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M. Lachièze-Rey

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Cosmology in the Solar System: Pioneer effect is not cosmological

Marc Lachièze-Rey

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Abstract

Does the Solar System and, more generally, a gravitationally bound system follow the cosmic expansion law ? Is there a cosmological influence on the dynamics or optics in such systems ? The general relativity theory provides an unique and unambiguous answer, as a solution of Einstein equations with a local source in addition to the cosmic fluid, and obeying the correct (cosmological) limiting conditions. This solution has no analytic expression. A Taylor development of its metric allows a complete treatment of dynamics and optics in gravitationally bound systems, up to the size of galaxy clusters, taking into account both local and cosmological effects. In the solar System, this provides an estimation of the (non zero) cosmological influence on the Pioneer probe: it fails to account for the “ Pioneer effect ” by about 10 orders of magnitude. We criticize contradictory claims on this topic.

1 Introduction

The Pioneer effect [1] is often expressed as an “ anomalous acceleration ” $a_{Pioneer} = c \frac{dz}{dt}$ in the direction of the Sun, felt by the Pioneer probe (and similar). The numerical coincidence with the quantity $c H_0^{-1}$ has led to recurrent claims of a possible cosmological origin. We show that (according to general relativity) this fails by several orders of magnitude. The reason is simple : the “ cosmic acceleration ” of a probe (at distance D) following the cosmic expansion is not $c H_0$ but $\approx H_0^2 D$. For the Pioneer probe, this amounts to about 10 orders of magnitude below $a_{Pioneer}$.

Although this conclusion can be expected from order of magnitude arguments (see section 3), it is the rigorous consequence of the more general analysis performed in [8]. We gave there the solution of general relativity which describes a gravitationally bound system (the Sun, a galaxy, a galaxy cluster, ...) in the cosmological environment: this is the solution of the Einstein equations with the local (like the Sun) and global (the homogeneous isotropic cosmic fluid) gravitational sources; the limiting conditions are not those of a flat (Minkowski) space-time, but tend asymptotically toward the desired cosmological model, like the present favorite Λ -CDM. This system has no analytical solution in general (excepted the Schwarzschild - de Sitter solution for a pure de Sitter cosmological model, and with spherical symmetry). However, [8] provided a

Taylor development of the resulting metric in small parameters: the local (Newtonian) gravitational potential ϕ , and a dimensionless parameter $r H_0/c$, expressing the characteristic dimension of the system in Hubble length units. This approximation replaces the unknown exact space-time solution by an *osculating space-time* ([8], see also [2]), which is shown to describe perfectly the Solar system, galaxies, and galaxy clusters.

This enlightens the question of the influence of cosmology in these systems (see also [3]). In particular, an inertial probe in radial motion in the Solar System feels a “cosmic acceleration” (which is defined below covariantly) $q_0 H_0^2 D$. Its magnitude is about 10 orders of magnitude below the Pioneer effect. In the last section, we analyze some works by [5], [6] and [4] about cosmological influence on the Pioneer probe.

2 The osculating metric

This section gives a short account of the results of [8], in the framework of general relativity.

The chrono-geometry of a gravitationally bound system is correctly described by the solution of the Einstein equations with the local (e.g., the Sun) and cosmic sources, which tends asymptotically toward the Friedmann-Lemaître model, with the cosmic expansion law involving the measured values of the cosmic parameters ($H_0, \Omega_M, \Omega_\Lambda$). No analytic solution exists in general but we define two small parameters:

- the local gravitational potential ϕ . A first (Newtonian) order analysis is sufficient if we are not interested in strong field (post-Newtonian) effects;
- and a small dimensionless parameter $r H_0$ (we chose now units where $c = 1$), characterizing the influence of cosmology. Estimations show that this parameter remains very small: from about 10^{-13} in the solar system to 10^{-3} in a galaxy cluster.

For the case of a central massive source, [8] have obtained the Taylor expansion of the metric (their equation 22) at first order in ϕ and second cosmological order:

$$g \approx \left(1 - 2 \frac{M}{r} + q_0 (H_0 r)^2\right) dt^2 - dr^2 \left(1 + 2 \frac{M}{r} + \Omega_0 (H_0 r)^2\right) - r^2 d\omega^2. \quad (1)$$

Here H_0 is the present Hubble constant (not the time varying Hubble parameter) and q_0 the present deceleration parameter. This osculating metric [8] at the position of the observer solves the Einstein equations at the desired order. It is written in a convenient static coordinate system. Putting $\phi = 0$ gives the pure Friedmann-Lemaître model in its static form, see [8]. Putting $H_0 = 0$ gives the usual Newtonian solution, without cosmological effects. The calculations assume spherical symmetry of the source. When the cosmology is pure de Sitter, this solution is exact.

The Pioneer effect is not cosmological

The motion of an inertial probe like Pioneer is described by the radial geodesic equation derived from the metric (1). So is the null radial geodesic equation corresponding to the light-rays from the probe to the observer. This provides the redshift $z(\tau)$ of the probe measured by the observer, as a function of his proper time τ , and its derivative $\dot{z} \equiv \frac{dz}{d\tau}$. All these quantities are covariantly defined.

A comparison with the pure Newtonian treatment (i.e., which neglects cosmology) must involve frame-invariant quantities only. This is the case for z , τ and \dot{z} . This latter observable quantity is called “acceleration”, and the difference with the pure Newtonian estimation, which measures the effect of cosmology, is called a_{cosmic} . The calculations [8] give $a_{cosmic} = q_0 H_0^2 r$, with q_0 the usual deceleration parameter, ten orders of magnitude below $a_{Pioneer}$. This expression is exact even at the second cosmological order. The motions of both the source and the electromagnetic signal (in the optical approximation) are treated as relativistic, in the space-time curved by the cosmology and by the local potential. All calculations are covariant.

2.1 Orbital motion: a modified Kepler law

To complete the discussion of local cosmological effects, we consider the case of orbital motion, assumed purely circular: $V^r = V^\varphi = 0$. From [8], we may explicit the geodesics equations in the metric above:

$$\begin{aligned} \left(1 - 2\frac{M}{r} + q_0 (H_0 r)^2\right) V^t V^t - r^2 V^\theta V^\theta &= 1, \\ \left(\frac{M}{r^2} + q_0 H_0^2 r\right) V^t V^t - r V^\theta V^\theta &= 0. \end{aligned}$$

They lead to

$$\begin{aligned} V^t &\approx 1 + \frac{3M}{2r}, \\ (r V^\theta)^2 &\approx \frac{M}{r} + q_0 (H_0 r)^2. \end{aligned}$$

These quantities are the components of a vector. They are not directly observables, nor covariant. However, we may characterize an orbit by its proper period T and proper circumference $C = 2\pi r$.

The proper period is the integral of proper time along a closed orbit.

$$T = \int_{orbit} ds = \int_0^{2\pi} d\theta \frac{ds}{d\theta} = \frac{2\pi}{V^\theta} = \frac{2\pi r}{\sqrt{\frac{M}{r} + q_0 (H_0 r)^2}}.$$

This may be expressed under the form of a modification of the Kepler law

$$T^2 (1 + \epsilon_{cosmo}) = \frac{C^3}{2\pi M}; \quad \epsilon_{cosmo} \equiv \frac{q_0 H_0^2 T^2}{(2\pi)^2}. \quad (2)$$

This accounts for the cosmological effect on a circular orbital motion in a gravitationally bound system (see also the discussions by [3] and references therein), at Newtonian order, and second cosmological order: the orbit is not expanding, nor shrinking (at first order). Cosmology slightly modifies its characteristics w.r.t. the pure Newtonian case. We have expressed this modification as a modification (2) of the Kepler law. This effect is far from being measurable in the Solar system.

2.2 Cosmological effects in gravitationally bounded systems

The osculating metric describes any gravitationally bound system, like galaxies, galaxy clusters..., clearly accounting for the cosmological influence. Although calculations are performed with spherical symmetry, there is no problem (beside technical) to extend them to any distribution of sources. The case of an object which is bound by non gravitational interactions, like a material rod, an hydrogen atom..., remains open. We summarise our main results

- In such a bound system, an inertial object suffers basically the same cosmological effect that it would suffer in the absence of local gravitational sources. This effect may be described by a “ cosmic acceleration ” which adds to the local terms. Note that this cosmic acceleration would be zero for a non accelerated expansion ($q_0 = 0$).
- For application to the Solar systems, galaxies, galaxy clusters, a first order expansion in the cosmological parameter is largely sufficient (our results are valid up to second order).
- For a probe in radial [inertial] motion, this acceleration is manifest as an additional term in the derivative of the redshift w.r.t. the observer’s proper time. This is qualitatively of the same nature of the Pioneer effect, although many orders of magnitude below (see also the following section and [4] for a kinematical cosmic contribution).
- For a probe in orbital [inertial] motion (i.e., a planet), the cosmic acceleration adds to the local terms and slightly modifies the orbit, whose characteristics remain however constant (at first order): it does not expand or shrink (up to second order) under the effect of cosmology. The difference with the pure Newtonian case may be expressed as a modification (2) of the Kepler law.
- In the Solar system, the cosmic acceleration is far from being measurable. It fails to account for the Pioneer effect.

The weak influence of cosmological effects, measured by $q_0 H_0^2 r$, increases with the size of the system. In external regions of galaxies, or galaxy clusters, precision analyses (like estimations of dark matter, gravitational lensing....) would require to take it into account. Since their signature differs from that of the local effects, it may be hoped that future observations will reach the precision allowing to distinguish them from the local effects, so providing new kinds of cosmological tests.

3 Different analyses of the Pioneer effect

For the moment, the approximate coincidence between $a_{Pioneer}$ and $c H_0$ has no explanation. We have shown that it cannot be explained by relativistic cosmology. A modification of our gravitational theory may possibly provide an explanation for the Pioneer effect ([9],[7] and references therein) but the answer would not be cosmological.

To complete this short note, we analyse some recent discussions about cosmic effects for the Pioneer probe. One is in the context of special relativity [5] ; another in that of a pure expanding cosmology [6]. Both neglect the local gravitational sources. We show

that even in such simplified approaches, the correct treatment leads to the conclusion that cosmology fails by several orders of magnitude to account for the Pioneer effect.

3.1 A special relativistic approach

In the framework of special relativity, a particle following an expansion law cannot be inertial, but is submitted to a “cosmic force”. The latter imprints an acceleration a_{cosmic} responsible for the Hubble law, that [5] compare to $a_{Pioneer}$.

The authors start from an application of the redshift law to the de Broglie length of a massive particle. Unfortunately, this appears to be incompatible with special relativity. The special relativistic formula for the redshift of a particle, $z = v/c = \dot{r}/c$, implies for the de Broglie length $\lambda = h/p = h/mv \propto \frac{1}{z}$: it does not follow the expansion law. Moreover, it is impossible to imagine any modification justifying formula (9) of [5], since the latter would imply $z \mapsto \infty$ for $v = 0$, instead of the correct formula $z = 1$. There is no way to reconcile an expansion law for the de Broglie length with special relativity.

In fact, for a particle following an Hubble law, the redshift $z = v = \frac{dD}{dt} = H D$. Derivation gives

$$\frac{dz}{dt} = a_{cosmic} = \frac{dH}{dt} D + H \frac{dD}{dt} = -q_0 H^2 D,$$

in accordance with the result above.

Note the correct cosmological evolution of the de Broglie length length of a *non massive* particle ($p = h\nu/c$),

$$\lambda = \frac{c}{\nu} = \frac{c}{\nu_0 + \delta\nu} = (1 + z) \lambda_0.$$

3.2 Pure cosmic expansion approach

A different approach is adopted in [6], who also neglect the local potential. The relativistic approach of their section (2), involving geodesics, gives the correct order of magnitude. Their section (3), however, invokes a possible cosmic explanation for the Pioneer acceleration. We show that it is based on a misinterpretation of the cosmological equation.

For an observer at cosmic time t_{obs} , the equation (25) of [6] must be written

$$1 + z(t_{obs}) = \frac{\lambda_1}{\lambda_0} = \frac{a(t_{obs})}{a[t_{source}(t_{obs})]}, \quad (3)$$

where $t_{source}(t_{obs})$ is the time of emission, by the source, of the light-ray reaching the observer at t_{obs} . It is solution of the null radial geodesic equation:

$$\int_{t_{source}}^{t_{obs}} \frac{dt'}{a(t')} = r, \quad (4)$$

with r the constant coordinate of the comoving source. Derivation of the latter gives

$$\frac{dt_{source}}{d(t_{obs})} = \frac{a(t_{source})}{a(t_{obs})}. \quad (5)$$

On the other hand, the Taylor development of (3) near t_{obs} gives

$$z(t_{obs}) = -H(t_{obs}) (t_{source} - t_{obs}) + H(t_{obs})^2 (t_{source} - t_{obs})^2 (1 + q_0/2). \quad (6)$$

Derivation gives

$$\begin{aligned} \frac{dz(t_{obs})}{dt_{obs}} &= -\frac{dH(t_{obs})}{dt_{obs}} (t_{source} - t_{obs}) - H(t_{obs}) \left(\frac{dt_{source}}{dt_{obs}} - 1 \right) \quad (7) \\ +2 H(t_{obs}) \frac{dH(t_{obs})}{dt_{obs}} (t_{source} - t_{obs})^2 (1 + q_0/2) &+ 2 H(t_{obs})^2 (t_{source} - t_{obs}) \left(\frac{dt_{source}}{dt_{obs}} - 1 \right) (1 + q_0/2), \end{aligned}$$

where the two last terms are at third order. Thus, after using (5) and (3), we obtain again (up to second order)

$$\frac{dz(t_{obs})}{dt_{obs}} = -q_0 r a(t_{obs}) H(t_{obs})^2.$$

This result could be derived more quickly : neglecting local gravitation, a comoving galaxy is at a proper distance $D(t) = a(t) r$ from the observer, with r the comoving coordinate of the source, constant by definition. The Hubble law follows: $V = \dot{D} = H a(t) r = H D(t)$. Derivation gives $\dot{V} = \dot{H} D + H \dot{D}$, which leads to a_{cosmic} as above.

Thus, at the required order, pure cosmological calculations (i.e., without local effects) are sufficient to provide the correct value of the cosmic acceleration. In the same context, [4] also exhibited an additional contribution of cosmological origin.

3.3 A cosmologic kinematical effect

In addition to the “ cosmological acceleration ” considered above, cosmology also imprints a kinematical signature of the time-derivative of the redshift, as it was recently shown by [4].

In the pure cosmological context (neglecting the local gravitation), described by the usual RW metric, the comoving observer has velocity $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. An arbitrary radially moving source has velocity $v = \begin{bmatrix} \gamma(t) \\ \sigma(t)/a(t) \end{bmatrix}$, with $\gamma(t)^2 - \sigma^2(t) = 1$, and $a(t)$ the usual cosmic scale factor. The comoving observer observes the source with a redshift $1 + z = \frac{\delta t_{obs}}{\delta t_{source}} = \frac{a(t_{obs})}{a(t_{source})} (\gamma + \sigma)$.

The case of signal reflection has been studied in more details by [4]: a signal emitted by the comoving observer at t_0 , reflected by a moving mirror at t_1 (with velocity v as above) is observed by the same comoving observer at t_2 with a redshift

$$1 + z_{20} = \frac{\delta t_2}{\delta t_0} = \frac{a(t_2)}{a(t_0)} (\gamma + \sigma)^2. \quad (8)$$

This is both the formula (17) and the formula (22) of [4] (they are identical for radial motion).

In the inertial case (relevant for the Pioneer probe), γ and σ remain constant. It is easy to calculate the derivative of the redshift (8) w.r.t. t_2 , which is the proper time

of the observer. Using the usual Taylor developments of the cosmic quantities, one obtains (at lowest order)

$$\frac{dz}{dt_2} \approx H_2^2 (1 + q) (t_0 - t_2) + z H_2,$$

with H_2 and q the Hubble constant and the deceleration parameter at observer's position.

The first term in the RHS is the cosmic acceleration discussed above. The second term represents the kinematical contribution pointed by [4]. As discussed by these authors, it is also negligible in the situation of the Pioneer probe.

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