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**MODELLING AND ANALYSIS OF STATIC TRANSMISSION ERROR –  
EFFECT OF WHEEL BODY DEFORMATION AND INTERACTIONS BETWEEN  
ADJACENT LOADED TEETH**

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## INTRODUCTION

The main source of excitation in gearboxes is generated by the meshing process. Researchers usually assume that transmission error and variation in gear mesh stiffness are responsible for noise radiated by the gearbox. Welbourn defined transmission error as the difference between actual position of the output toothed wheel and the position it would occupy if the gear drive were perfect [1]. Its characteristics depend on the instantaneous situations of the meshing tooth pairs. Under load at very low speed (static transmission error), these situations result from tooth deflections and manufacturing errors.

Under operating conditions, the mesh stiffness variations (due to variations in the length of contact line and tooth deflections) and the excitation located at the mesh point generate dynamic mesh force which is transmitted to the housing through shafts and bearings. Noise radiated by the gearbox is closely related to the vibratory level of the housing.

Predicting the static transmission error is a necessary condition to reduce noise radiated from the gearbox. This paper deals with estimation of static transmission error and mesh stiffness variations of spur and helical gears. For these purposes, two distinct numerical tools have been developed. They are based on a 3D finite element analysis of tooth deflections, using two different modellings and solvers. Compliance matrix associated with nodes of tooth flanks are computed for each toothed wheel. Two methods are used to solve the static equilibrium of the gear pair, in order to estimate load distribution and static transmission error, for a set of successive positions of the driving wheel. Different modellings of a generic gear pair have been built in order to analyse the effect of wheel body deformation and interactions between adjacent loaded teeth. Results obtained using each method have been compared.

## I- CALCULATION PROCEDURE

### 1.1 Three-dimensional finite element modelisation of toothed wheels

Calculation of the static transmission error firstly requires estimation of the loaded teeth deflections. In order to evaluate these required quantities, Tavakoli proposed to model gear tooth using a non uniform cantilever beam [2]. Tobe used a cantilever plate [3], while numerous authors developed 2D finite element tooth modelling [4].

Unfortunately, hypothesis related to these models can not be justified because characteristic dimensions of gear teeth are neither representative of a beam nor a plate, and tooth bending behaviour changes along gear width.

Furthermore, authors who used 3D finite element tooth modelling usually assumed that wheel bodies were rigid and that deformation of a tooth pair was not affected by load applied on the adjacent loaded tooth pairs [5], although these hypothesis have not been validated.

These remarks led each author to develop original 3D finite element modellings for each toothed wheel [6, 7]. Gear geometry (involute profile, fillet) is defined accurately owing to the noticeable influence of gear design parameters on the tooth deflection. For each toothed wheel, several adjacent teeth and the whole body are modelled. Each wheel is assumed to be locked on rigid shaft.

Method 1 uses 3D solid elements with 20 nodes per element and 3 degrees of freedom per node. Model of each toothed wheel has about 8000 nodes, 1500 elements and 17000 degrees of freedom. Figure 1 displays model of a 35/49 helical gear [6].

A normal unitary load is applied on each node of the tooth flank. For each loaded position, the normal displacement of all the other nodes is calculated after subtracting the local deformation. This operation allows to extract only the compliance due to bending and shear deformation, excluding the contact deformation. Then, the compliance matrix  $\underline{\underline{H}}^{u,F}(\mathbf{w}=\mathbf{0})$  associated with nodes of the tooth flanks is built. It is defined as :  $\underline{\underline{u}} = \underline{\underline{H}}^{u,F}(\mathbf{w}=\mathbf{0}) \cdot \underline{\underline{F}}$ . This procedure is successively applied to the pinion and the gear.

Method 2 uses 3D solid elements with 8 nodes per element. Model of each toothed wheel has about 6500 nodes, 5000 elements and 18000 degrees of freedom [7]. The main difference compare with method 1 is the calculation, from 3D analysis, of curve fitting of displacement, both in axial and radial directions. Furthermore, a procedure allows method 2 to calculate cross-coupling compliance functions between all loaded teeth pairs while method 1 only takes account of the cross-coupling compliance parameters of tooth N with previous tooth (N-1) and next tooth (N+1).

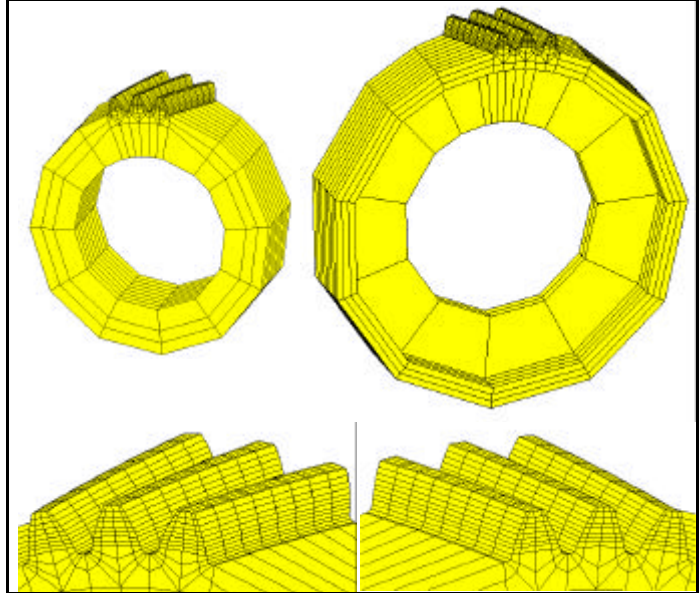


Figure 1 : Modelling of the gear pair.

## 1.2 Calculation of the static transmission error

Static transmission error is expressed as linear displacement  $\delta$  at the pitch point.  $\delta$  is calculated for a set of successive position  $\theta$  of the driving wheel, in order to evaluate its evolution against time. For each value of  $\theta$ , a cinematic analysis of gear mesh allows to determine the location of contact line for each loaded tooth pair. These contact lines are discretized. The matrix equation which governs static equilibrium of the gear pair can be written as follows :

$$\left\{ \begin{array}{l} \underline{\underline{H}}^{u,F}(\mathbf{w}=\mathbf{0}) \cdot \underline{\underline{F}} = \underline{\underline{d}}(\mathbf{q}) - \underline{\underline{e}} - \underline{\underline{hertz}}(\underline{\underline{F}}) \\ \underline{\underline{S}} \underline{\underline{F}}[i] = \underline{\underline{F}}_{total} \end{array} \right. \quad (1)$$

- $\underline{\underline{H}}^{u,F}(\mathbf{w}=\mathbf{0})$  is the compliance matrix related to the nodes of contact lines. Using method 1, components of the matrix are linear combination of components of the compliance matrix of each toothed wheel previously calculated. Using method 2, they are directly obtained from the curve fitting of displacement.
- $\underline{\underline{d}}(\mathbf{q})$  is the vector corresponding to linear displacement of the gear related to the pinion. This linear displacement is identical for the whole nodes of contact lines.
- $\underline{\underline{F}}$  is the load vector induced by the input torque.
- $\underline{\underline{e}}$  is the vector corresponding to the initial distance from the gear surface to the pinion surface. It is calculated from tooth surface modifications and manufacturing errors. (Inclination and deviation induced by static deformations of shafts, bearings and housing should be added to manufacturing errors).
- $\underline{\underline{hertz}}$  is the hertzian deformation vector calculated for each loaded tooth pair using Hertzian theory.

Unknown quantities of system (1) are the  $N$  components of the load vector  $\underline{F}$  and the value of  $\underline{d}(\mathbf{q})$ . System (1) is non linear because Hertzian deformations are non linear and because each loaded teeth pair is progressively going into contact : the total length of lines of contact grows with the applied load (geometric non linearity).

For each position of the driving wheel, both methods 1 and 2 use similar iterative procedures to solve the static equilibrium of the gear pair and to calculate the load distribution on the contact lines and the static transmission error.

## II- NUMERICAL RESULTS

A generic 35/49 helical gear pair fitting out a truck gearbox have been considered, in order to study the main physical phenomena (figure 2). Figures 4, 5 and 7 display results obtained with method 2 considering an input torque equal to 1300 Nm. Results and conclusions obtained with method 1 are similar (see part 2.4).

Number of teeth	35 / 49
Normal module (mm)	3.5
Normal pressure angle	22.5°
Nominal helix angle	21.539°
Facewidth (mm)	36.5
Rim thickness (mm)	36.5 / 9.124
Center distance (mm)	158
Tooth addendum (mm)	3.300 / 3.297
Tooth dedendum (mm)	4.725 / 4.200
Tool radius (mm)	0.875/1.225
Shaft diameter (mm)	77 / 95
Rim diameter (mm)	77/ 155
Transverse / overlap /	1.373 / 1.219 /
Total contact ratio	2.592

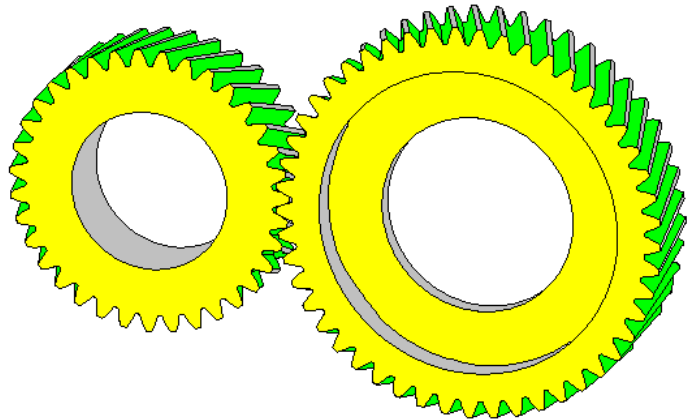


Figure 2 : 35 /49 helical gear.

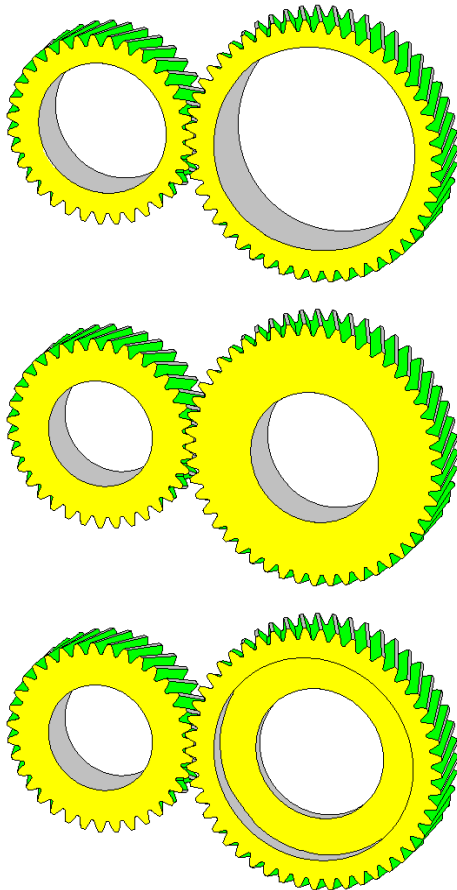
### 2.1 Effect of wheel body deformation

Among all deflections external to the tooth, the closest one is the wheel body deformation under load. Three distinct modellings of the gear pair have been compared in order to estimate the influence of this parameter. First, bodies of the driving and driven wheels are supposed to be rigid (model 1). Second, driving and driven wheel bodies are supposed to be full (model 2). Third, the thin-rimmed body of the driven wheel is taken into account (model 3).

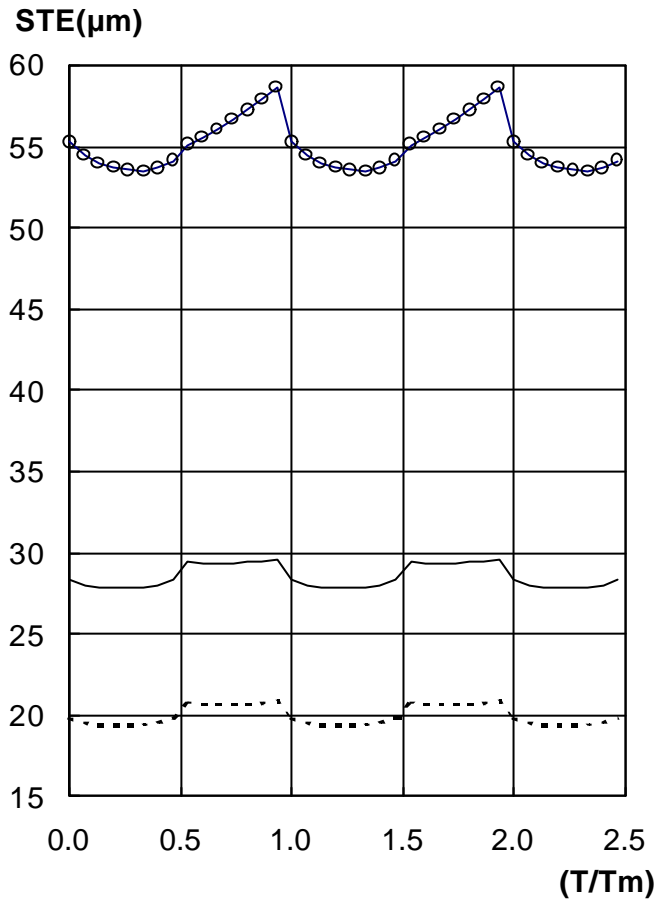
For a gear pair without manufacturing errors, static transmission error is  $T_m$ -periodic ( $T_m$  meshing period).

Figure 4 firstly shows that elasticity of wheel bodies induces an increasing of the static transmission error mean value (20.1  $\mu\text{m}$  for model 1, 28.7  $\mu\text{m}$  for model 2 and 55.3  $\mu\text{m}$  for model 3). Furthermore, load applied on each tooth induces bending deformation of the wheel bodies and angular displacement of the loaded teeth. Amplitude of the wheels deformation depends on load distribution and so, on the position of the driving wheel. Therefore, elasticity of

wheel bodies modifies the contact between loaded tooth pairs and induces a change of static transmission error variations. Its peak to peak value is equal to 1.4  $\mu\text{m}$  with rigid wheel bodies (model 1), 1.8  $\mu\text{m}$  with full bodies (model 2), and 5.2  $\mu\text{m}$  with a thin-rimmed driven wheel (model 3). Reducing the rim thickness could lead to excessive transmission error variations, especially for high input torque.



**Figure 3 :** Model 1 : Rigid wheel bodies.  
 Model 2 : Full wheel bodies.  
 Model 3 : Thin-rimmed driven wheel.



**Figure 4 :** Static transmission error.  
 Model 1 (-----); Model 2 (———) ; Model 3 (oooooooo).

## 2.2 Interactions between adjacent loaded teeth

When a normal unitary load is applied on a node of the tooth flank, bending deformations not only extend over the loaded tooth but also over the adjacent teeth. As several couples of teeth can mesh simultaneously, there may be interactions between adjacent loaded teeth.

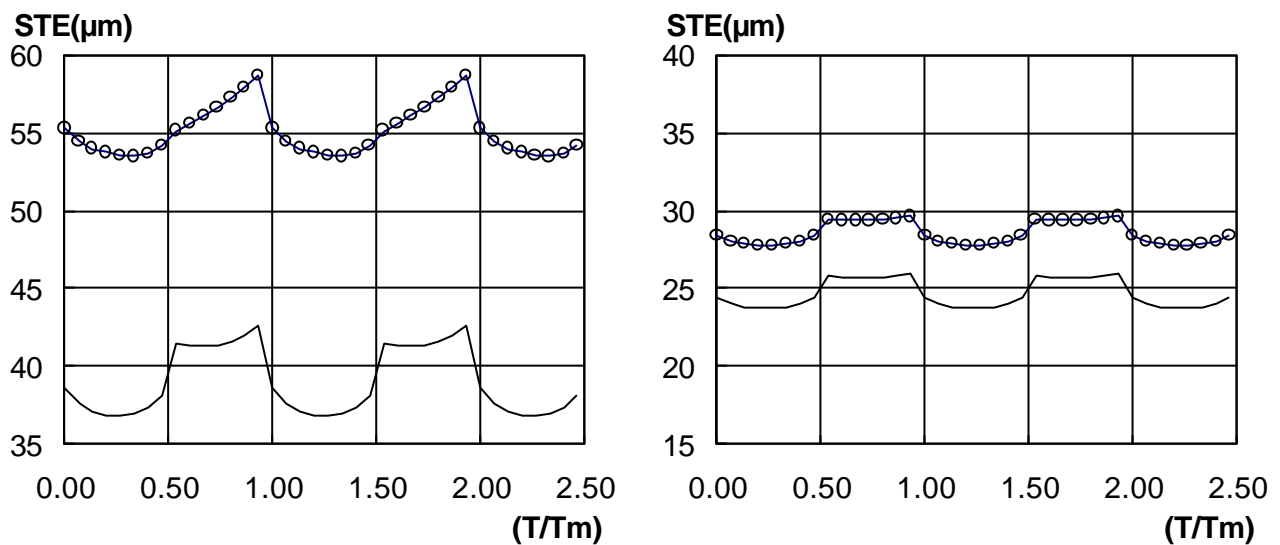
The compliance matrix  $\underline{H}^{u,F}(\mathbf{w}=\mathbf{0})$  related to the nodes of contact lines has been modified in order to compare static transmission errors of the gear pair (model 3) with and without interactions.

Figures 5a shows that interactions between adjacent loaded teeth must be taken into account to correctly predict characteristics of static transmission error. In fact, interactions lead to an

increasing of the mean value of static transmission error (from 39.4  $\mu\text{m}$  to 55.3  $\mu\text{m}$ ) and to a decreasing of its peak to peak value (from 5.8  $\mu\text{m}$  to 5.2  $\mu\text{m}$ ).

Figure 5b displays the effect of interactions for the gear pair with full wheel bodies (model 2). It shows that the effect of interactions of a gear fitted out with full wheel bodies is less significant than interactions of a thin-rimmed gear. The mean value of static transmission error only increases from 24.8  $\mu\text{m}$  to 28.7  $\mu\text{m}$  and its peak to peak value only decreases from 2.2  $\mu\text{m}$  to 1.8  $\mu\text{m}$ .

Furthermore, results obtained show that it is not possible to evaluate transmission error and mesh stiffness of a gear pair from the stiffness of a tooth pair alone, in spite of this method is usually used. Bending deformation of a loaded tooth is not only induced by the load applied on the tooth but also by the loads applied on the other teeth.



(a) Model 3 (Thin rimmed driven wheel).

(b) Model 2 (full wheel bodies).

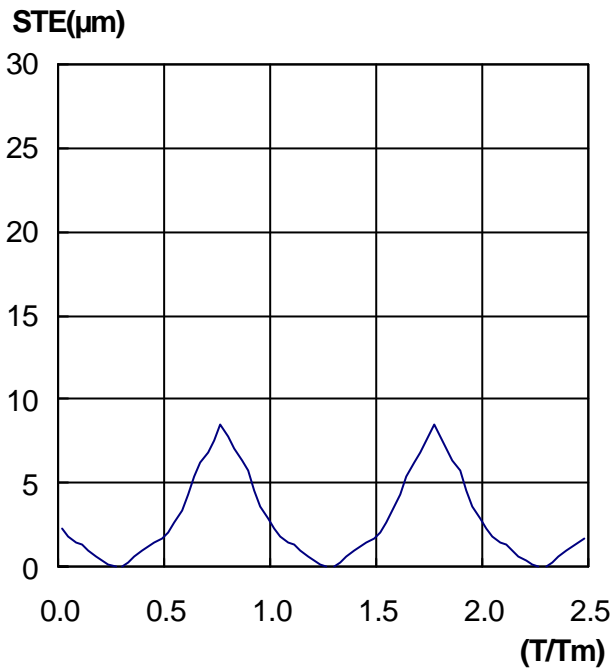
**Figure 5 :** Static transmission error. With (ooooooo) or without (—) interactions between adjacent loaded teeth.

### 2.3 Tooth modifications

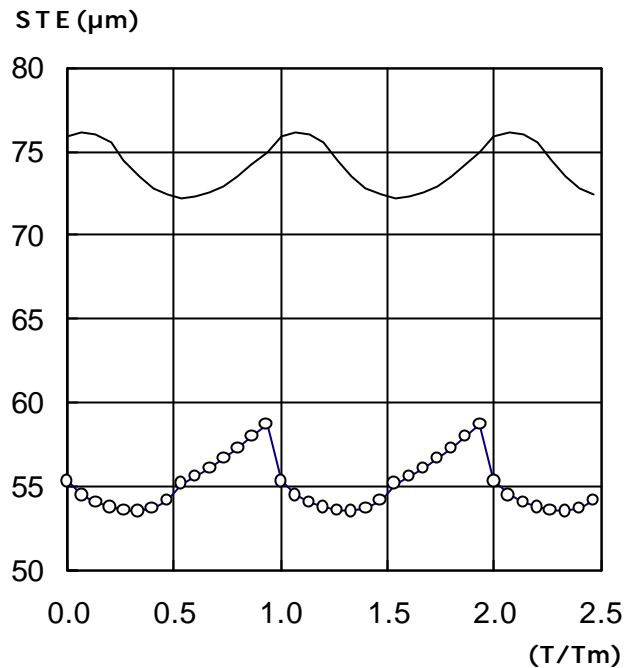
For unmodified tooth surface and ideal operating conditions, unloaded static transmission error (cinematic error) is nul and variations of static transmission error under load simply increase with the input torque. Evolution of static transmission error versus torque is very much dependant of the tooth modifications. Developed numerical tools allow to analyse the effect of different tooth modification types and to select the correct micro-geometry. Depending on the application, one can choose to optimize static transmission error for a given torque or for a range of torques.

For each toothed wheel, parabolic tip relief (20  $\mu\text{m}$ ), parabolic root relief (20  $\mu\text{m}$ ) and parabolic symetric crowning (10  $\mu\text{m}$ ) have been introduced. Figure 6 displays the unloaded static transmission error. Its peak to peak value is equal to 8.1  $\mu\text{m}$ .

Figure 7 displays static transmission error of the modified gear pair. Its mean value has increased (from 55.3  $\mu\text{m}$  to 74.0m) while its peak to peak value has decreased (from 5.2  $\mu\text{m}$  to 3.9  $\mu\text{m}$ ).



**Figure 6 :** Unloaded static transmission error (cinematic error).



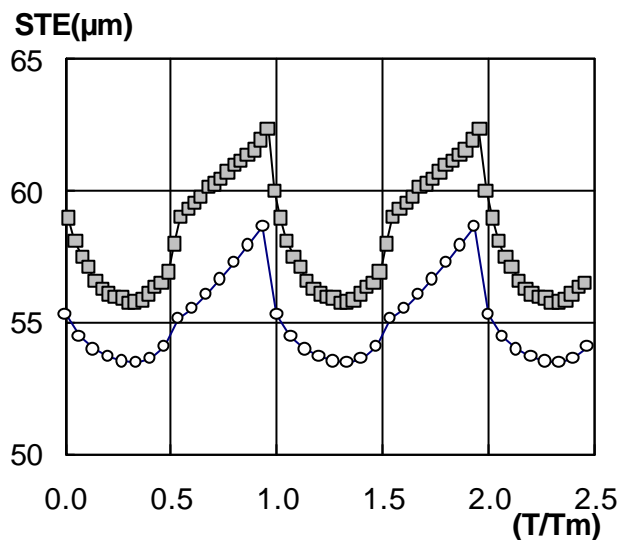
**Figure 7 :** Static transmission error under load with (—) and without (ooooooo) tooth modifications.

## 2.4 Comparisons between methods 1 and 2

Results obtained with methods 1 and 2 have been compared in order to validate both methods and conclusions.

Figure 8 displays static transmission error for the unmodified gear pair fitted out with a thin-rimmed driven wheel (model 3). It shows that the results obtained with method 1 are similar to those obtained with method 2, even if the mean value and the peak to peak value of static transmission error are a little bit higher.

Figure 9 compares results obtained for the unmodified gear pair modelled with models 1, 2 and 3, with or without interactions between adjacent loaded teeth. It allows to validate both methods 1 and 2. (Differences do not exceed 4 μm for the static transmission error mean value and 1 μm for its peak to peak value).

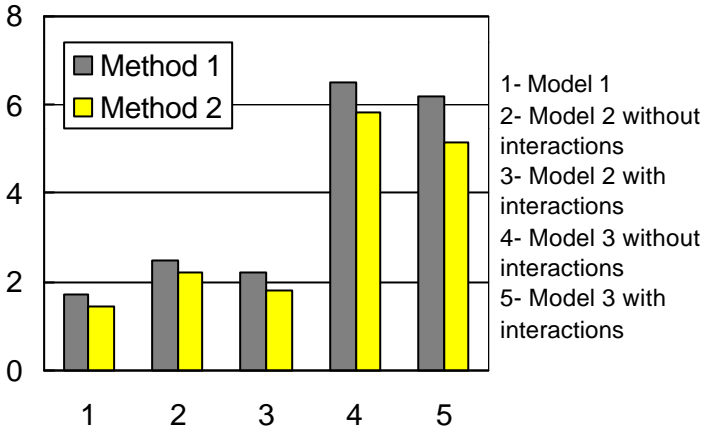


**Figure 8 :** Static Transmission Error computed with method 1 (■) and method 2 (o).

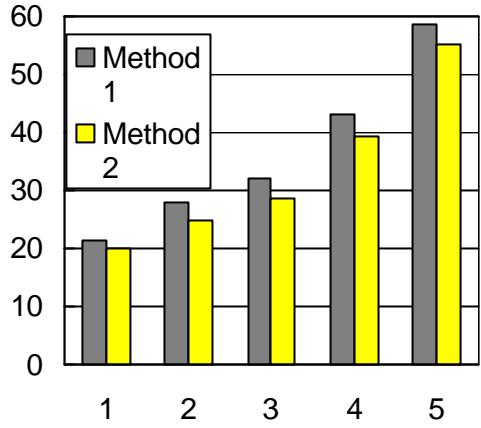
The main differences between results obtained with methods 1 and 2 concern the modified gear pair. As illustrated in figure 10, evolutions of peak to peak static transmission error with the input torque are not exactly the same. Nevertheless, both methods show that the tooth modifications allow to obtain moderate static transmission error variations for a large range of torques (from 250 Nm to 1300 Nm).



**STE peak to peak ( $\mu\text{m}$ )**

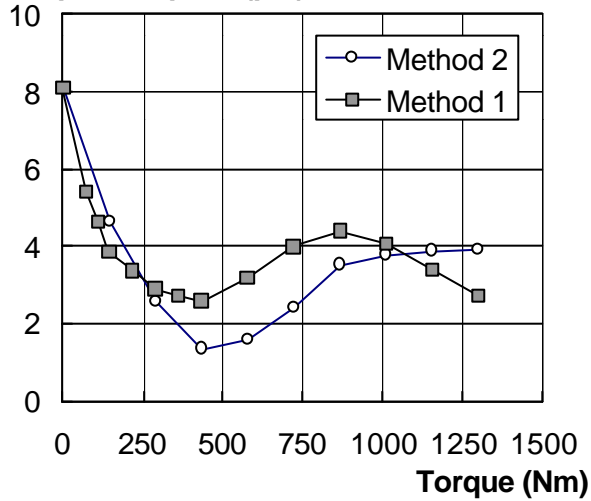


**STE Mean Value ( $\mu\text{m}$ )**

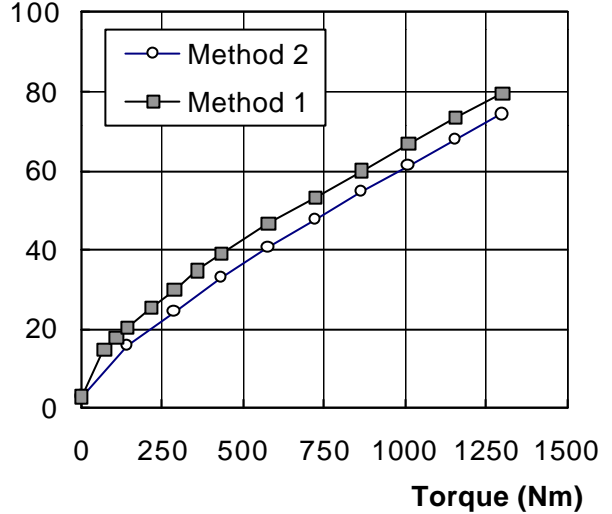


**Figure 9 :** Comparison between method 1 and method 2 for unmodified gear pair (peak to peak value and mean value). Torque=1300 Nm.

**STE peak to peak ( $\mu\text{m}$ )**



**STE Mean Value ( $\mu\text{m}$ )**



**Figure 10 :** Evolution of static transmission error of the modified gear pair with the input torque (peak to peak value and mean value).

## CONCLUSION

Two numerical methods using 3D finite element modelling of toothed wheels have been developed to analyse the main static transmission error characteristics, for spur and helical gear pairs. Numerous simulations allowed to validate both methods and showed that a correct prediction of transmission error (both mean value and peak to peak value) needed an accurate modelling of the whole toothed wheels. Elasticity of wheel bodies modifies the contact between loaded tooth pairs and the static transmission error variations. Furthermore, interactions between adjacent loaded teeth must be taken into account. Simulations showed that these conclusions were valid especially for gear pair fitted out with thin-rimmed wheel bodies. Finally, all physical phenomena contributing to bending deformation of loaded teeth must be well integrated.

Developed numerical methods allow to optimize the static transmission error characteristics, for a given torque or for a range of torques, by introducing the suitable tooth modifications. Numerous gear body shapes and micro-geometries can be compared in order to minimize static transmission error variations. This technic offers interesting possibilities at the first steps of the development of a transmission system and can also successfully be used to improve existing components.

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