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The relaxation time of a chiral quantum R-L circuit

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We report on the GHz complex admittance of a chiral one dimensional ballistic conductor formed by edge states in the quantum Hall regime. The circuit consists of a wide Hall bar (the inductor L) in series with a tunable resistor (R) formed by a quantum point contact. Electron interactions between edges are screened by a pair of side gates. Conductance steps are observed on both real and imaginary parts of the admittance. Remarkably, the phase of the admittance is transmission-independent. This shows that the relaxation time of a chiral R-L circuit is resistance independent. A current and charge conserving scattering theory is presented that accounts for this observation with a relaxation time given by the electronic transit time in the circuit.

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Violation of classical electro kinetic laws is a hallmark of quantum transport. In the dc regime, it is well known that transport is non-local over the electronic coherence length. This leads to the non-additivity of parallel conductances [1] and to quantum composition laws to relate impurity scattering to resistance. Recently a similar manifestation of quantum coherence has been reported by Gabelli et al.[2, 3] in the ac regime where the resistance which determines the RC-charge relaxation time of a mesoscopic capacitor is found to be quantized at half of a resistance quantum. This observation, in agreement with predictions of Büttiker, Thomas, and Prêtre [4, 5], establishes the concept of a charge relaxation resistance [6] different from the standard dc Landauer resistance. A second fundamental dynamical time scale is the L/R -time of a mesoscopic circuit which in macroscopic conductors is determined by the ratio of the inductance and the resistance of the sample.

Here we investigate a series combination of an inductive and resistive element and demonstrate that macroscopic kinetics does not account for the correct ac response. In this case, chirality is responsible for the observed non-classical behavior. The inductive conductor is a 2D electron gas (2DEG) Hall bar [7] in the Quantum Hall regime. It is followed by a quantum point contact [8] (QPC) which controls the number of transmitted channels and their transmission and thus represents the resistance contribution. In a quantizing magnetic field, this geometry allows to investigate the relaxation time of the quantum R-L circuit made of the *kinetic* inductance of the electrons in edge states [9, 10, 11] of the Hall bar in series with the resistance of the QPC. Theory [12] predicts that edge channels that connect two reservoirs contribute to the impedance inductively due to kinetic effects, whereas edge channels which are reflected

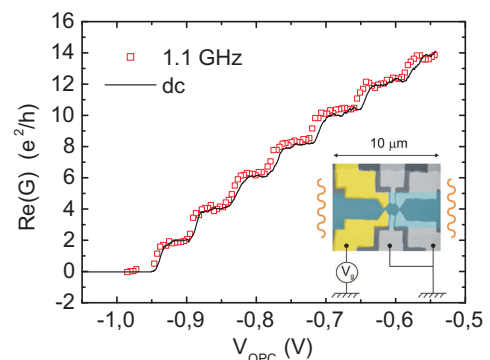


FIG. 1: Quantized steps in the dc conductance and RF transmission of the circuit as function of the QPC gate voltage. The curves have been slightly shifted along the voltage axis for clarity. Temperature and magnetic field are respectively 50 mK and 0.224 T.

to the same reservoir from which they are incident, contribute capacitively. Importantly, in the present set-up, the inter-edge coupling is reduced due to the large bar width and further minimized by using side gates strongly coupled to the edge states. Our main result is that this relaxation time is not the classical L/R time but the electronic transit time of the circuit.

In the work presented here the sample is still short compared to the wave length of an edge-magneto-plasmon. Previous experimental investigations of the electromagnetic response of Hall bars [13, 14, 15, 16] have addressed the regime where the response is well accounted for by collective excitations called edge-magneto-plasmons [17] with wave-length short compared to the dimensions of the sample. Refs.[13, 15] have extensively studied the time domain and Ref.[16] the frequency domain.

In this letter we report on phase-resolved impedance measurements of a quantum R-L circuit in the edge state regime at GHz frequency and milli-Kelvin temperatures.

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With increasing QPC transmission, dc-like conductance steps are observed on both quadratures of the admittance. Remarkably, the admittance phase is independent of the number of transmitted modes and of their transmission. This shows that the relaxation time of the chiral R-L circuit is resistance independent. A current and charge conserving scattering theory extending Ref.[12] is presented that accounts for this observation with a relaxation time given by the electronic dwell time in the circuit.

The sample is a $50\mu\text{m}$ long and $6\mu\text{m}$ wide Hall bar made in a GaAs/AlGaAs electron gas of nominal density $n_s = 1.3 \times 10^{11} \text{ cm}^{-2}$ and mobility $\mu = 3 \times 10^6 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$. A magnetic field of $B = 0.224\text{T}$ and $B = 0.385\text{T}$ is applied in the spin degenerate quantum Hall regime (filling factors $N = 24$ and $N = 14$ respectively) so that edge states are well developed. The bar is interrupted in its middle by a pair of quantum point contacts (inset of Fig.1) but only the first QPC is active with a negative voltage bias ($V_g \sim -1\text{V}$); the electron gas density beneath it is fully depleted and the gate to 2DEG capacitance is small. The gate of the second QPC is grounded and widely overlaps the electron gas. This results in a large gate-2DEG capacitance $c_g \sim 30 \text{ fF}$ (for a gate length $l_g \sim 10\mu\text{m}$) which efficiently screens the inter-edge interactions. In particular we estimate $c_g \gg c_H$, with $c_H \sim 1 \text{ fF}$ the edge-to-edge capacitance for the full Hall bar length. Long and wide leads (not shown in Fig.1) connect the sample to the contact pads. Due to their large inductance and stray capacitance the leads acts as Π -filters above their cut-off frequency which introduces two additional π phase shifts in series.

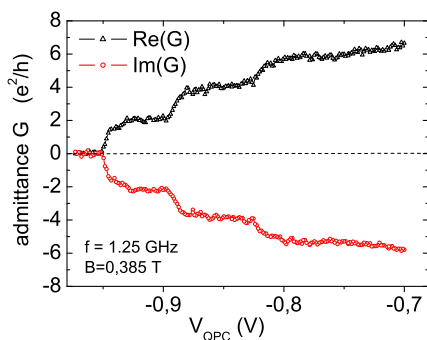


FIG. 2: Real and imaginary parts of the RF admittance of the quantum Hall conductor as function of the QPC voltage at $T = 50\text{mK}$. Both signals show steps at the opening of the first conducting channel. The negative imaginary part corresponds to a negative emittance which is characteristic of an inductive behavior.

The sample is mounted in between and connected to two impedance-matched 50Ω coplanar lines. The sample impedance being large ($\gtrsim 10k\Omega$), the RF conductance is simply proportional to the RF transmission of the set-up. The reference phase is calibrated by assigning

a purely capacitive admittance ($\simeq 40\text{fF}$) to the sample at the pinch-off. This is corroborated by the vanishing of the dc conductance.

Figure 1 shows the real part $Re(G)$ at the opening of the QPC. The large filling factor in the Hall bar ($N = 24$) allows the QPC to control the transmission of a large number of edge states (more than seven doubly degenerate channels). As can be seen in the figure, the transmission measured at RF frequency is proportional to the dc one. In the following we shall assign the value $2e^2/h$ to the $Re(G)$ steps as a calibration of our set-up.

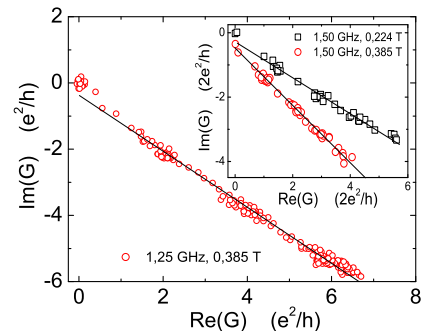


FIG. 3: Main frame: Nyquist representation of the data of Fig.2 showing that the admittance phase is constant as function of the number of transmitted channels and of their transmission. Point accumulation corresponds to the admittance plateaus in Fig.2. Insert : similar measurements obtained at two different magnetic fields showing the linear increase of the admittance phase with magnetic field.

Figure 2 shows $Re(G)$ and $Im(G)$ at $N = 14$ for the opening of the first three channels. Note that $Im(G)$ is negative which shows an inductive behavior of the sample. In addition $Re(G)$ and $Im(G)$ show similar regular steps as function of QPC transmission. The inductance step amplitude is $\simeq 1\mu\text{H}$. In fact both quadratures are mutually proportional as can be seen in the Nyquist plot of Fig.3. This corresponds to a transmission-independent phase factor, $\tan(\omega\tau) = -Im(G)/Re(G)$, and relaxation time τ . This phase is too large to be an experimental artefact. It is well explained by a constant relaxation time, in strong contrast with a classical R-L circuit where the time constant ($\sim L/R$) is proportional to the dc conductance. As an additional information, the inset depicts the linear magnetic field dependence of τ . These are the main results of our experiment. We propose below an interpretation relying on the theory developed by T. Christen and one of us for the low frequency admittance of chiral conductors [12].

In Ref.[12] the emittance, $E = Im(G/\omega)$, has been calculated for the case of a Hall bar with fully transmitted and/or fully reflected edge states. The calculation takes into account both inter-edge coupling and coupling to side gates. Here we consider the case of a quantum Hall bar coupled to side gates in series with a quantum

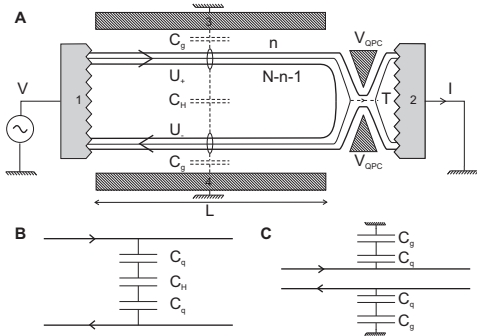


FIG. 4: A: Schematics of a quantum Hall bar with N edge states in series with a quantum point contact (QPC) with n fully transmitted channels and one partially transmitted channel. Electrochemical equivalent circuit of a Hall bar in the limit of weak edge to gate coupling (panel B) and weak interedge coupling (panel C). Notations are specified in the text.

point contact which controls the number of transmitted channels and their transmission (see fig.4A). Let l_g be the length of the bar and N the number of filled Landau levels. For simplicity we do not take into account spin degeneracy in the calculations. Let n be the number of fully transmitted modes and T the transmission of the partially transmitted one so that $(N-n-1)$ modes are totally reflected. The bar is sandwiched between two ohmic contacts across which the admittance is measured. The length l_g is small enough that propagation effects can be neglected. Thus charging of edge states is uniform but might differ on the upper and lower branch of the edge state. Thus we can assume that the edge states on the left upper side (labeled $+$) of the sample experience the same electrostatic potential U_+ and all the edge states on the left lower side (labeled $-$) experience the potential U_- . The upper and lower edge states on the left side are equally coupled to side gates with capacitance c_g and the long range electrostatic interactions between the upper and lower edges are described by a capacitance c_H (see fig.1.B). For simplicity, we take all left edge states to have the same density of states, $\nu = l_g/hv_D$, where v_D is the drift velocity. v_D is the ratio of the confining electric field to the applied magnetic field and is therefore $\propto N$. The quantum capacitance per channel is given by $c_q = e^2\nu = l_g e^2/hv_D$.

The low frequency response of the conductor is of the form $dI_\alpha(\omega)/dV_\beta(\omega) = G_{\alpha\beta} - i\omega E_{\alpha\beta} + \dots$ where α, β label current contacts indices 1 and 2 and gate indices 3 and 4. $G_{\alpha\beta}$ is non-zero only for current contacts. $E_{\alpha\beta}$ is a four terminal emittance matrix for the quantum conductor with its gates. According to Ref.[12], the emittance is

$$E_{\alpha\beta} = e^2 \sum_{k=+,-} \left[\frac{dN_{\alpha k\beta}}{dE} - \frac{dN_{\alpha k}}{dE} u_{k,\beta} \right], \quad (1)$$

where $\frac{dN_{\alpha k\beta}}{dE}$ is the partial density of states of carriers injected in contact β that reach the upper edge $k = +$,

(or the lower edge $k = -$), and exit the sample through contact α . $\frac{dN_{\alpha k}}{dE} = \sum_\beta \frac{dN_{\alpha k\beta}}{dE}$ is the emissivity of region $k = \pm$ irrespective of the contact from which the carriers are incident. The characteristic potential $u_{k,\beta}$ relates the change of the electrostatic potential of conductor k to that of the electrochemical potential of contact β . In our geometry the only non-zero partial density of states are:

$$\begin{aligned} \frac{dN_{1,\pm,1}}{dE} &= (R + N - (n+1))\nu \\ \frac{dN_{2,+1}}{dE} &= \frac{dN_{1,-,2}}{dE} = (T + n)\nu \end{aligned}$$

from which we find the emissivities $\frac{dN_{\alpha k\beta}}{dE}$. We next need to find the characteristic potentials $u_{k,\beta}$ on the upper and lower edges of the conductor $k = \pm$ for each of the four contacts $\beta = 1, 2, 3, 4$. To this end we follow closely Ref.[12] and find the emittance matrix. The two-terminal admittance measurement considered in this work is determined by the matrix element [18] $E_{2,1} \equiv E$. We find

$$E = -c_{\mu g} \frac{(T+n)}{N} - \eta c_{\mu H} \frac{(T+n)^2}{N^2} \quad (2)$$

where

$$c_{\mu g} = \frac{N c_q c_g}{c_g + N c_q}, \quad c_{\mu H} = \frac{N c_q c_H}{2c_H + N c_q} \quad (3)$$

$$\eta = \frac{1}{1 + c_g/N c_q} \times \frac{1}{1 + c_g/(2c_H + N c_q)} \quad (4)$$

are respectively the electrochemical capacitance between one edge and its side-gate and the mutual capacitance of the edge states across the Hall bar. The dimensionless coefficient η is smaller than unity and vanishes for strong gate coupling ($c_g \gg c_q$). Note that $v_D \propto N$, so that $N\nu$, $c_{\mu H}$ and $c_{\mu g}$ do not depend on N .

In the case of our experiment where inter-edge coupling is weak ($c_H \ll c_q \lesssim c_g$), $E \simeq -c_{\mu g}(T+n)/N$, and we obtain

$$G(\omega) = G_0 \left(1 - i\omega \frac{\hbar}{e^2} \frac{c_{\mu g}}{N} \right) \quad (5)$$

where $G_0 = (n+T)e^2/h$ is the Landauer dc conductance. Remarkably $G(\omega)$ exhibits a transmission independent phase in agreement with the experiment. Here the negative bar emittance can be interpreted with the equivalent circuit in Fig.1C in terms of leakage currents to the gate. It can be shown that the addition of the Hall bar impedance (obtained by setting $T = 1$, $N = n$ in Eq.(5)) and the QPC impedance (obtained by setting $l_g = 0$ in Eq.(5)) gives a different result with a transmission-dependent phase factor. This corresponds to the situation where a fictitious reservoir is inserted between the two components which amounts essentially to break the chirality of the experiment. We thus observe a violation of classical laws which is here a pure effect of chirality. The phase factor (independent of transmission) is given by the transit time of electrons through the

Hall bar $\tau = l_g/\widetilde{v}_D$, where $\widetilde{v}_D = v_D + \frac{e^2 N l_g}{h c_g}$ is the drift velocity which takes into account the screening by the side gate. Note that \widetilde{v}_D is proportional to total number of edge states N and thus inversely proportional to the magnetic field. This magnetic field dependence is clearly observed in the experiment as shown in the inset of Fig.3. From the slope ($\simeq -0.89$) of the Nyquist diagram obtained at $B = 0.385T$ (main frame), and the length of the gate $l_g \sim 10\mu\text{m}$, we obtain an estimate $\widetilde{v}_D \sim 10^5$ m/s of the drift velocity. This value is in order of magnitude agreement with the numbers in the literature. [15].

To put these results into perspective it is worth considering the opposite situation of strong inter-edge coupling ($c_g \ll c_q \lesssim c_H$). This corresponds to the case of a non-chiral quantum wire exemplified by single walled carbon nanotubes. In this limit we obtain

$$G(\omega) = G_0 \left(1 - i\omega G_0 \frac{\hbar^2}{e^4} c_{\mu H} \right). \quad (6)$$

It corresponds to the lowest order development for the admittance of the Landauer resistance $1/G_0$ in series with the Hall-bar electrochemical inductance $L_\mu = \frac{\hbar^2}{e^4} c_{\mu H} / N^2$. In this non-chiral situation classical laws are recovered. The negative Hall bar emittance can be interpreted with the equivalent circuit in Fig.4B, in terms of displacement counter-currents proportional to frequency. For very strong inter-edge coupling ($c_H \rightarrow \infty$), L_μ reduces to the usual kinetic inductance of a quantum wire, $L_{kin} = \frac{\hbar^2}{e^2} \frac{\nu}{2N}$ [19, 20]. The experimental signature of this regime would be the linear variation of the admittance

phase, or the time response of the circuit with transmission, which corresponds to a circle arch in the Nyquist diagram.

In conclusion, we have provided phase resolved measurements of the admittance of a quantum Hall bar coupled to gates in series with a quantum point contact. This realizes the simplest *chiral* quantum R-L circuit. We observe quantized steps in both the active and reactive parts of the admittance with a remarkable transmission-independent phase. The phase is directly related to the transit time of the electrons in the Hall bar. This interpretation is further supported by the expected magnetic field dependence of the transit time. Our measurements are well described by a scattering theory in the limit of strong side-gate coupling, allowing for a direct determination of the electronic transit-time. Our work demonstrates that interesting novel transport quantities such as the mesoscopic analogs of the RC and L/R-times become accessible in the GHz range provided the measurement is carried out on a sample with properties that can be tuned over a wide range for instance as here with a QPC.

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