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We show that the expansion of an initially confined interacting 1D Bose-Einstein condensate can exhibit Anderson localization in a weak random potential. For speckle potentials used in quantum gases, the Fourier transform of the correlation function has a finite support and in 1D there is a mobility edge $k_{\rm m} = 1/\sigma_{\rm R}$, where $\sigma_{\rm R}$ is the correlation length of the disorder. Then, for the initial healing length of the expanding condensate $\xi_{\rm in} > \sigma_{\rm R}$ the localization is exponential, and for $\xi_{\rm in} < \sigma_{\rm R}$ it changes to algebraic.

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Disorder in quantum systems can have dramatic effects, such as strong Anderson localization of non-interacting particles in random media [1]. The main paradigm of Anderson localization (AL) is that the suppression of transport is due to a destructive interference of particles (waves) which multiply scatter from the modulations of a random potential. AL is thus expected to occur when the interference effects play a central role in the multiple scattering process [2]. In 3D, this requires the particle wavelength larger than the scattering mean free path, l, as pointed out by Ioffe and Regel [3]. One then finds a *mobility edge* at momentum $k_{\rm m} = 1/l$, below which AL can appear. In 1D and 2D all single-particle quantum states are predicted to be localized [4, 5, 6], except for certain types of disorder (see below and Ref. [7]). The subtle question is whether and how the interaction between particles can cause delocalization and transport, and there is a long-standing discussion of this issue for the case of electrons in solids [8].

Ultracold atomic gases can shed new light on these problems owing to an unprecedented control of interactions, a perfect isolation from a thermal bath, and the possibilities of designing accurately controlled random [9, 10, 11, 12, 13] or quasi-random [14] potentials. Of particular interest are the studies of localization in Bose gases [15, 16] and the understanding of the interplay between interactions and disorder in Bose and Fermi gases [17, 18]. Localization of expanding Bose-Einstein condensates (BEC) in random potentials has been reported in Refs. [11, 12, 13]. However, this effect is *not* related to AL, but rather to the fragmentation of the core of the BEC, and to strong single reflections from large modulations of the random potential in the tails [11]. Numerical calculations [11, 19, 20] confirm this scenario for parameters relevant to the experiments of Ref. [11, 12, 13].

In this Letter, we show that the expansion of a 1D interacting BEC can exhibit AL in a random potential without large or wide modulations. Here, in contrast to the situation in Refs. [11, 12, 13], the BEC is not significantly affected by a single reflection. For this *weak disorder* regime we have identified the following localization scenario on the basis of numerical calculations and the toy model described below.

At short times, the disorder does not play a significant role, atom-atom interactions drive the expansion of the BEC and determine the long-time momentum distribution, $\mathcal{D}(k)$. According to the scaling theory [21], $\mathcal{D}(k)$ has a high-momentum cut-off at $1/\xi_{\rm in}$, where $\xi_{\rm in} = \hbar/\sqrt{4m\mu}$ and μ are the initial healing length and chemical potential of the BEC, and m is the atom mass. As soon as the density is significantly decreased, the long-time expansion is governed by the scattering of almost non-interacting particles from the random potential. Each wave with momentum k undergoes Anderson localization on a momentum-dependent localization length $L(k)$. For speckle potentials used in the studies of quantum gases [10, 11, 12, 13] the Fourier transform of the correlation function has a finite support and in 1D there is a mobility edge $k_{\rm m} = 1/\sigma_{\rm R}$, where $\sigma_{\rm R}$ is the correlation length of the disorder (see below). Thus, the density of the localized BEC will be determined by the superposition of localized waves with momenta $k < \min(1/\xi_{\rm in}, 1/\sigma_{\rm R})$. Asymptotic behavior of the density profile shows that if the high-momentum cut-off is provided by the momentum distribution $\mathcal{D}(k)$ (for $\xi_{\rm in} > \sigma_{\rm R}$), the BEC is *exponentially* localized. Otherwise, if the cut-off is provided by the mobility edge (for $\xi_{\rm in} < \sigma_{\rm R}$) the localization is *algebraic*. These findings pave the way to observe AL in experiments similar to those of Refs. [11, 12, 13].

We consider a 1D Bose gas with repulsive short-range interactions, characterized by the 1D coupling constant g and trapped in an external harmonic potential $V_{\rm ho}(z) = m\omega^2 z^2/2$. The finite size of the sample introduced by the trapping potential provides a low-momentum cut-off for the phase fluctuations, and in the weakly interacting regime ($n \gg mg/\hbar^2$, where n is the 1D density), the gas forms a true BEC at sufficiently low temperatures [22].

We treat the BEC wave function $\psi(z,t)$ using the mean-field Gross-Pitaevskii equation (GPE). In the presence of a superimposed homogeneous random potential $V(z)$ [23], this equation reads:

$$i\hbar\partial_t\psi = \left[\frac{-\hbar^2}{2m}\partial_z^2 + V_{\rm ho}(z) + V(z) + g|\psi|^2 - \mu\right]\psi, \quad (1)$$

where ψ is normalized by $\int {\rm d}z|\psi|^2 = N$, with N being the number of atoms. It can be assumed without loss of gener-

ality that the average of $V(z)$ over the disorder, $\langle V\rangle$, vanishes, while the correlation function $C(z)=\langle V(z')V(z'+z)\rangle$ can be written as $C(z) = V_{\rm R}^2 c(z/\sigma_{\rm R})$, where the reduced correlation function $c(u)$ has unity height and width. So, $V_{\rm R} = \sqrt{\langle V^2\rangle}$ is the standard deviation, and $\sigma_{\rm R}$ is the correlation length of the disorder.

The properties of the correlation function depend on the model of disorder and are central features for AL. Although most of the discussion below is general, we mainly refer to a 1D speckle random potential [24] similar to the ones used in experiments with cold atoms [9, 10, 11, 12, 13]. It is a random potential with a truncated negative exponential single-point distribution [24]:

$$\mathcal{P}[V(z)] = \frac{\exp[-(V(z)+V_{\rm R})/V_{\rm R}]}{V_{\rm R}}\,\Theta\left(\frac{V(z)}{V_{\rm R}}+1\right), \quad (2)$$

where Θ is the Heaviside step function, and with a correlation function which can be controlled almost at will [13]. For a speckle potential produced by diffraction through a 1D square aperture [13, 24], we have

$$C(z) = V_{\rm R}^2 c(z/\sigma_{\rm R}); \qquad c(u) = \sin u/u. \quad (3)$$

The key point is that the Fourier transform of $C(z)$ has a finite support:

$$\widehat{C}(k) = V_{\rm R}^2\sigma_{\rm R}\widehat{c}(k\sigma_{\rm R});\ \widehat{c}(\kappa) = \sqrt{\pi/2}(1-\kappa/2)\Theta(1-\kappa/2), \quad (4)$$

so that $\widehat{C}(k) = 0$ for $k > 2/\sigma_{\rm R}$. This is a general property of speckle potentials, related to the way they are produced using finite-size diffusive plates [24].

We now consider the expansion of the BEC, using the following toy model. Initially, the BEC is assumed to be at equilibrium in the trapping potential $V_{\rm ho}(z)$ and in the absence of disorder. In the Thomas-Fermi regime (TF) where $\mu \gg \hbar\omega$, the initial BEC density is an inverted parabola, $n(z) = (\mu/g)(1 - z^2/L_{\rm TF}^2)\Theta(1 - |z|/L_{\rm TF})$, with $L_{\rm TF} = \sqrt{2\mu/m\omega^2}$ being the TF half-length. The expansion is induced by abruptly switching off the confining trap at time $t = 0$, still in the absence of disorder. Assuming that the condition of weak interactions is preserved during the expansion, we work within the framework of the GPE (1). Repulsive atom-atom interactions drive the short-time ($t \lesssim 1/\omega$) expansion, while at longer times ($t \gg 1/\omega$) the interactions are not important and the expansion becomes free. According to the scaling approach [21], the expanding BEC acquires a dynamical phase and the density profile is rescaled, remaining an inverted parabola:

$$\psi(z,t) = \left(\psi[z/b(t),0]/\sqrt{b(t)}\right)\exp\{{\rm i}mz^2\dot{b}(t)/2\hbar b(t)\}, \quad (5)$$

where the scaling parameter $b(t) = 1$ for $t = 0$, and $b(t) \simeq \sqrt{2}\omega t$ for $t \gg 1/\omega$ [11].

We assume that the random potential is abruptly switched on at a time $t_0 \gg 1/\omega$. Since the atom-atom interactions are no longer important, the BEC represents a superposition of almost independent plane waves:

$$\psi(z,t) = \int \frac{{\rm d}k}{\sqrt{2\pi}}\widehat{\psi}(k,t)\exp({\rm i}kz). \quad (6)$$

The momentum distribution $\mathcal{D}(k)$ follows from Eq. (5). Importantly, $\mathcal{D}(k)$ is stationary and has a high-momentum cutoff at the inverse healing length $1/\xi_{\rm in}$:

$$\mathcal{D}(k) = |\widehat{\psi}(k,t)|^2 \simeq \frac{3N\xi_{\rm in}}{4}(1-k^2\xi_{\rm in}^2)\Theta(1-k\xi_{\rm in}), \quad (7)$$

with the normalization condition $\int_{-\infty}^{+\infty}{\rm d}k\mathcal{D}(k) = N$.

According to the Anderson theory [1], k-waves will exponentially localize as a result of multiple scattering from the modulations of the random potential. Thus, components $\exp({\rm i}kz)$ in Eq. (6) will become localized functions $\phi_k(z)$. At large distances the function $\phi_k(z)$ decays exponentially, so that $\ln|\phi_k(z)| \simeq -\gamma(k)|z|$, with $\gamma(k) = 1/L(k)$ being the Lyapunov exponent, and $L(k)$, the localization length. The AL of the BEC occurs when the independent k-waves have localized. Assuming that the phases of the functions $\phi_k(z)$, which are determined by the local properties of the random potential and by the time t_0, are random uncorrelated functions for different momenta, the BEC density is given by

$$n_0(z) \equiv \langle|\psi(z)|^2\rangle = 2\int_0^\infty {\rm d}k\mathcal{D}(k)\langle|\phi_k(z)|^2\rangle, \quad (8)$$

where we have taken into account that $\mathcal{D}(k) = \mathcal{D}(-k)$ and $\langle|\phi_k(z)|^2\rangle = \langle|\phi_{-k}(z)|^2\rangle$.

We now briefly outline the properties of the functions $\phi_k(z)$ following from the theory of localization of single particles. For a weak random potential, using the phase formalism approach [23] the state with momentum k is written in the form:

$$\phi_k(z) = \ r(z)\sin[\theta(z)]\,;\ \partial_z\phi_k = kr(z)\cos[\theta(z)]\,, \quad (9)$$

and the Lyapunov exponent is obtained from the relation $\gamma(k) = -\lim_{|z|\to\infty}\langle\log[r(z)]/|z|\rangle$. If the disorder is sufficiently weak, then the phase is approximately kz and solving the Schrödinger equation up to first order in $|\partial_z\theta(z)/k - 1|$, one finds [23],

$$\gamma(k) \simeq (\sqrt{2\pi}/8\sigma_{\rm R})(V_{\rm R}/E)^2(k\sigma_{\rm R})^2\widehat{c}(2k\sigma_{\rm R}), \quad (10)$$

where $E = \hbar^2k^2/2m$. Such a perturbative approach assumes the inequality

$$V_R\sigma_{\rm R} \ll (\hbar^2k/m)(k\sigma_{\rm R})^{1/2}, \quad (11)$$

which is equivalent to $\gamma(k) \ll k$. Typically, Eq. (11) means that the random potential does not comprise large or wide peaks.

Deviations from a pure exponential decay of ϕ_k are obtained using diagrammatic methods [25], and one has

$$\langle|\phi_k(z)|^2\rangle \ = \ \frac{\pi^2\gamma(k)}{2}\int_0^\infty {\rm d}u\ u\sinh(\pi u)\times \quad (12)$$
$$\left(\frac{1+u^2}{1+\cosh(\pi u)}\right)^2 {\rm e}^{-2(1+u^2)\gamma(k)|z|},$$

where $\gamma(k)$ is given by Eq. (10). Note that at large distances ($\gamma(k)|z| \gg 1$), Eq. (12) reduces to $\langle|\phi_k(z)|^2\rangle \simeq \left(\pi^{7/2}/64\sqrt{2\gamma(k)}|z|^{3/2}\right)\exp\{-2\gamma(k)|z|\}$.

Equations (10) and (12) show that the localization effect is closely related to the properties of the correlation function of the random potential. In particular, if the Fourier transform of the correlation function $\widehat{C}(k)$ has a high-momentum cut-off $k_{\rm max}$, a mobility edge appears at $k_{\rm m} = k_{\rm max}/2$. This means that only particles with momenta $k < k_{\rm m}$ become localized. For the 1D speckle potential, from Eqs. (4) and (10) we find

$$\gamma(k) = \gamma_0(k)(1-k\sigma_{\rm R})\Theta(1-k\sigma_{\rm R});\ \gamma_0(k) = \frac{\pi\sigma_{\rm R}}{2k^2\xi_{\rm in}^4}\left(\frac{V_{\rm R}}{4\mu}\right)^2. \quad (13)$$

According to Eq. (13), one has $\gamma(k) > 0$ only for $k\sigma_{\rm R} < 1$ so that the mobility edge is $k_{\rm m} = 1/\sigma_{\rm R}$. We stress that the existence of a mobility edge is a *general* feature of optical speckle potentials, owing to the finite support of the Fourier transform of their correlation function.

We then use Eqs. (7), (12) and (13) for calculating the density profile of the localized BEC from Eq. (8). Since the high-momentum cut-off of the momentum distribution $\mathcal{D}(k)$ is $1/\xi_{\rm in}$ [see Eq. (7)], and for the speckle potential the mobility edge is $1/\sigma_{\rm R}$, the upper bound of integration in Eq. (8) is $k_{\rm c} = \min\{1/\xi_{\rm in}, 1/\sigma_{\rm R}\}$. As the density profile $n_0(z)$ is a sum of functions $\langle|\phi_k(z)|^2\rangle$ which decay exponentially with a rate $2\gamma(k)$, the long-tail behavior of $n_0(z)$ is mainly determined by the components with the smallest $\gamma(k)$. These are the components with momenta close to the high-momentum cut-off $k_{\rm c}$. Therefore, when calculating the integral in Eq. (8), one may use Taylor series for $\mathcal{D}(k)$ and $\gamma(k)$ at k close to $k_{\rm c}$. In the following, we limit ourselves to the leading order terms.

For $\xi_{\rm in} > \sigma_{\rm R}$, the high-momentum cut-off $k_{\rm c}$ in Eq. (8) is set by the momentum distribution $\mathcal{D}(k)$ and is equal to $1/\xi_{\rm in}$. In this case all functions $\langle|\phi_k(z)|^2\rangle$ have a finite Lyapunov exponent, $\gamma(k) > \gamma(1/\xi_{\rm in}) > 0$, and the whole BEC wave function is *exponentially localized*. For the long-tail behavior of $n_0(z)$, from Eqs. (7), (8) and (12) we obtain:

$$n_0(z) \propto |z|^{-7/2}\exp\{-2\gamma(1/\xi_{\rm in})|z|\};\quad \xi_{\rm in} > \sigma_{\rm R}. \quad (14)$$

Equation (14) assumes the inequality $\gamma(1/\xi_{\rm in})|z| \gg 1$, which can be alternatively written as $\gamma_0(k_{\rm c})(1-\sigma_{\rm R}/\xi_{\rm in})|z| \gg 1$.

For $\xi_{\rm in} < \sigma_{\rm R}$, the momentum cut-off is provided by the mobility edge, so that the Lyapunov exponents of $\langle|\phi_k(z)|^2\rangle$ in Eq. (8) do not have a finite lower bound. Then the localization of the BEC becomes *algebraic* and it is only *partial*. The part of the BEC wave function, corresponding to the waves with momenta in the range $1/\sigma_{\rm R} < k < 1/\xi_{\rm in}$, continues to expand. Under the condition $\gamma_0(k_{\rm c})(1-\xi_{\rm in}^2/\sigma_{\rm R}^2)|z| \gg 1$ for the asymptotic density distribution of localized particles, Eqs. (8) and (12) yield:

$$n_0(z) \propto |z|^{-2};\quad \xi_{\rm in} < \sigma_{\rm R}. \quad (15)$$

Far tails of $n_0(z)$ will be always described by the asymptotic relations (14) or (15), unless $\xi_{\rm in} = \sigma_{\rm R}$. In the special case of $\xi_{\rm in} = \sigma_{\rm R}$, or for $\xi_{\rm in}$ very close to $\sigma_{\rm R}$ and at distances where $\gamma_0(k_{\rm c})|(1-\xi_{\rm in}^2/\sigma_{\rm R}^2)z| \ll 1$, still assuming that $\gamma_0(k_{\rm c})|z| \gg 1$ we find $n_0(z) \propto |z|^{-3}$.

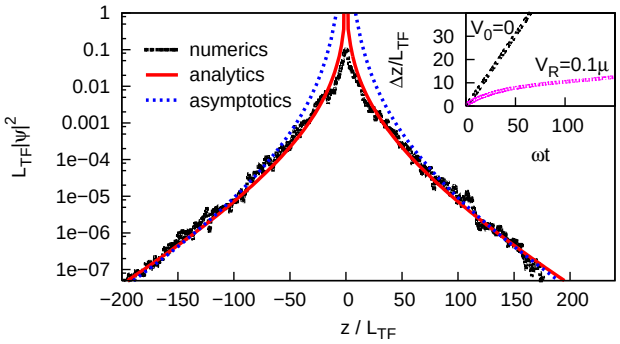

Figure 1: (color online) Density profile of the localized BEC in a speckle potential. Shown are the numerical data (black points), the fit of the result from Eqs. (7), (8) and (12) [red solid line], and the fit of the asymptotic formula (14) [blue dotted line]. Inset: Time evolution of the rms size of the BEC. The parameters are $V_{\rm R} = 0.1\mu$, $\xi_{\rm in} = 0.01L_{\rm TF}$, and $\sigma_{\rm R} = 0.78\xi_{\rm in}$.

Since the typical momentum of the expanding BEC is $1/\xi_{\rm in}$, according to Eq. (11), our approach is valid for $V_{\rm R} \ll \mu(\xi_{\rm in}/\sigma_{\rm R})^{1/2}$. For a speckle potential, the typical momentum of the waves which become localized is $1/\sigma_{\rm R}$ and for $\xi_{\rm in} < \sigma_{\rm R}$ the restriction is stronger: $V_R \ll \mu(\xi_{\rm in}/\sigma_{\rm R})^2$. These conditions were not fulfilled, neither in the experiments of Refs. [11, 12, 13], nor in the numerics of Refs. [11, 19, 20].

We now present the results of numerical calculations for the expansion of a 1D interacting BEC in a speckle potential, performed on the basis of Eq. (1). The BEC is initially at equilibrium in the combined random plus confining harmonic potential, and the expansion of the BEC is induced by switching off abruptly the confining potential at time $t = 0$ as in Refs. [11, 12, 16]. The differences from the model discussed above are that the random potential is already present for the initial stationary condensate and that the interactions are maintained during the whole expansion. This, however, does not significantly change the physical picture.

The properties of the initially trapped BEC have been discussed in Ref. [18] for an arbitrary ratio $\xi_{\rm in}/\sigma_{\rm R}$. For $\xi_{\rm in} \ll \sigma_{\rm R}$, the BEC follows the modulations of the random potential, while for $\xi_{\rm in} \gtrsim \sigma_{\rm R}$ the effect of the random potential can be significantly smoothed. In both cases, the weak random potential only slightly modifies the density profile [18]. At the same time, the expansion of the BEC is strongly suppressed compared to the non-disordered case. This is seen from the time evolution of the rms size of the BEC, $\Delta z = \sqrt{\langle z^2\rangle - \langle z\rangle^2}$, in the inset of Fig. 1. At large times, the BEC density reaches an almost stationary profile. The numerically obtained density profile in Fig. 1 shows an excellent agreement with a fit of $n_0(z)$ from Eqs. (7), (8) and (12), where a multiplying constant was the only fitting parameter. Note that Eq. (8) overestimates the density in the center of the localized

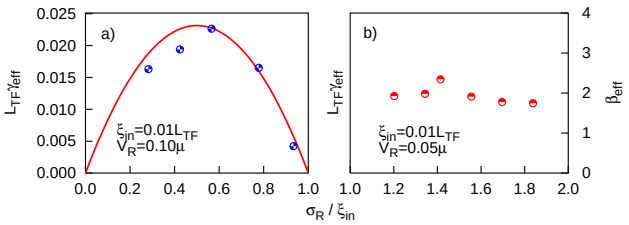

Figure 2: (color online) a) Lyapunov exponent $\gamma_{\rm eff}$ in units of $1/L_{\rm TF}$ for the localized BEC in a speckle potential, in the regime $\xi_{\rm in} > \sigma_{\rm R}$. The solid line is $\gamma(1/\xi_{\rm in})$ from Eq. (13). b) Exponent of the power-law decay of the localized BEC in the regime $\xi_{\rm in} < \sigma_{\rm R}$. The parameters are indicated in the figure.

BEC, where the contribution of waves with very small k is important. This is because Eq. (13) overestimates $\gamma(k)$ in this momentum range, where the criterion (11) is not satisfied.

We have also studied the long-tail asymptotic behavior of the numerically obtained density profiles. In the regime where $\xi_{\rm in} > \sigma_{\rm R}$, we have performed fits of $|z|^{-7/2}{\rm e}^{-2\gamma_{\rm eff}|z|}$ to the numerical data. The obtained $\gamma_{\rm eff}$ are in excellent agreement with $\gamma(1/\xi_{\rm in})$ following from the prediction of Eq. (14), as shown in Fig. 2a. For $\xi_{\rm in} < \sigma_{\rm R}$, we have fitted $|z|^{-\beta_{\rm eff}}$ to the data. The results are plotted in Fig. 2b and show that the long-tail behavior of the BEC density is compatible with a power-law decay with $\beta_{\rm eff} \simeq 2$, in agreement with the prediction of Eq. (15).

In summary, we have shown that in weak disorder the expansion of an initially confined interacting 1D BEC can exhibit Anderson localization. Importantly, the presence of a mobility edge for 1D speckle potentials can change localization from exponential to algebraic. Our results draw prospects for the observation of Anderson localization of matter waves in experiments and we would like to raise an interesting problem for future studies. The expanding and then localized BEC is an excited Bose-condensed state as it has been made by switching off the confining trap. Therefore, the remaining small interaction between atoms should cause the depletion of the BEC and the relaxation to a new equilibrium state. The question is how the relaxation process occurs and to which extent it modifies the localized state. For 1D Bose gases with short-range interactions the relaxation rate should depend on the amplitude of the disorder, since in the absence of an external potential this is an integrable system where the momentum distribution of an excited state does not change.

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