

# Inverse acoustic scattering by small-obstacle expansion of misfit function

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## ABSTRACT

The present study is set in the general framework of inverse scattering of scalar (e.g. acoustic) waves. To identify hidden obstacles from external measurements (e.g. overspecified boundary data) associated with the scattering of known incident waves by the unknown object(s), it is customary to invoke iterative algorithms such as gradient-based optimization procedures. The numerical solution of the forward scattering problem associated with an assumed obstacle configuration is often a computationally demanding task. Besides, iterative inversion algorithms are sensitive to the choice of initial “guess” (number of components, initial location, shape and size of obstacle(s)). This has prompted the definition of preliminary probing techniques, which aim at delineating in a computationally fast way the hidden obstacle(s), namely the linear sampling [2], not pursued here, or the concept of topological sensitivity [1, 3]. If  $J$  denotes the cost function used for solving the inverse problem, then in 3D situations the topological derivative  $\mathcal{T}_3(\mathbf{x}_s)$  associated with the nucleation of a small obstacle of volume  $O(\varepsilon^3)$  and specified shape appears through the expansion

$$J(\varepsilon, \mathbf{x}_s) - J(0) = \varepsilon^3 \mathcal{T}(\mathbf{x}_s) + o(\varepsilon^3) \quad (1)$$

In this communication, an extension of the topological derivative is presented, whereby  $J(\varepsilon, \mathbf{x}_s)$  is expanded further in powers of  $\varepsilon$ . Specifically, the expansion to order  $O(\varepsilon^6)$  for 3D acoustic scattering by a hard obstacle of size  $\varepsilon$  is presented. The choice of order  $O(\varepsilon^6)$  is important for cost functions  $J$  of least-squares format. In particular, the expansion of  $J$  for any *centrally-symmetric* infinitesimal hard obstacle of radius  $\varepsilon$  centered at  $\mathbf{x}_s$  is found to have the form

$$J(\varepsilon, \mathbf{x}_s) - J(0, \mathbf{x}_s) = \varepsilon^3 \mathcal{T}_3(\mathbf{x}_s) + \varepsilon^5 \mathcal{T}_5(\mathbf{x}_s) + \varepsilon^6 \mathcal{T}_6(\mathbf{x}_s) + o(\varepsilon^6) = J(0, \mathbf{x}_s) + J_6(\varepsilon, \mathbf{x}_s) + o(\varepsilon^6) \quad (2)$$

The previously known topological derivative  $\mathcal{T}_3(\mathbf{x}_s)$  and the new coefficients  $\mathcal{T}_5(\mathbf{x}_s)$ ,  $\mathcal{T}_6(\mathbf{x}_s)$  have explicit expressions in terms of the relevant acoustic Green’s function. Expansions of the form (2) offer the option of minimizing the approximate polynomial expression  $J_6(\varepsilon, \mathbf{x}_s)$ . This is a simple and inexpensive task, which can be performed for locations  $\mathbf{x}_s$  spanning a search grid, thereby defining a (approximate) global search procedure. The values of  $\mathbf{x}_s$  and  $\varepsilon$  leading to an absolute minimum of  $J_6(\varepsilon, \mathbf{x}_s)$  over the search grid then constitute the best estimate of the hidden scatterer furnished by this procedure, and might provide e.g. a useful initial guess for an iterative inversion algorithm. Results of numerical experiments in 3D conditions based on this idea will be presented at the conference.

## References

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