\!R^2$ *such that, putting* $m(x) = x - x_0$*, we have :*

$$(m \cdot \nu) \leq 0 \quad \forall x \in \Gamma_0 \quad \textit{and} \quad (m \cdot \nu) \geq \delta^{-1}, \quad \forall x \in \Gamma_1.$$

Then for any smooth initial data $u_0 \in D(\mathcal{A})$*, there exist a constant* $M > 0$ *depending only on* $\| u_0 \|_{D(\mathcal{A})}$ *such that the energy* $E(t)$ *of the system (1.6) has a polynomial decay rate:*

$$E(t) \leq \; E(0)\frac{2M}{M+t}, \quad \forall \; t \geq 0. \tag{1.7}$$

We next consider the strong stability. Notice that the resolvent $(I + \mathcal{A})^{-1}$ is not compact in the energy space $\mathcal{H}$. Therefore, the classic methods such as Lasalle's invariance principle [22] or the spectral decomposition theory of Sz-Nagy-Foias [4] don't work in that case. We have meet some technical difficulties to use more general criteria in [3] and [8] of strong stability of C^0-semigroup in the absence of compactness of the resolvent. The following result is a consequence of Theorem 1.1 and the contraction of the semigroup $e^{t\mathcal{A}}$.

Theorem 1.2 *For any usual initial data* $u_0 \in \mathcal{H}$*, the energy* $E(t)$ *of the system (1.6) asymptotically decreases to zero:*

$$\lim_{t \to +\infty} E(t) = 0. \tag{1.8}$$

The result of Theorem 1.2 is optimal in the sense that the energy $E(t)$ has no uniform decay rate in the general case. Moreover, using a spectral approach, we can prove that the result of Theorem 1.1 is optimal for the one-dimensional problem (see [24]).

2 Well-posedness of problem.

Throughout this paper, we assume that $\Gamma = \overline{\Gamma}_0 \bigcup \overline{\Gamma}_1$ of class C^4 such that $\overline{\Gamma}_0 \bigcap \overline{\Gamma}_1 = \{\emptyset\}$. We first define the following spaces :

$$X = L^2(\Gamma_1), \quad \| \eta \|_X^2 = \int_{\Gamma_1} |\eta|^2 \, d\Gamma,$$

$$V = \left\{ y \in H^1(\Omega) \; : \; y = 0 \text{ on } \Gamma_0 \right\}, \quad \| y \|_V^2 = \int_\Omega (y^2 + \gamma \mid \nabla y \mid^2) dx,$$

$$W = \left\{ y \in H^2(\Omega) \ : \ y = \frac{\partial y}{\partial \nu} = 0 \text{ on } \Gamma_0 \right\}, \quad \| \, y \, \|_W^2 = \int_\Omega a(y,y)dx.$$

For all $(y,z) \in H^4(\Omega) \times H^2(\Omega)$, let us recall Green's formula (see [10]):

$$\int_\Omega \Delta^2 yz dx = \int_\Omega a(y,z)dx$$

$$- \int_\Gamma \left(\Delta y + (1-\mu) B_1 y \right) \frac{\partial z}{\partial \nu} d\Gamma + \int_\Gamma \left(\frac{\partial \Delta y}{\partial \nu} + (1-\mu) \frac{\partial B_2 y}{\partial \tau} \right) z d\Gamma. \tag{2.1}$$

Now let y be a smooth solution of the system (1.1)-(1.2). Multiplying (1.1) by a function $\phi \in W$ and using Green's formula (2.1), we get :

$$\int_\Omega (y_{tt}\phi + \gamma \nabla y_{tt} \nabla \phi) dx + \int_\Omega a(y,\phi)dx + \int_{\Gamma_1} \eta \frac{\partial \phi}{\partial \nu} d\Gamma + \int_{\Gamma_1} \xi \phi d\Gamma + \int_{\Gamma_1} \frac{\partial y_t}{\partial \tau} \cdot \frac{\partial \phi}{\partial \tau} d\Gamma = 0. \tag{2.2}$$

Then we define the linear operators $A, D_2 \in \mathcal{L}(W;W')$, $C \in \mathcal{L}(V;V')$, $B \in \mathcal{L}(X;V')$ and $D_1 \in \mathcal{L}(X;W')$ by

$$\begin{aligned}
&< Ay, \phi >_{W' \times W} = (y,\phi)_W, && \forall \ y, \ \phi \in W, \\
&< B\xi, \phi >_{V' \times V} = (\xi,\phi)_X, && \forall \ \xi \in X, \ \forall \phi \in V, \\
&< Cy, \phi >_{V' \times V} = (y,\phi)_V, && \forall \ y, \ \phi \in V, \\
&< D_1\eta, \phi >_{W' \times W} = (\eta, \partial_\nu \phi)_X, && \forall \ \eta \in X, \ \forall \phi \in W, \\
&< D_2 y, \phi >_{W' \times W} = (\partial_\tau y, \partial_\tau \phi)_X, && \forall \ y, \ \phi \in W.
\end{aligned}$$

By virtue of Lax-Milgram's theorem (see [5]), we see that A, C is the canonical isomorphism from W onto W' respectively, from V onto V'. Thanks to Sobolev's embeddings (see [1]), we check easily that B, D_1, D_2 are continuous operators for the corresponding topologies.

Assume that $Ay + D_1\eta + D_2 z \in V'$, then we can formulate the variational equation (2.2) as :

$$y_{tt} + C^{-1}(Ay + D_1\eta + D_2 y_t + B\xi) = 0, \quad \text{in } V.$$

We define the energy space $\mathcal{H} = W \times V \times X \times X$, endowed with the usual inner product :

$$(u, \widetilde{u})_{\mathcal{H}} = \left\{ \int_\Omega (z\widetilde{z} + \gamma \nabla z \nabla \widetilde{z}) dx + \int_\Omega a(y,\widetilde{y})dx + \int_{\Gamma_1} (\eta\widetilde{\eta} + \xi\widetilde{\xi}) d\Gamma \right\},$$

$$\forall u = (y,z,\eta,\xi), \quad \widetilde{u} = (\widetilde{y},\widetilde{z},\widetilde{\eta},\widetilde{\xi}) \in \mathcal{H}. \tag{2.3}$$

Next we introduce the linear unbounded operator $\mathcal{A}$ as follows :

$$D(\mathcal{A}) = \left\{ (y,z,\eta,\xi)^T \in \mathcal{H} : z \in W \text{ and } Ay + D_1\eta + D_2 z \in V' \right\}, \tag{2.4}$$

$$\mathcal{A}u = \begin{pmatrix} -z \\ C^{-1}(Ay + D_1\eta + D_2 z + B\xi) \\ -\dfrac{\partial z}{\partial \nu} + \eta \\ -z + \xi \end{pmatrix}, \quad \forall u \in D(\mathcal{A}). \tag{2.5}$$

Moreover, denoting by $u = (y, y_t, \eta, \xi)^T$ the state of the system (1.1)-(1.2), we can formulate the system as an evolutionary equation :

$$u_t + \mathcal{A}u = 0, \qquad u(0) = u_0 \in \mathcal{H}. \tag{2.6}$$

It is easy to prove that $\mathcal{A}$ is a maximal monotone operator in the energy space $\mathcal{H}$. Then $\mathcal{A}$ generates a C_0 semigroup $S_{\mathcal{A}}(t)$ of contractions on the energy space $\mathcal{H}$ (see [5]).

Proposition 2.1 *If $u = (y, z, \eta, \xi) \in D(\mathcal{A}^2)$, then we have:*

$$y \in H^{5/2}(\Omega) \cap W. \tag{2.7}$$

Proof. Given $u_0 = (y_0, z_0, \eta_0, \xi_0) \in D(\mathcal{A})$, we solve the equation $\mathcal{A}u = u_0$. This means that

$$\begin{cases} z = -y_0, \\ Ay + D_1\eta + D_2 z + B\xi = Cz_0, \\ \eta - \dfrac{\partial z}{\partial \nu} = \eta_0, \\ \xi - z = \xi_0. \end{cases} \tag{2.8}$$

Since C is the canonic isomorphism from V onto V' and since $z_0 \in V$, then eliminating z, η and ξ in (2.8), we find that y satisfies :

$$Ay + D_1\left(\eta_0 - \frac{\partial y_0}{\partial \nu}\right) + B\left(\xi_0 - y_0\right) = Cz_0 + D_2 y_0, \quad \text{in } W'. \tag{2.9}$$

Using Green's formula (2.1), we interpret equation (2.9) into the following variational equation :

$$\int_\Omega \left(\Delta^2 y + \gamma\Delta z_0 - z_0\right)\phi dx + \int_{\Gamma_1} \left[\Delta y + (1-\mu)B_1 y + \left(\eta_0 - \frac{\partial y_0}{\partial \nu}\right)\right]\frac{\partial \phi}{\partial \nu} d\Gamma$$

$$- \int_{\Gamma_1} \left[\frac{\partial \Delta y}{\partial \nu} + (1-\mu)\frac{\partial B_2 y}{\partial \tau} - (\xi_0 - y_0) - \frac{\partial^2 y_0}{\partial \tau^2} + \gamma\frac{\partial z_0}{\partial \nu}\right]\phi d\Gamma = 0, \quad \forall \phi \in W. \tag{2.10}$$

It follows that

$$\begin{cases} \Delta^2 y = z_0 - \gamma\Delta z_0 \ \in L^2(\Omega) \ , \\ \Delta y + (1-\mu)B_1 y = -(\eta_0 - \dfrac{\partial y_0}{\partial \nu}) \in L^2(\Gamma_1) \ , \\ \dfrac{\partial \Delta y}{\partial \nu} + (1-\mu)\dfrac{\partial B_2 y}{\partial \tau} = (\xi_0 - y_0) - \gamma\dfrac{\partial z_0}{\partial \nu} + \dfrac{\partial^2 y_0}{\partial \tau^2} \in H^{-1}(\Gamma_1). \end{cases} \tag{2.11}$$

Then, thanks to the elliptic theory (see [13]), we deduce that $y \in H^{5/2}(\Omega)$. The proof is thus complete.

Proposition 2.2 *The resolvent $(I+\mathcal{A})^{-1}$ of the operator $-\mathcal{A}$ is non-compact in the energy space $\mathcal{H}$.*

Proof Let $u_n = (y_n, z_n, \eta_n, \xi_n)$ be a bounded sequence in $\mathcal{H}$ such that

$$\| u_n \|_{\mathcal{H}} \leq 1, \quad \forall\, n \geq 0.$$

Let :

$$\widetilde{u}_n = (I+\mathcal{A})^{-1} u_n, \quad \forall n \geq 0.$$

Then $\widetilde{u}_n = (\widetilde{y}_n, \widetilde{z}_n, \widetilde{\eta}_n, \widetilde{\xi}_n)$ is determined by the following system :

$$\begin{cases} \widetilde{z}_n = \widetilde{y}_n - y_n, \\ 2\widetilde{\eta}_n = \eta_n + \dfrac{\partial \widetilde{z}_n}{\partial \nu}, \\ 2\widetilde{\xi}_n = \xi_n + \widetilde{z}_n, \end{cases} \tag{2.12}$$

where $\widetilde{y}_n$ is the unique solution of the variational equation :

$$(\widetilde{y}_n, \phi)_W + (\widetilde{y}_n, \phi)_V + \int_{\Gamma_1} \nabla \widetilde{y}_n \nabla \phi d\Gamma + \int_{\Gamma_1} \widetilde{y}_n \phi d\Gamma = (y_n + z_n, \phi)_V$$

$$+ \int_{\Gamma_1} \nabla y_n \nabla \phi d\Gamma + \int_{\Gamma_1} y_n \phi d\Gamma - \int_{\Gamma_1} \eta_n \frac{\partial \phi}{\partial \nu} d\Gamma - \int_{\Gamma_1} \xi_n \phi d\Gamma, \quad \forall\, \phi \in W \tag{2.13}$$

from which we get easily that

$$\| \widetilde{y}_n \|_W \leq C \| u_n \|_{\mathcal{H}} < +\infty, \quad \forall n \geq 1.$$

The using the trace theorems, we can find positive constant C such that for all $n \geq 1$ we have :

$$\left\| \frac{\partial \widetilde{z}_n}{\partial \nu} \right\|_{H^{1/2}(\Gamma_1)} + \| \widetilde{z}_n \|_{H^{3/2}(\Gamma_1)} \leq C \| \widetilde{z}_n \|_W \leq C \Big(\| \widetilde{y}_n \|_W + \| y_n \|_W \Big) < +\infty.$$

Since the imbeddings from $H^{1/2}(\Gamma_1)$ and $H^{3/2}(\Gamma_1)$ into $L^2(\Gamma_1)$ are compact, without lass of generality, we can assume that $\dfrac{\partial \widetilde{z}_n}{\partial \nu}$ and $\widetilde{z}_n$ converge strongly in $L^2(\Gamma_1)$. Now let $(\xi_n)_{n\geq 0}$ be an orthonormal sequence in $L^2(\Gamma_1)$. By Bessel's inequality we know that

$$\xi_n \rightharpoonup 0, \quad \text{in } L^2(\Gamma_1)$$

and non subsequence of ξ_n converges strongly in $L^2(\Gamma_1)$. Therefore, non subsequence of $\widetilde{\xi}_n = \dfrac{1}{2}(\xi_n + \widetilde{z}_n)$ converges strongly in $L^2(\Gamma_1)$. This proves that $(I+\mathcal{A})^{-1}$ is non-compact in $\mathcal{H}$.

3 Polynomial energy decay rate and strong stability.

In this section, we first establish the polynomial energy decay rate for smooth solution of equation (2.6). Next we prove the strong stability for the weak solution. We assume that there exist $\delta > 0$ and $x_0 \in I\!R^2$ such that, putting $m(x) = x - x_0$, we have :

$$(m \cdot \nu) \geq \delta^{-1}, \quad \forall\, x \in \Gamma_1 \text{ and } (m \cdot \nu) \leq 0, \quad \forall\, x \in \Gamma_0. \tag{3.1}$$

Lemma 3.1 *Let $v_1, v_2, \widetilde{v}_2, \hat{v}_2 \in L^2(\Gamma_1)$ and y satisfy the following conditions :*

$$\begin{cases} y \in W, \quad \Delta^2 y \in L^2(\Omega) & \\ \Delta y + (1-\mu) B_1 y = v_1 & \text{on } \Gamma_1, \\ \dfrac{\partial \Delta y}{\partial \nu} + (1-\mu)\dfrac{\partial B_2 y}{\partial \tau} = v_2 + \dfrac{\partial \hat{v}_2}{\partial \tau} + \widetilde{v}_2 & \text{on } \Gamma_1, \\ y = \dfrac{\partial y}{\partial \nu} = 0 & \text{on } \Gamma_0. \end{cases} \tag{3.2}$$

Then we have :

$$-\int_\Omega \Delta^2 y (m \cdot \nabla y) dx \leq -\frac{1}{2}\int_\Omega a(y,y) dx$$
$$+C_0 \int_{\Gamma_1} (|v_1|^2 + |v_2|^2 + |\hat{v}_2|^2) d\Gamma - \int_{\Gamma_1} \widetilde{v}_2 (m \cdot \nabla y) d\Gamma, \tag{3.3}$$

where C_0 is a positive constant depending only on the domain Ω.

Proof. Assume that $v_1, \hat{v}_2 \in H^{3/2}(\Gamma_1)$ and $v_2, \widetilde{v}_2 \in H^{1/2}(\Gamma_1)$. Then we have $y \in H^4(\Omega)$. Using Green's formula (2.1) we have :

$$\int_\Omega \Delta^2 y (m \cdot \nabla y) dx = \int_\Omega a(y,y) dx$$
$$+\int_\Gamma \left(\frac{\partial \Delta y}{\partial \nu} + (1-\mu)\frac{\partial B_2 y}{\partial \tau} \right)(m \cdot \nabla y) d\Gamma - \int_\Gamma \left(\Delta y + (1-\mu) B_1 y \right) \frac{\partial (m \cdot \nabla y)}{\partial \nu} d\Gamma$$
$$+\frac{1}{2}\int_\Gamma (m \cdot \nu) \left\{ \left(\frac{\partial^2 y}{\partial x_1^2}\right)^2 + \left(\frac{\partial^2 y}{\partial x_2^2}\right)^2 + 2\mu \frac{\partial^2 y}{\partial x_1^2}\frac{\partial^2 y}{\partial x_2^2} + 2(1-\mu)\left(\frac{\partial^2 y}{\partial x_1 \partial x_2}\right)^2 \right\}. \tag{3.4}$$

Since $y = \dfrac{\partial y}{\partial \nu} = 0$ on Γ_0, it follows that

$$\nabla y = 0, \quad B_1 y = 0, \quad \frac{\partial (m \cdot \nabla y)}{\partial \nu} = (m \cdot \nu)\Delta y \text{ on } \Gamma_0, \tag{3.5}$$

$$\left(\frac{\partial^2 y}{\partial x_1^2}\right)^2 + \left(\frac{\partial^2 y}{\partial x_2^2}\right)^2 + 2\mu \frac{\partial^2 y}{\partial x_1^2}\frac{\partial^2 y}{\partial x_2^2} + 2(1-\mu)\left(\frac{\partial^2 y}{\partial x_1 \partial x_2}\right)^2 = (\Delta y)^2 \text{ on } \Gamma_0. \tag{3.6}$$

On the other hand, we have :

$$\left(\frac{\partial^2 y}{\partial x_1^2}\right)^2 + \left(\frac{\partial^2 y}{\partial x_2^2}\right)^2 + 2\mu \frac{\partial^2 y}{\partial x_1^2}\frac{\partial^2 y}{\partial x_2^2} + 2(1-\mu)\left(\frac{\partial^2 y}{\partial x_1 \partial x_2}\right)^2$$

$$\geq (1-\mu)\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}. \tag{3.7}$$

Inserting (3.5)-(3.7) into (3.4) gives

$$\int_\Omega \Delta^2 y(m\cdot\nabla y)dx \geq \int_\Omega a(y,y)dx$$

$$+\int_{\Gamma_1}\Big(v_2+\widetilde{v}_2\Big)(m\cdot\nabla y)d\Gamma-\int_{\Gamma_1} v_1\frac{\partial(m\cdot\nabla y)}{\partial\nu}d\Gamma-\int_{\Gamma_1}\hat{v}_2\frac{\partial(m\cdot\nabla y)}{\partial\tau}d\Gamma$$

$$+\frac{(1-\mu)}{2\delta}\int_{\Gamma_1}\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}. \tag{3.8}$$

A direct computation gives

$$\Big|\frac{\partial(m.\nabla y)}{\partial\nu}\Big| \leq \Big|\frac{\partial y}{\partial\nu}\Big|+R\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}^{\frac{1}{2}}, \tag{3.9}$$

$$\Big|\frac{\partial(m.\nabla y)}{\partial\tau}\Big| \leq \Big|\frac{\partial y}{\partial\tau}\Big|+R\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}^{\frac{1}{2}} \tag{3.10}$$

where $R=\| m \|_{L^\infty(\Omega)}$ is the diameter of Ω. Then for any $\lambda>0$, it follows that

$$\int_{\Gamma_1} v_1\frac{\partial(m\cdot\nabla y)}{\partial\nu}d\Gamma \geq -\frac{\lambda}{2}\int_{\Gamma_1}|v_1|^2\,d\Gamma-\frac{1}{\lambda}\int_{\Gamma_1}\Big|\frac{\partial y}{\partial\nu}\Big|^2\,d\Gamma$$

$$-\frac{R^2}{\lambda}\int_{\Gamma_1}\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}d\Gamma, \tag{3.11}$$

$$\int_{\Gamma_1}\hat{v}_2\frac{\partial(m\cdot\nabla y)}{\partial\tau}d\Gamma \geq -\frac{\lambda}{2}\int_{\Gamma_1}|\hat{v}_2|^2\,d\Gamma-\frac{1}{\lambda}\int_{\Gamma_1}\Big|\frac{\partial y}{\partial\tau}\Big|^2\,d\Gamma$$

$$-\frac{R^2}{\lambda}\int_{\Gamma_1}\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}d\Gamma, \tag{3.12}$$

$$\int_{\Gamma_1} v_2(m\cdot\nabla y)d\Gamma \geq -\frac{\lambda}{2}\int_{\Gamma_1}|v_2|^2\,d\Gamma-\frac{R^2}{2\lambda}\int_{\Gamma_1}|\nabla y|^2\,d\Gamma. \tag{3.13}$$

Inserting (3.11)-(3.13) into (3.8), we obtain that

$$\int_\Omega \Delta^2 y(m\cdot\nabla y)dx \tag{3.14}$$

$$\geq \int_\Omega a(y,y)dx+\int_{\Gamma_1}\widetilde{v}_2(m\cdot\nabla y)d\Gamma-\frac{\lambda}{2}\int_{\Gamma_1}\Big(|v_1|^2+|\hat{v}_2|^2+|v_2|^2\Big)d\Gamma$$

$$+\Big(\frac{(1-\mu)}{2\delta}-\frac{2R^2}{\lambda}\Big)\int_{\Gamma_1}\Big\{\Big(\frac{\partial^2 y}{\partial x_1^2}\Big)^2+\Big(\frac{\partial^2 y}{\partial x_2^2}\Big)^2+2\Big(\frac{\partial^2 y}{\partial x_1\partial x_2}\Big)^2\Big\}d\Gamma-\frac{2+R^2}{2\lambda}\int_{\Gamma_1}|\nabla y|^2\,d\Gamma.$$

We obtain (3.3) by taking $\lambda > 0$ sufficiently large in (3.14) :

$$\lambda \geq \frac{4\delta R^2}{(1-\mu)} \quad \text{and} \quad \frac{2+R^2}{2\lambda}\int_{\Gamma_1} |\nabla y|^2 \, d\Gamma \leq \frac{1}{2}\int_\Omega a(y,y)dx, \quad \forall y \in W.$$

The case v_1, v_2, $\widetilde{v}_2$, $\hat{v}_2 \in L^2(\Gamma_1)$ can be completed by the standard density arguments (see Lemma 3.1 in Rao [18]). The proof is thus complete.

Lemma 3.2 *Let $u_0 \in D(\mathcal{A}^2)$ and y be a smooth solution of the system (1.1)-(1.3). Then for all $0 \leq S \leq T < \infty$ we have :*

$$\int_S^T \int_\Omega y_t^2 E(t)dxdt + \frac{1}{2}\int_S^T \int_\Omega a(y,y)E(t)dxdt \leq M_1 E(S)E(0) \tag{3.15}$$

where M_1 is a positive constant depending on $\| u_0 \|_{D(\mathcal{A})}$.

Proof. Multiplying the equation (1.1) with $m \cdot \nabla y E(t)$ we obtain :

$$\int_S^T \int_\Omega y_{tt}(m\cdot\nabla y)E(t)dxdt - \gamma\int_S^T \int_\Omega \Delta y_{tt}(m\cdot\nabla y)E(t)dxdt =$$

$$-\int_S^T \int_\Omega \Delta^2 y(m\cdot\nabla y)E(t)dxdt. \tag{3.16}$$

Integrating by parts on the first term in (3.16) gives :

$$\int_S^T \int_\Omega y_{tt}(m\cdot\nabla y)E(t)dxdt = \left[\int_\Omega y_t(m\cdot\nabla y)E(t)\right]_S^T - \int_S^T \int_\Omega y_t(m\cdot\nabla y)E_t(t)dxdt$$

$$+\int_S^T \int_\Omega y_t^2 E(t)dxdt - \frac{1}{2}\int_S^T \int_{\Gamma_1} (m\cdot\nu)y_t^2 E(t)d\Gamma dt. \tag{3.17}$$

Using Poincarré's inequality, there exists a constant $C_1 > 0$ such that :

$$\int_\Omega y_t(m\cdot\nabla y)dx \leq C_1 E(t), \quad \forall t \geq 0.$$

Then it follows that :

$$\left[\int_\Omega y_t(m\cdot\nabla y)E(t)dx\right]_S^T - \int_S^T \int_\Omega y_t(m\cdot\nabla y)E_t(t)dxdt$$

$$\geq -C_1\Big(E^2(S) + E^2(T)\Big) + C_1\int_S^T E_t(t)E(t)dt \geq -3C_1E^2(S). \tag{3.18}$$

Inserting (3.18) into (3.17) gives :

$$\int_S^T \int_\Omega y_{tt}(m\cdot\nabla y)E(t)dxdt$$

$$\geq \int_S^T \int_\Omega y_t^2 E(t) dx dt - 3C_1 E^2(S) - \frac{R}{2} \int_S^T \int_{\Gamma_1} y_t^2 E(t) d\Gamma dt. \tag{3.19}$$

Since $u_0 \in D(\mathcal{A}^2)$, from Proposition 2.1 and semigroupe theory (see [17]), we have :

$$\begin{cases} y \in C^0(I\!R^+; H^{5/2}) \cap C^2(I\!R^+; W) \cap C^3(I\!R^+; V), \\ \eta, \xi \in C^2(I\!R^+; L^2(\Gamma_1)). \end{cases} \tag{3.20}$$

Then we can integrate by parts on the second term in (3.16) :

$$-\gamma \int_S^T \int_\Omega \Delta y_{tt} (m \cdot \nabla y) E(t) dx dt = \tag{3.21}$$

$$-\gamma \int_S^T \int_{\Gamma_1} \frac{\partial y_{tt}}{\partial \nu} (m \cdot \nabla y) E(t) d\Gamma dt + \gamma \left[\int_\Omega \nabla y_t \nabla (m \cdot \nabla y) E(t) \right]_S^T$$

$$-\gamma \int_S^T \int_\Omega \nabla y_t \nabla (m \cdot \nabla y) E_t(t) dx dt - \frac{\gamma}{2} \int_S^T \int_{\Gamma_1} (m \cdot \nu) \left| \nabla y_t \right|^2 E(t) d\Gamma dt.$$

Using Cauchy-Schwarz's inequality, we get :

$$\left| \gamma \int_\Omega \nabla y_t \nabla (m \cdot \nabla y) dx \right| \leq C_2 E(t), \quad \forall t \geq 0.$$

It follows that

$$\gamma \left[\int_\Omega \nabla y_t \nabla (m \cdot \nabla y) E(t) dx \right]_S^T \geq -2C_2 E^2(S), \tag{3.22}$$

$$-\gamma \int_S^T \int_\Omega \nabla y_t \nabla (m \cdot \nabla y) E_t(t) dx dt \geq -\frac{C_2}{2} E^2(S). \tag{3.23}$$

Inserting (3.22) and (3.23) onto (3.21) we obtain that

$$-\gamma \int_S^T \int_\Omega \Delta y_{tt} (m \cdot \nabla y) E(t) dx dt \geq -\gamma \int_S^T \int_{\Gamma_1} \frac{\partial y_{tt}}{\partial \nu} (m \cdot \nabla y) E(t) d\Gamma dt$$

$$-\frac{5}{2} C_2 E^2(S) - \frac{\gamma R}{2} \int_S^T \int_{\Gamma_1} \left| \nabla y_t \right|^2 E(t) d\Gamma dt. \tag{3.24}$$

On the other hand, using (3.20) we have :

$$\begin{cases} \Delta^2 y \in L^2(\Omega), & \\ \Delta y + (1-\mu) B_1 y = v_1 & \text{on } \Gamma_1, \\ \dfrac{\partial \Delta y}{\partial \nu} + (1-\mu) \dfrac{\partial B_2 y}{\partial \tau} = v_2 - \dfrac{\partial \hat{v}_2}{\partial \tau} + \widetilde{v}_2 & \text{on } \Gamma_1, \\ y = \dfrac{\partial y}{\partial \nu} = 0 & \text{on } \Gamma_0, \end{cases} \tag{3.25}$$

with

$$v_1 = -\eta \in L^2(\Gamma_1), \;\; v_2 = \xi \in L^2(\Gamma_1), \; \widetilde{v}_2 = \gamma \frac{\partial y_{tt}}{\partial \nu} \in L^2(\Gamma_1) \text{ and } \hat{v}_2 = -\frac{\partial y_t}{\partial \tau} \in L^2(\Gamma_1).$$

Applying Lemma 3.1 to (3.25) we get :

$$-\int_S^T \int_\Omega \Delta^2 y(m\cdot\nabla y)E(t) \leq -\frac{1}{2}\int_S^T \int_\Omega a(y,y)E(t)dxdt$$

$$+C_0\int_S^T \int_{\Gamma_1}\left(\eta^2+\xi^2+\left|\frac{\partial y_t}{\partial\tau}\right|^2\right)E(t)d\Gamma dt-\gamma\int_S^T\int_{\Gamma_1}\frac{\partial y_{tt}}{\partial\nu}(m\cdot\nabla y)E(t)d\Gamma dt. \quad (3.26)$$

Using (1.3) we have :

$$|y_t|^2+|\nabla y_t|^2 \leq 2\left(|\eta_t|^2+|\xi_t|^2+|\eta|^2+|\xi|^2+\left|\frac{\partial y_t}{\partial\tau}\right|^2\right), \quad \text{on } \Gamma_1. \quad (3.27)$$

Inserting (3.19), (3.24), (3.26) and (3.27) into (3.16) gives that

$$\int_S^T\int_\Omega y_t^2E(t)dxdt+\frac{1}{2}\int_S^T\int_\Omega a(y,y)E(t)dxdt \leq 3(C_1+C_2)E(S)E(0)$$

$$+\Big[(\gamma+1)R+C_0\Big]E(S)\int_S^T\int_{\Gamma_1}\left(\eta^2+\xi^2+\left|\frac{\partial y_t}{\partial\tau}\right|^2+\eta_t^2+\xi_t^2\right)d\Gamma dt. \quad (3.28)$$

On the other hand, from (1.5) we have :

$$\int_S^T\int_{\Gamma_1}\left(\eta^2+\xi^2+\left|\frac{\partial y_t}{\partial\tau}\right|^2\right)d\Gamma dt=-\int_S^T E_t(t)dt \leq E(S). \quad (3.29)$$

Since $u_0 \in D(\mathcal{A}^2)$ then differentiating the system (2.6) with respect to the variable t and using (3.29) we have :

$$\int_S^T\int_{\Gamma_1}\left(\eta_t^2+\xi_t^2\right)d\Gamma dt \leq E_1(S) \quad (3.30)$$

where the energy of high order $E_1(t)$ is defined by

$$E_1=\frac{1}{2}\parallel u_t(t)\parallel^2=\frac{1}{2}\parallel \mathcal{A}u(t)\parallel^2 .$$

Finally inserting (3.29) and (3.30) into (3.28) we obtain (3.15), with

$$M_1=3(C_1+C_2)+\Big[(\gamma+1)R+C_0\Big]\left(1+\frac{E_1(0)}{E(0)}\right).$$

The proof is complete.

Lemma 3.3 *Let $u_0 \in D(\mathcal{A}^2)$ and y be a smooth solution of the system (1.1)-(1.3). Then for all $0 \leq S \leq T < \infty$, we have*

$$\int_S^T\int_\Omega y_t^2E(t)dxdt+\gamma\int_S^T\int_\Omega|\nabla y_t|^2E(t)dxdt-\int_S^T\int_\Omega a(y,y)E(t)dxdt$$

$$\leq M_2E(S)E(0)+\frac{1}{2}\int_S^T E^2(t)dt \quad (3.31)$$

where M_2 is a positive constant independent of u_0.

Proof. Multiplying equation (1.1) with $yE(t)$ and using (2.1) we obtain that :

$$\int_S^T \int_\Omega y_t^2 E(t)dxdt + \gamma \int_S^T \int_\Omega |\nabla y_t|^2 E(t)dxdt - \int_S^T \int_\Omega a(y,y)E(t)dxdt =$$

$$\left[\int_\Omega y_t y E(t)dx\right]_S^T - \int_S^T \int_\Omega y_t y E_t(t)dxdt + \gamma \left[\int_\Omega \nabla y_t \nabla y E(t)dx\right]_S^T$$

$$-\gamma \int_S^T \int_\Omega \nabla y_t \nabla y E_t(t)dxdt + \int_S^T \int_{\Gamma_1} \xi y E(t)d\Gamma dt + \int_S^T \int_{\Gamma_1} \eta \frac{\partial y}{\partial \nu} E(t)d\Gamma dt$$

$$+ \int_S^T \int_{\Gamma_1} \frac{\partial y_t}{\partial \tau}\frac{\partial y}{\partial \tau} E(t)d\Gamma dt. \tag{3.32}$$

Using Poincarré's inequality, there exists a constant $C_3 > 0$ such that :

$$\left[\int_\Omega y_t y E(t)dx\right]_S^T - \int_S^T \int_\Omega y_t y E_t(t)dxdt \leq C_3\Big(E^2(S) + E^2(T)\Big)$$

$$-C_3 \int_S^T E_t(t)E(t)dt \leq 3C_3 E^2(S). \tag{3.33}$$

Similarly we have :

$$\gamma\left[\int_\Omega \nabla y_t \nabla y E(t)dx\right]_S^T - \gamma \int_S^T \int_\Omega \nabla y_t \nabla y E_t(t)dxdt \leq 3C_4 E^2(S). \tag{3.34}$$

Inserting (3.33) and (3.34) onto (3.32) we obtain that

$$\int_S^T \int_\Omega y_t^2 E(t)dxdt + \gamma \int_S^T \int_\Omega |\nabla y_t|^2 E(t)dxdt - \int_S^T \int_\Omega a(y,y)E(t)dxdt \leq 3(C_3+C_4)E^2(S)$$

$$+ \int_S^T \int_{\Gamma_1} \xi y E(t)d\Gamma dt + \int_S^T \int_{\Gamma_1} \eta \frac{\partial y}{\partial \nu} E(t)d\Gamma dt + \int_S^T \int_{\Gamma_1} \frac{\partial y_t}{\partial \tau}\frac{\partial y}{\partial \tau} E(t)d\Gamma dt. \tag{3.35}$$

On the other hand, thanks to Sobolev's imbedding we obtain that

$$\int_{\Gamma_1} y^2 d\Gamma \leq C_5 E(t), \quad \int_{\Gamma_1} |\nabla y|^2 d\Gamma \leq C_5 E(t), \quad \forall t \geq 0. \tag{3.36}$$

Choosing $\varepsilon > 0$ sufficiently small and using (3.36) we have :

$$\int_S^T \int_{\Gamma_1} \xi y E(t)d\Gamma dt \leq \frac{1}{2\varepsilon}\int_S^T \int_{\Gamma_1} \xi^2 E(t)d\Gamma dt + \frac{\varepsilon}{2}\int_S^T \int_{\Gamma_1} y^2 E(t)d\Gamma dt$$

$$\leq \frac{1}{2\varepsilon}E(S)E(0) + \frac{\varepsilon}{2}C_5 \int_S^T E^2(t)dt. \tag{3.37}$$

Similarly we have :

$$\int_S^T \int_{\Gamma_1} \eta \frac{\partial y}{\partial \nu} E(t) d\Gamma dt \leq \frac{1}{2\varepsilon} E(S)E(0) + \frac{\varepsilon}{2} C_5 \int_S^T E^2(t) dt \tag{3.38}$$

and

$$\int_S^T \int_{\Gamma_1} \frac{\partial y_t}{\partial \tau} \frac{\partial y}{\partial \tau} E(t) d\Gamma dt \leq \frac{1}{2\varepsilon} E(S)E(0) + \frac{\varepsilon}{2} C_5 \int_S^T E^2(t) dt. \tag{3.39}$$

Finally inserting (3.37), (3.38) and (3.39) into (3.35) we obtain that :

$$\begin{aligned} &\int_S^T \int_\Omega y_t^2 E(t) dx dt + \gamma \int_S^T \int_\Omega |\nabla y_t|^2 E(t) dx dt - \int_S^T \int_\Omega a(y,y) E(t) dx dt \\ &\qquad \leq \left[3(C_3 + C_4) + \frac{3}{2\varepsilon} \right] E(S)E(0) + \frac{3\varepsilon}{2} C_5 \int_S^T E^2(t) dt. \end{aligned} \tag{3.40}$$

Choosing $\varepsilon = \dfrac{1}{3C_5}$ in (3.40) we obtain (3.31).

Proof of Theorem 1.1. We first assume that $u_0 \in D(\mathcal{A}^2)$. Combining (3.15) and (3.31) we obtain that :

$$\int_S^{+\infty} E^2(t) dt \leq M E(0) E(S), \quad \forall S \geq 0 \tag{3.41}$$

where M is given by

$$M = C\left(1 + \frac{\| u_0 \|^2_{D(\mathcal{A})}}{\| u_0 \|^2_E}\right), \quad \text{and} \quad C > 0 \text{ is independent of } u_0.$$

Thanks to a classical result of Haraux (see [9]) we obtain that

$$E(t) \leq \; E(0) \frac{2M}{M+t}, \quad \forall \; t \geq 0. \tag{3.42}$$

Finally by the density arguments we prove the polynomial decay rate (3.35) for all $u_0 \in D(\mathcal{A})$.

Proof of Theorem 1.2. Let $u_0 \in \mathcal{H}$, there exists a sequence $(u_0^n)_{n \geq 0}$ in $D(\mathcal{A})$ converges to u_0 in $\mathcal{H}$. Then $\forall \varepsilon > 0$ there exists $n_\varepsilon > 0$ such that

$$\| u_0 - u_0^{n_\varepsilon} \|_{\mathcal{H}} < \frac{\varepsilon}{2}.$$

From Theorem 1.1 there exists $T > 0$ such that

$$\| S_{\mathcal{A}}(t) u_0^{n_\varepsilon} \|_{\mathcal{H}} < \frac{\varepsilon}{2}, \quad t > T. \tag{3.43}$$

This implies that

$$\| S_{\mathcal{A}}(t) u_0 \|_{\mathcal{H}} \leq \| S_{\mathcal{A}}(t) u_0^{n_\varepsilon} \|_{\mathcal{H}} + \| u_0 - u_0^{n_\varepsilon} \|_{\mathcal{H}} \leq \varepsilon, \quad \forall t > T.$$

It follows that

$$\| S_{\mathcal{A}}(t) u_0 \|_{\mathcal{H}} \to 0, \quad \forall t \to +\infty.$$

The proof is thus complete.

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