Numerical reconstruction of interfacial defects and interface thermal resistances using thermal measurements

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Abstract

Thermography provides an experimental mean to control the quality of interface in laminates or in coatings. It is thus important to link thermal measurements obtained on free surfaces to the possible interfacial defects. In this paper a numerical inverse method is proposed to localize and quantify the defects from measured thermal data. The identification problem leads to a non-linear minimization problem solved using gradient methods.

1. Introduction

In elaboration of fiber composites or coatings, a very important point is the realisation of a perfect interface between the fiber and the matrix or between the coating and its substrate. Some interfacial defects may appear like delamination or like presence of undesirable matter between layers of the laminates. It is very important to localize such defects and also to determine their intensity (quality of the interface). As defects are located inside the materials, they cannot be observed or detected directly but it is already possible to obtain qualitative information from measurements done on free surfaces away from the interface. We present in this paper a numerical method to identify and *quantify* interfacial defects in materials using thermal data usually obtained using non destructive testing methods like infrared thermography.

2. Modelisation of interfacial defects

The problem adressed here is a two layer material as shown *figure 1.* The boundary surface S_m is plane and infinite as well as the interface S_i located at a known depth. Moreover, the second layer extends to infinity in depth. Infrared thermography is a means to detect possible interfacial defects. To do so, a known thermal flux is applied on the surface S_m and in the same time, a thermal detector measures the temperature on this surface. The modelling of this problem is indicated on *figure 2.*. The two homogeneous layers occupy respectively the geometrical domains Ω_1 and Ω_2. For each domain Ω_n, the thermal properties are the specific heat c_n, the density ρ_n and the thermal conductivity k_n. To model the interface and its defects, we define an interface thermal resistance $R(x)$, that is, the temperature jump is proportional to the heat flux $q(x,t)$ across S_i :

$$\|T\|(x,t) = -R(x)\, q(x,t) \tag{1}$$

where

$$q(x,t) = -k_{\pm} \frac{\partial T}{\partial y}(x, h_{\pm}, t)$$

$T(x,y,t)$ is the temperature field and $\|\cdot\|$ denotes the jump operator across S_i : $\|f\|(x) = f(x,h_2,t) - f(x,h_1,t)$.

Besides, the temperature evolution in the conductive medium is governed by the well-known homogeneous transient heat conduction equation:

$$\mathrm{div}(k_n \nabla T) - \rho_n c_n \frac{\partial T}{\partial t} = 0 \quad n = 1,2 \tag{2}$$

The known thermal flux applied on S_m is:

$$g(x,t) = -k_1 \frac{\partial T}{\partial y}(x,t) \tag{3}$$

and as in any transient problem we need to precise also the initial conditions which are:

$$T(x,y,0) = 0 \quad \text{in } \Omega_1 \cup \Omega_2 \tag{4}$$

3. Identification problem

The identification problem consists in determining the interface resistance $R(x)$ from the measurable data e.g. the temperature $T_m(x,t)$ for $x \in S_m$ and $t \in [0,\tau]$. τ represents the duration of the measurements.

If $T_R(x,y,t)$ denotes the temperature field governed by equations (1), (2), (3) and (4), one can remark that the temperature measurement on S_m acts as a overspecified data and allows one to reconstruct the interface resistance $R(x)$. To do so, $R(x)$ is treated as a minimizer of the least-squares functional $J(R)$:

$$J(R) = \frac{1}{2} \int_0^{\tau} \int_{S_m} \left[T(x,t) - T_m(x,t) \right]^2 dS\, dt \tag{5}$$

under the constraint

$$T(x,t) = T_R(x,0,t) \quad \text{in } S_m \times [0,\tau] \tag{6}$$

Hence the reconstruction problem leads to a non-linear constrained least-squares problem.

4. Calculation of the gradient for the minimization

As the minimization problem has no explicit solution, a way to minimize the functional J is to use non-linear optimisation algorithms. The most efficient ones, e.g. conjugate gradient or BFGS variable metric [1], need not only $J(R)$ but also its gradient with respect to the design variable R. The latter could be computed using finite difference

formulae, but it is far safer, from the point of view of accurancy and efficiency, to use an exact expression of the gradient.

However, differentiation of (5) is by no means a trivial matter, since J depends *functionally* on R.

Let the set V of scalar functions $v(x,y,t) \quad \{(x,y) \in \Omega_1 \cup \Omega_2 ; t \in [0,\tau]\}$ be defined as:

$$V = \left\{ v(x,y,t), v \in L^2, \nabla v \in L^2, v(x,y,0) = 0 \right\} \tag{7}$$

The temperature field T_R solves the following variational formulation, which is equivalent to equations (1), (2), (3) and (4); find $T \in V$ such that:

$$\forall w \in V \quad \int_\Omega \left\{ k \nabla T \cdot \nabla w + \frac{\partial T}{\partial t} w \right\} d\Omega = \int_{S_m} g w \, dS + \int_{S_i} \frac{\|T\| \|w\|}{R} dS \tag{8}$$

In order to express the gradient of the functional J under the constraint (6), let us introduce the Lagrangian $\mathcal{L}$:

$$\mathcal{L}(T,w,R) = \int_0^\tau \left\{ \tfrac{1}{2} \int_{S_m} (T - T_m)^2 \, dS + \int_{S_m} g w \, dS + \int_{S_i} \frac{\|T\| \|w\|}{R} dS \right.$$
$$\left. - \int_\Omega \left(k \nabla T \cdot \nabla w + \frac{\partial T}{\partial t} w \right) d\Omega \right\} dt \tag{9}$$

One has evidently $J(R) = \mathcal{L}(T,w,R)$ if the constraint (6) is satisfied. The gradient of J will thus be investigated by means of the gradient of $\mathcal{L}$. As $\mathcal{L}$ is a functional, we introduce an evolution of the functions T, w, R defined by the parameter η and the pseudo-velocities $\dot{T}$, $\dot{w}$, $\dot{R}$:

$$T(\eta) = T + \eta \dot{T} \qquad w(\eta) = w + \eta \dot{w} \qquad R(\eta) = R + \eta \dot{R} \tag{10}$$

Then the derivative $\partial \mathcal{L} / \partial \eta$ of the functional $\mathcal{L}$ considered as a function of η is a linear form of the functions $\dot{T}$, $\dot{w}$, $\dot{R}$. In the following, all derivatives with respect to η will be implicitely taken for $\eta = 0$. The result is a sum of three linear forms as follows:

$$\frac{\partial \mathcal{L}}{\partial \eta} = \mathcal{L}_{,T}(\dot{T}) + \mathcal{L}_{,w}(\dot{w}) + \mathcal{L}_{,R}(\dot{R}) \tag{11}$$

$$\mathcal{L}_{,T}(\dot{T}) = \int_0^\tau \left\{ \int_{S_m} (T - T_m) \dot{T} \, dS + \int_{S_i} \frac{\|\dot{T}\| \|w\|}{R} dS - \int_\Omega \left(k \nabla \dot{T} \cdot \nabla w + \frac{\partial \dot{T}}{\partial t} w \right) d\Omega \right\} dt$$

$$\mathcal{L}_{,w}(\dot{w}) = \int_0^\tau \left\{ \int_{S_m} g \dot{w} \, dS + \int_{S_i} \frac{\|T\| \|\dot{w}\|}{R} dS - \int_\Omega \left(k \nabla T \cdot \nabla \dot{w} + \frac{\partial T}{\partial t} \dot{w} \right) d\Omega \right\} dt$$

$$\mathcal{L}_{,R}(\dot{R}) = - \int_0^\tau \int_{S_i} \frac{\|T\| \|w\|}{R^2} \dot{R} \, dS \, dt \tag{12}$$

where $\mathcal{L}_{,f}$ represents the linear tangent application with respect to the field f.

The gradient of J is also determined by its linear form of the function $\dot{R}$:

$$\frac{\partial J}{\partial \eta} = J_{,R}(\dot{R})$$

and the important result is:

$$\text{if} \quad \begin{cases} \mathcal{L}_{,T} = 0 \\ \mathcal{L}_{,w} = 0 \end{cases} \quad \text{then} \tag{13}$$

$$\frac{\partial J}{\partial \eta} = \frac{\partial \mathcal{L}}{\partial \eta} \quad \text{or} \quad J_{,R} = \mathcal{L}_{,R} \tag{14}$$

The second condition of (13) is obtained when T is the solution of the problem governed by equations. (1), (2), (3) and (4). The first condition of (13) gives a variational equation on w and its solution, w_R , is called the *adjoint state*. A detailed examination of this equation shows that w is equivalently governed by the following final-boundary value problem:

$$[\![w]\!](x,t) = -R(x)q(x,t) \qquad \text{where} \qquad q(x,t) = -k_{\pm}\frac{\partial w}{\partial y}(x,h_{\pm},t) \tag{15}$$

$$\operatorname{div}(k_n \nabla w) + \rho c_n \frac{\partial w}{\partial t} = 0 \quad n = 1,2 \tag{16}$$

$$k_1 \frac{\partial w}{\partial y}(x,0,t) = -\left[T(x,0,t) - T_m(x,t)\right] \quad \text{on } S_m \tag{17}$$

$$T(x,y,\tau) = 0 \quad \text{in } \Omega_1 \cup \Omega_2 \tag{18}$$

It is apparent in (16) that w is governed by the *backward* heat equation: the evolution problem for w must be solved from $t = \tau$ to $t = 0$.

5. Numerical identification

5.1. Resolution procedure

To realize the identification of the interface resistance R, we propose to use a transient thermal code which can take into account jumps of temperature at the interface. The minimisation of J is performed by a succession of iterations. Each iteration, in which R is fixed, consists in four steps:

— direct computation of the temperature T following the equations (1), (2), (3), and (4);

— reverse computation of the adjoint state w from equations (15), (16), (17), and (18);

— computation of $J_{,R}$ from equations (12) and (14);

— modification of R which leads to a decrease of J.

One must remark that the computation of the adjoint state w can be done with the same code that computes the temperature T if the following transformation is respected:

$$w(x,y,t) = \omega(x,y,\tau - t)$$

5.2. 1D results

A first investigation was done on a 1D model to test the identification method using the calculation of the gradient of *J*. All the thermal fields were assumed to be uniform along the *x*-axis and to depend only on *y* and *t*. Simulated data with a known resistance *R* were used for the imposed flux *g* and for the measured temperature T_m. Several cases were studied depending on the conductivity of the two layers, on the simulated resistance and on the noise added to the measured temperature (see *table 1*). To indicate the effects of noise, the reconstructed value of *R* is given within a range corresponding to twice the value of J_{min}. With no noise the reconstruction is always perfect whereas the presence of noise leads to inaccurate values especially if the duration of the measurements is short.

6. Conclusions

The detection of internal interfacial defects found in laminates or coatings can be performed with the identification method presented above based on a minimization problem. This numerical method is all the more efficient since it quantifies the defects through the thermal resistance *R* and allows the use of optimization algorithms with gradient. We propose an exact expression of the gradient which needs no more computation than a classical transient thermal problem.

This method of defect identification may be generalized to more complicated geometries and to other boundary conditions. It is also possible to consider the depth location of the interface as an unknown function and this leads to the calculation of the gradient of *J* with respect to a surface location.

Futhermore, the inverse problem decripted above is ill-posed [2] due to the nature of the integral operators involved [3] and due to unavoidable noise of the measured data. As a consequence, the procedure needs to be *regularized* in order to stabilize its solution and avoid erroneous results caused by noised data. The regularization consists in introducing a *stabilization functional* $S(R)$ which incorporates a priori informations like physically relevant range of values, rough estimate of the location of defects, expected regularities...

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Table 1. – Results of a 1D example.

τ	k_1	k_2	Simulated R	Noise 0%	Noise 1%	Noise 3%
12	1	1	1	1 1—1	1 0.84—1.14	1 0.54—1.49
12	10	1	1	1 1—1	1 0.94—1.05	1 0.83—1.19
12	1	10	1	1 1—1	1 0.92—1.06	1 0.75—1.23
12	1	1	3	3 3—3	3 2.46—3.47	3 1.7—4.9
6	1	1	1	1 1—1	1.02 0.57—1.68	
4	1	1	1	1 1—1	1 0.16—6.5	

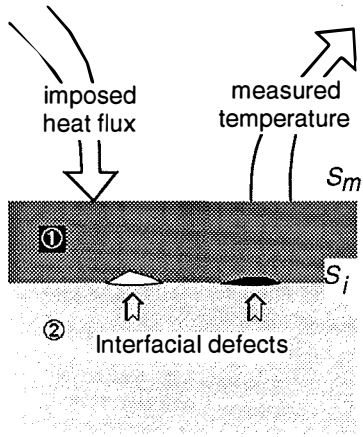

Fig. 1. – Thermography on a laminate with interfacial defects.

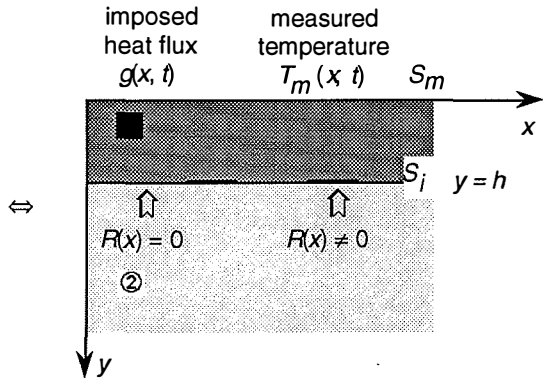

Fig. 2. – Modelisation of the interfacial defects.