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Localized modes in a one-dimensional diatomic chain of coupled spheres

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This paper presents the propagation of waves along a one-dimensional "diatomic" chain made up to welded spheres, i.e., with two steel spheres of different diameters alternating. First, a theoretical analysis is presented, which gives the vibration modes of an infinite chain, leading to two low-frequency branches, separated by a band gap. A theoretical analysis is then performed on a finite chain, containing an even or an odd number of spheres. Depending on the parity of the number of spheres in the finite chain and on the ratio between the masses of the spheres, it points out that localized modes may appear in the band gap. The theoretical results have been validated by a comparison between numerical and experimental results. Many applications of such systems can therefore be found: acoustic filters, noise and vibration isolation, acoustic wave guiding, etc.
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I. INTRODUCTION

There is a growing interest in the study of the propagation of acoustic waves in periodic samples, such as aggregates or multilayered structures, in particular, because there can be frequency ranges in which waves cannot propagate: in these systems, propagation of ultrasound or acoustic phonons is forbidden due to the existence of band gaps.[1–8] This fact is analogous to photonic band gaps for electromagnetic waves. Therefore, it is possible to predict that such systems can be applied for acoustic filters, noise and vibration isolation, acoustic wave guiding, etc. Recently, the study of phononic crystals is going further with the study of wave phenomena such as localization,[9–12] where the vibration amplitude is attenuated along the structure. By introducing a defect in the crystal, a narrow passband appears in the stop band.[9] Similar studies have also been performed in disordered systems.[13] In multilayered structures on silicon substrates, localized surface modes are observed in the structure depending on the nature of the last layer.[14] Bria and Djafari-Rouhani[15] have presented a theoretical analysis of finite one-dimensional phononic crystal structures, embedded between two substrates, which can exhibit an omnidirectional reflection band.

To go further in the comprehension of vibrations in aggregates and multilayered structures, the study of linear chains of spheres has been proposed. A simplified model has revealed the existence of allowed and forbidden frequency bands.[5,9,13,16] In a previous paper,[17] a quantitative analysis of the vibration modes in a finite set of identical coupled spheres was presented, with the help of an analogy with phonons in solid-state physics. Numerical modeling using the finite element method[18,19] was compared with experiments and a good agreement was observed. An interesting extension of the previous work is to consider a "diatomic" chain of spheres, with two spheres of different diameters alternating, made of the same material. This can lead to the presence or not of localized modes in the band gap.

The work presented in this paper concerns the low-frequency modes. The chain contains spheres that are welded and the weld is supposed to be the same between two adjacent spheres. The theoretical formulation is presented, first in the case of an infinite diatomic chain of spheres, leading to two low-frequency branches, separated by a band gap. Vibration modes in a finite chain are then studied. In this case, an even or an odd number of spheres in the chain is considered. This study points out that localized modes may be observed in the band gap, between the first two low-frequency branches. In Sec. III B, experimental results on various chains of spheres are compared to numerical results, confirming the theoretical analysis.

II. THEORETICAL APPROACH

The vibration modes of a single sphere and of a one-dimensional chain of identical spheres have been previously studied in detail.[15,16,20–22] Here, it is extended to the case of a diatomic chain of spheres, which means with spheres of alternately large and small diameters, made up of the same material. Many references present the case[23,24] of an infinite diatomic chain, thus, only the main results are recalled in this paper. A part of the investigated chain is shown in Fig. 1. The spheres are referenced as type 1 and type 2, alternating. They are characterized by their radius r_1 and r_2, and their masses m_1 and m_2. On Fig. 1, m_1 is less than m_2. The opposite situation can also be envisaged. In this case, the sphere on the left is the biggest one. The period of the structure is defined by d, which is a little bit smaller than $2r_1+2r_2$ due to the weld between adjacent spheres. The weld, characterized by r_w,[17] is supposed to be the same between two adjacent spheres. The first part of this section presents the case of an infinite diatomic chain of spheres. The vibration modes of a finite chain are presented in the second part.

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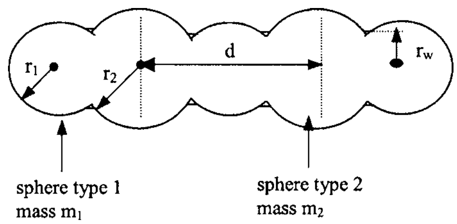

FIG. 1. Definition of the characteristic distances in a "diatomic" chain of welded spheres (r_1, r_2, d, and r_w).

A. Case of an infinite diatomic chain of spheres

For the sake of simplicity, equations of motion are written considering only the interaction between adjacent spheres. Coupling constants between spheres are identical. Due to the periodicity of the system, the wave number k is introduced for designating the vibration modes. k belongs to the first Brillouin zone, which is given by $[-\pi/d, +\pi/d]$. For each k value, vibration frequencies ω exist that lead to dispersion curves $\omega(k)$. In that configuration, these curves present two low-frequency branches, instead of one for the "mono-atomic" chain. These branches are named "acoustical branch" and "optical branch." They are solutions to the following equation:[23,24]

$$m_1 m_2 \omega^4 - 2C(m_1 + m_2)\omega^2 + 2C^2(1 - \cos kd) = 0, \qquad (1)$$

where C designates the coupling constant. This equation is completely symmetric in m_1 and m_2. The waves corresponding to the lower branch (i.e., *acoustical branch*) are in phase. In particular, at $k=0$, the corresponding frequency is equal to zero: the structure is translating. The branch starts at the origin and increases to $(2C/m_2)^{1/2}$ for $kd=\pi$, considering $m_1 < m_2$.

On the other hand, in the upper branch (i.e., *optical branch*), the motion of two adjacent spheres is out of phase. The branch starts at $[2C(1/m_1 + 1/m_2)]^{1/2}$ for $k=0$ and decreases to $(2C/m_1)^{1/2}$ for $kd=\pi$. The value of C is fitted with the help of the frequencies at $k=0$ and $k=\pi/d$. Taking account of the masses, the parameter C is enough to make the fitting.

In the stop band, for ω between $(2C/m_2)^{1/2}$ and $(2C/m_1)^{1/2}$, real frequency ω^2 exists for complex values of k, with real part equal to $\pm\pi/d$: the motion is attenuated along the chain, the vibrations amplitude decreases from one sphere to the other. Corresponding modes are named *localized modes*.[24] Notice that the stop band width is proportional to the quantity $m_1^{-1/2} - m_2^{-1/2}$.

The propagation of plane acoustic waves in such infinite and periodic structures is studied with the help of the finite element method,[18,19] using only the mesh of one unit cell, thanks to the Bloch-Floquet relations. Due to the symmetry of the structure, an axisymmetrical model is used that only requires bidimensional elements. The unit cell is meshed and divided into elements connected by nodes. In this study, isoparametric elements are used, with a quadratic interpolation along the element sides. The calculation provides dispersion curves from which results of physical interest can be easily extracted: identification of propagation modes, cutoff frequencies, passbands, and stopping bands.[21,22] Figure 2 presents the lowest branches of the dispersion curve of an infinite diatomic chain of steel spheres, the diameters of which are, respectively, 10 and 8 mm, and the masses are 4.08 and 2.09 g. The weld between spheres is characterized by r_w $=0.8$ mm. The equation of each branch can be determined with the help of Eq. (1). The first and last values of the acoustical and optical branches are equal to $(2C/m_2)^{1/2}$, $(2C/m_1)^{1/2}$, and $[2C(1/m_1 + 1/m_2)]^{1/2}$. A stop band appears between 55.0 and 77.2 kHz. For $k=\pi/d$, the lower value corresponds to $(2C/m_2)^{1/2}$ and the upper value corresponds to $(2C/m_1)^{1/2}$. Thus, one can deduce the value of $C=6.2 \times 10^6$ N/m. For given masses m_1 and m_2, if the weld between adjacent spheres increases, the value of C increases, too (for $r_w=1$ mm, $C=8\times 10^6$ N/m).

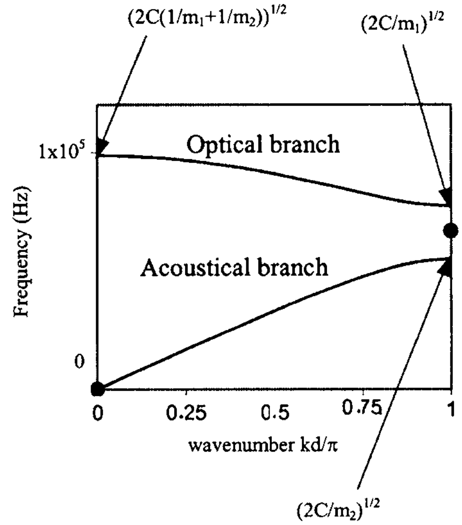

FIG. 2. Numerical results of the first branches of the dispersion curves of an infinite "diatomic" chain of steel spheres in the first Brillouin zone, with reduced scale. Diameter of the steel spheres: 10 and 8 mm, alternating. The marks are numerical results of a set of two welded spheres.

Higher branches related to other modes (Rayleigh, breathing, and whispering Gallery) are not reproduced on Fig. 2. They are approximately flat: in that case, frequencies of the Rayleigh modes are quite different for the 8-mm sphere and for the 10-mm sphere. This frequency difference considerably limits the coupling between spheres. There could be coupling between two neighboring identical spheres, but the intermediate sphere acts as a plug, thus branches are flat.

B. Case of a finite diatomic chain of spheres

Let us study the vibration modes of a diatomic finite chain. The chain contains N spheres, with alternate spheres of type 1 and type 2. If N is even, it is written as $N=2p$: the first sphere of the chain is type 1 and the last one is type 2. If N is odd, it is written as $N=2p+1$ and the first and the last spheres of the chain are both of the same type. In the following calculation, we suppose that the first sphere of the chain on the left is type 1. m_1 can be smaller or not than m_2. Let us designate by u_s the vibration amplitude of the sphere s of type 1, and v_s the vibration amplitude of the sphere s of type 2.

1. Even number of spheres in the chain

When N is even, writing the equations of motion gives a system, the size of which is N:

$$\omega^2 \begin{pmatrix} m_1 & 0 & \cdots & 0 & 0 \\ 0 & m_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & m_1 & 0 \\ 0 & 0 & \cdots & 0 & m_2 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_p \\ v_p \end{pmatrix} = C \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_p \\ v_p \end{pmatrix}. \quad (2)$$

Solving the system gives the N discrete frequencies of the corresponding mode. The resonance frequency 0 is always a solution to the system. The calculation of the corresponding eigenvector gives an identical vibration amplitude along the chain ($u_s = v_s$ for any s), the N spheres are all translating. One can notice that $\omega_{\mathrm{loc}} = [C(1/m_1 + 1/m_2)]^{1/2}$ is always a solution to the system. Its position is in the stop band described in Sec. II A, thus it is referenced as a localized mode. The calculation of the corresponding eigenvector shows that the vibration amplitude is decreasing from one extremity of the chain to the other. Considering $m_2 > m_1$, its normalized vibration amplitude is 1 on the smallest sphere at one end of the chain (m_1), then the amplitude is m_1/m_2 for the next two spheres, is m_1^2/m_2^2 for the next two spheres, etc., and finally is m_1^p/m_2^p for the last sphere at the other end (m_2). The vibration amplitude is greater on the extremity of the small sphere (m_1), and much smaller on the other extremity (m_2).

One can easily understand the presence of this mode in the stop band using a transition from the mono-atomic chain of spheres to the diatomic chain of spheres. We consider an infinite chain with identical welded spheres, regularly spaced at a distance d'. Dispersion curves are drawn in the first Brillouin zone, which is given by $[-\pi/d', +\pi/d']$. In a previous paper,[17] we have shown that the vibration modes of a finite chain can be deduced from the dispersion curves of an infinite chain, thanks to a quantification of the wave number. For the first low-frequency branch, the corresponding values of k are $k = s\pi/2pd'$, where $2p$ is the number of spheres in the chain and $s = 0, \ldots, 2p-1$. One can notice that a mode is placed in the middle of the Brillouin zone, for $s = p$. For the mono-atomic chain of spheres, if we use a double periodicity, the curve is duplicated due to Brillouin-zone folding. Thanks to the quantification of the wave number, the mode previously located in the middle of the Brillouin zone is now at the boundary of the Brillouin mode, where the curve is duplicated. The transition from the mono-atomic chain to the diatomic chain opens the gap. Therefore, a vibration mode appears in the stop band.

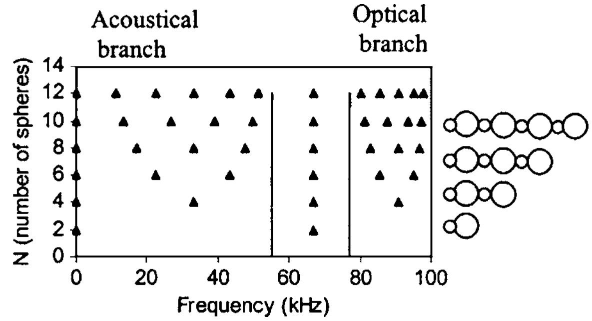

FIG. 3. Frequencies of the first modes of a chain with an even number of spheres ($N = 2p$), diameter=10 and 8 mm, alternating. The vertical lines correspond to the stop band limits (55.0 and 77.2 kHz).

To confirm the existence of a mode in the stop band, different diatomic chains of spheres, with various even number of spheres, have been meshed. As previously, the smallest sphere is 8 mm in diameter, the biggest sphere is 10 mm in diameter. The vibration modes are calculated using a modal analysis.[19] In Fig. 2, numerical results of a set of two welded steel spheres (10 and 8 mm) are presented (black dots). The set of two spheres is supposed to have both free ends. One mode is at the origin, one other mode is localized in the stop band. Figure 3 presents the position of the low-frequency modes in the frequency range ($f < 100$ kHz), for various diatomic chains of spheres (8 and 10 mm in diameter), with an even number of spheres ($2 \leqslant N \leqslant 12$). These modes are calculated with the help of the finite element method. The vertical lines correspond to the stop band limits. For a chain made of N spheres, N modes are obtained. There is always one mode for $f = 0$ and one mode at the same position in the stop band (66.8 kHz). The distribution of the other modes on the acoustical branch and on the optical branch is also plotted.

The normalized displacement in the chain direction is drawn for different values of N (Fig. 4) at the frequency of the localized mode (66.8 kHz). The extremity of the chain with the smallest sphere is always on the left. It clearly shows that the displacement is decreasing along the chain, which is a classical characteristic of a localized mode.

2. Odd number of spheres in the chain

In that case, we suppose that the first and the last spheres of the chain are type 1. When N is odd ($N = 2p+1$), writing the equations of motion gives a system, the size of which is N:

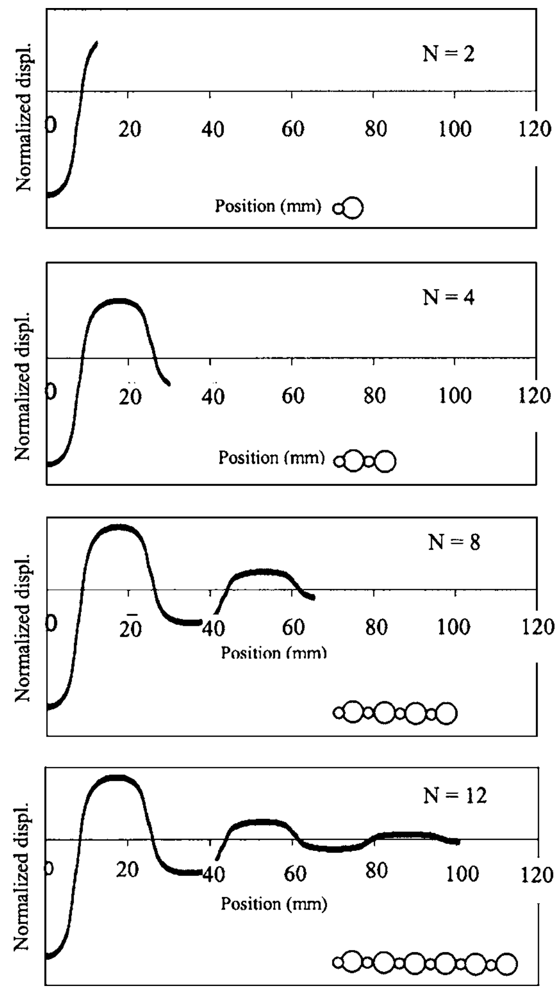

FIG. 4. Normalized displacement in the direction of the chain for $N=2$, 4, 8, and 12 spheres at the frequency of the localized mode. The smallest sphere is on the left.

$$\omega^2 \begin{pmatrix} m_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & m_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & m_1 & 0 & 0 \\ 0 & 0 & \cdots & 0 & m_2 & 0 \\ 0 & 0 & \cdots & 0 & 0 & m_1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_p \\ v_p \\ u_{p+1} \end{pmatrix} = C \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 2 & -1 & 0 \\ 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & \cdots & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_p \\ v_p \\ u_{p+1} \end{pmatrix}. \tag{3}$$

Solving the previous system gives the vibration modes of the finite chain.

For $N=3$, the resonance frequencies are 0, for which the three spheres have the same vibration amplitude, $(C/m_1)^{1/2}$ and $[C(1/m_1+2/m_2)]^{1/2}$. Depending on the ratio m_2/m_1, these last two modes can either or not be in the stop band described in Sec. II A.

Analytical solutions of the system have been calculated for $N=5$. The first resonance frequency is 0, for which the five spheres have the same vibration amplitude. The four other ω^2 solutions are: $C[\alpha+2-(\alpha^2+4)^{1/2}]/2$; $C[3\alpha+2-(\alpha^2+4)^{1/2}]/2$, $C[\alpha+2+(\alpha^2+4)^{1/2}]/2$, and $C[3\alpha+2+(\alpha^2+4)^{1/2}]/2$ with $\alpha=m_2/m_1$. Once again, depending on the value of the mass ratio α, two of these modes can either or not be in the stop band. For N greater or equal to 7, system (3) has been numerically solved. Many cases have been tested and lead to the following conclusions:

(i) If $m_1 > m_2$, [the first (type 1—m_1) and the last (type 1—m_1) spheres are the biggest ones], then no vibration modes appear in the stop band.

(ii) If $m_1 < m_2$, [the first (type 1—m_1) and the last (type 1—m_1) spheres are the smallest ones], then two vibration modes may appear in the stop band, depending on the mass ratio m_2/m_1.

- If $2 < m_2/m_1$, then two vibration modes appear in the stop band. Each of these two modes is localized at each extremity of the chain, particularly if the chain is long.
- If $3/2 < m_2/m_1 < 2$, then two vibration modes appear in the stop band only if the chain contains five spheres or more, there are no modes in the stop band for $N=3$.
- If $4/3 < m_2/m_1 < 3/2$, then two vibration modes appear in the stop band only if the chain contains seven spheres or more. There are no modes in the stop band for $N=3$ and $N=5$.
- More generally, if $(p+1)/p < m_2/m_1 < p/(p-1)$, then two vibration modes appear in the stop band only if the chain contains $2p+1$ spheres or more.

It means that, when the chain contains an odd number of spheres, localized modes can appear only if the first and the last spheres are the smallest ones. If the mass ratio is much greater than one, then two vibration modes appear in the stop band, whatever the number of spheres in the chain. If the mass ratio is close to one, then two vibration modes appear in the stop band only if the chain is long enough.

To confirm the existence of two modes or no mode in the stop band, various diatomic chains of spheres, with an odd number of spheres, have been meshed. As previously, the biggest sphere is 10 mm in diameter, the smallest sphere 8 mm in diameter. Thus, the mass ratio is equal to 1.95. The vibration modes are calculated using a modal analysis.[19] Figure 5 presents the position of the modes in the frequency band for various diatomic chains of spheres, with an odd number of spheres, when the smallest sphere is at both extremities [Fig. 5(a)] or the biggest sphere is at both extremities [Fig. 5(b)]. These modes are calculated with the help of the finite element method. For a chain made up of N spheres, N modes are obtained. There is always one mode at the origin. When the biggest sphere is at both extremities of the chain, no mode appears in the stop band. When the smallest

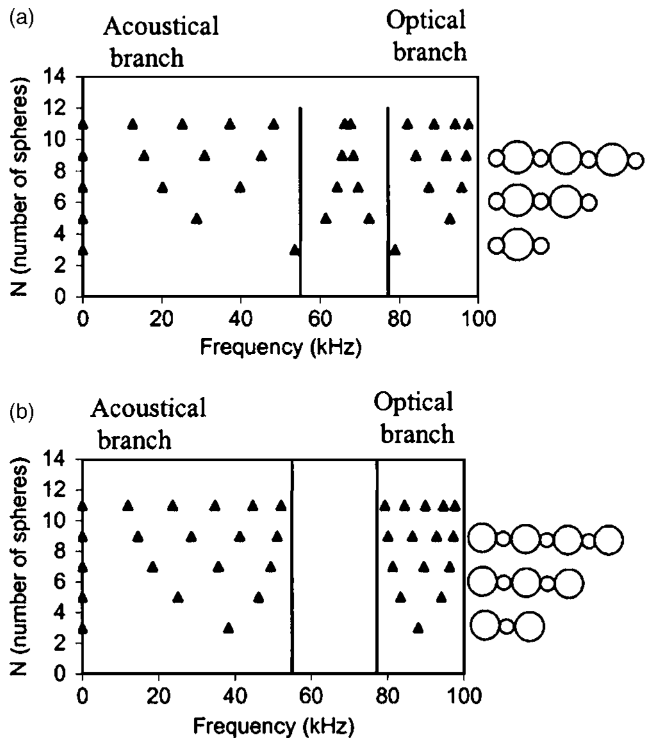

FIG. 5. Frequency of the first modes of a chain with an odd number of spheres ($N=2p+1$), when a small sphere (diameter=8 mm) is at each extremity (a), when a big sphere (diameter=10 mm) is at each extremity (b). The vertical lines correspond to the stop band limits (55.0 and 77.2 kHz).

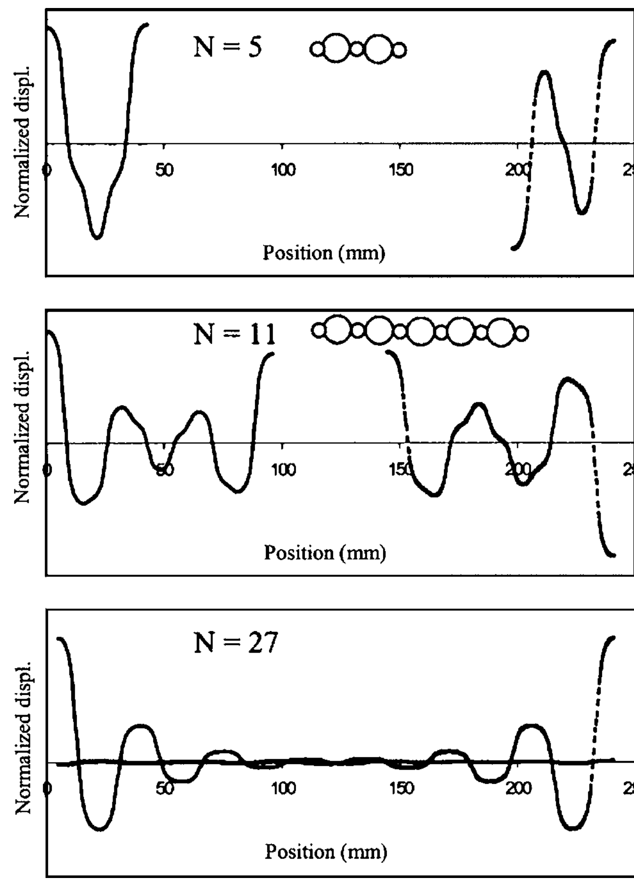

FIG. 6. Normalized displacement in the direction of the chain for 5, 11, and 27 spheres at the two frequencies of the localized mode. The small sphere is at each extremity. One mode is starting on the left, the other is starting on the right.

sphere is at both extremities of the chain, there are two modes in the stop band (for N greater than 5) and one can notice that, as the chain becomes longer, the two modes in the stop band become closer. For a chain made up of three spheres with two small spheres at the extremities, no localized modes are obtained, according to the previous model of the chain of atoms (mass ratio=1.95). It also shows the distribution of the other modes on the acoustical branch and on the optical branch.

The displacement in the chain direction is drawn for various numbers of spheres in the chain (Fig. 6) at the frequency of the localized modes [for $N=5$ at 66.4 and 72.3 kHz—Fig. 6(a), for $N=11$ at 66.2 and 67.7 kHz—Fig. 6(b), and for $N=27$ at 66.8 and 67.2 kHz—Fig. 6(c)]. One displacement is starting on the left side, the other is starting on the right. They only appear if the smallest sphere is at both extremities. As the chain becomes longer, these plots clearly show that the motion is localized on the small sphere at each extremity. The displacement is slowly decreasing when the chain is short.

III. EXPERIMENTS

A. Experimental setup and description of the samples

To generate the vibration modes of a diatomic chain, longitudinal broadband transmitters excited by short ultrasonic pulses are used. The transmitted temporal signal is visualized on the screen of an oscilloscope and the power spectrum of the windowed average signal (typically 500-μs length in time) is then calculated. Sometimes, unexpected peaks are observed in the experimental spectra. They are due to the propagation of a shear component which confirms that the longitudinal transducers are not perfectly polarized especially on the periphery of the piezoelectric sensor. No glue was used at the contact points between the transmitters and the samples. The static strength applied at the extremities of the samples does not affect the position of the peaks but only—as expected—the amplitude of the signal. So nonlinear effects have not been pointed out with this kind of experiments.

The fabrication of the samples is quite delicate. They were made up of spheres welded with a spot welding process. A very high intensity goes through the pilled beads and at each contact point a circular roll appears around the welding area, the radius of which is designated by r_w. This distance characterizes the coupling between the spheres and it was experimentally verified that—for samples used in the experiments—the coupling was the same at each contact point. Various samples were made with steel calibrated spheres of 6, 8, and 10 mm to check the theoretical conclusions drawn in Sec. II.

B. Experimental results on a finite set of spheres

In this section, the experimental results are presented and compared with the numerical results. They are concerned with a one-dimensional chain made up of two different spheres alternating. The frequency spectra of the trans-

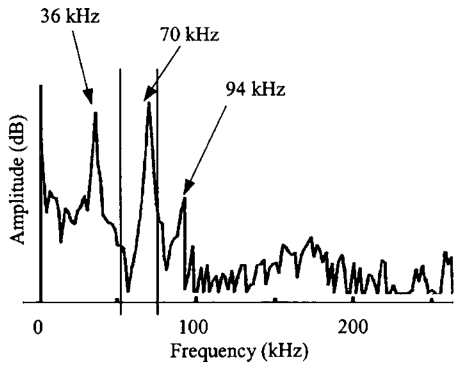

FIG. 7. Experimental frequency spectrum for a chain of four spheres whose diameters are 8 and 10 mm, alternating. The vertical lines correspond to the stop band limits (55.0 and 77.2 kHz).

mitted signal are analyzed and the positions of the main peaks are compared with the numerical calculations. The influence of the number of spheres in the sample is more specifically investigated. The study is mainly limited to the first low-frequency modes.

1. Case of an even number of spheres in the chain (mass ratio=1.95)

An example of experimental frequency spectrum is given in Fig. 7 for a chain made up of four spheres (10/8/10/8). The biggest sphere is 10 mm and the smallest sphere is 8 mm in diameter, giving a mass ratio equal to 1.95. We may distinguish three peaks at low frequency. These peaks are associated to the modes of the acoustical and optical branches. One mode is at a zero frequency and is not reproduced. Table I presents a comparison between the experimental and the numerical values of the low-frequency modes (in kHz) for two chains: (10/8) and (10/8/10/8). There is a reasonably good agreement between numerical and experimental results considering that the frequency resolution of the experimental setup is 2 kHz. Discrepancies may be due to the measurement of the distance r_w or to a slight misalignment of the beads during the welding process. It is interesting to notice that, as expected by the theory, one mode appears in the stop band (55.0–77.2 kHz for r_w =0.8 mm).

We experimentally notice that in the power spectrum, the amplitude of the peaks which corresponds to the optical branch is systematically smaller than the amplitude of the modes associated to the acoustical branch.

TABLE I. Comparison between experimental and numerical values of the low-frequency mode (in kHz) for different chains with an even number of spheres. Stop band spreads out between 55.0 and 77.2 kHz. The frequency in bold type corresponds to the localized mode. Mass ratio=1.95.

Number of spheres	Numerical (kHz)	Experimental (kHz)	%
2	**66.8**	**62**	7.2
Oo			
4	33.2	36	8.4
OoOo	**66.8**	**70**	4.7
	90.9	94	3.4

TABLE II. Comparison between experimental and numerical values of the low-frequency mode (in kHz) for different chains with an odd number of spheres, with a small sphere at each extremity (a), or with a large sphere at each extremity (b). Stop band spreads out between 55.0 and 77.2 kHz. The frequency in bold type corresponds to the localized modes. Mass ratio =1.95.

Number of spheres	Numerical (kHz)	Experimental (kHz)	%
(a) 3	53.6	56	4.5
oOo	79.0	80	1.3
5	28.9	30	3.8
oOoOo	**61.4**	**62**	0.9
	72.3	**72**	0.4
	92.9	92	0.9
(b) 3	38.3	40	4.4
OoO	88.1	90	2.1
5	25.1	26	3.6
OoOoO	46.2	46	0.4
	83.5	80	4.1
	94.2	94	0.2

2. Case of an odd number of spheres in the chain (mass ratio=1.95<2)

Table II presents a comparison between the experimental and the numerical values of the low-frequency modes for different chains with an odd number (N=3 and 5) of spheres, with a small sphere (8 mm in diameter) at each extremity [Table II(a)] or with a large sphere (10 mm in diameter) at each extremity [Table II(b)]. Once again, r_w is equal to 0.8 mm. The mode at zero frequency is not reproduced in the table. There is a very good agreement between numerical and experimental results (less than 4.5%). As expected localized modes appear for N=5 only if there is a small sphere at both extremities.

3. Case of an odd number of spheres in the chain (mass ratio=4.60>2)

Table III presents a comparison between the experimental and numerical data for two samples made up of three steel spheres of 6 and 10 mm in diameter, such that the mass ratio is equal to 4.6. The contact between spheres is characterized by r_w=0.6 mm. The results confirm that for the sample (6/10/6) [Table III(a)] two modes are detected in the stop band (47.4–102.7 kHz) and that no mode exists in the stop

TABLE III. Comparison between experimental and numerical values of the low-frequency mode (in kHz) for different chains with an odd number of spheres (diameter=10 and 6 mm), with a small sphere at each extremity (a), or with a large sphere at each extremity (b). Stop band spreads out between 47.4 and 102.7 kHz. The frequency in bold type corresponds to the localized modes. Mass ratio=4.6.

Number of spheres	Numerical (kHz)	Experimental (kHz)	%
(a) 3	**71.4**	**70**	2
oOo	**88.0**	**82**	6.8
(b) 3	33.1	32	3.2
OoO	109.9	104	5.3

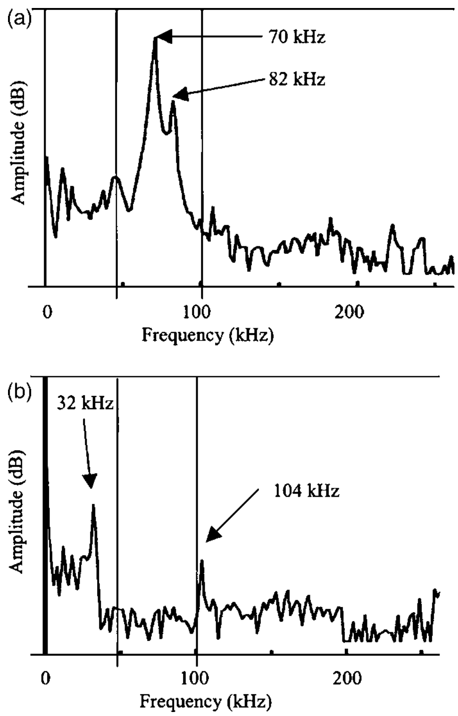

FIG. 8. Experimental frequency spectrum for a chain of three spheres whose diameters are 6 and 10 mm, alternating. The vertical lines correspond to the stop band limits (47.4 and 102.7 kHz). (a) Chain 6/10/6 and (b) chain 10/6/10.

band for the sample (10/6/10) [Table III(b)]. Figure 8 presents the corresponding experimental frequency spectrum. It clearly shows two peaks in the band gap in the first case [Fig. 8(a)] and no peak in the band gap in the second case [Fig. 8(b)]. These experimental observations are in agreement with the theoretical calculations and confirm the previous conclusions obtained from the numerical calculations.

IV. CONCLUSION

In this work, the vibrational response of finite one-dimensional periodic structure to an ultrasonic longitudinal pulse has been numerically and experimentally investigated. In particular, one-dimensional "diatomic" chains made up of welded spheres, i.e., with two steel spheres of different diameter alternating have been analyzed in terms of frequency spectrum and the influence of the parity of the number of the spheres in the chain is underlined. The possibility to generate sonic stop bands and passbands was demonstrated. Under certain conditions on the ratio between the mass of the two spheres, we have numerically and experimentally verified the existence of localized modes. This study lets us give a coherent physical interpretation of the number and existence of these so-called localized modes. All these results should contribute to the analysis of the elastic wave propagation in periodic systems such as finite aggregates or granular media. This work also suggests that such samples may be used as frequency filters to decouple resonators from their substrate, with an appropriate number of layers and appropriate mass ratio.

In the future, this investigation should be extended to two- and three-dimensional periodic, inhomogeneous, and finite lattice. The variations of the amplitude of the localized modes with the length of the samples should be analyzed and the influence of the boundary conditions on the frequency response could be investigated, too.

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