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**EFFECTS OF GEARBOX DESIGN PARAMETERS  
ON THE VIBRATORY RESPONSE OF ITS HOUSING**

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## INTRODUCTION

Noise radiated by a gearbox is directly related to the vibratory level of its housing. The aim of this study is to analyse the transfer mechanisms between the static transmission error of a gear pair and the dynamic responses of gear and housing of a gearbox. For this purpose, a generic single-stage gearbox is considered. An integrated model has been built, including gear, shafts, bearings and housing in order to compute its dynamic response. The methodology developed and the numerical results allow to compare several bearing types and housing designs.

## I- MODEL OF THE GEARBOX

In order to analyse the main dynamic phenomena, a generic gearbox is studied. It is fitted out with a 49/49 helical gear pair and a (450x280x160 mm) steel housing which is 10 mm thick. Helical gear, shafts, and housing are discretised using finite element method. Toothed wheels are coupled by a 12x12 stiffness matrix defined from the geometrical characteristics of the gear pair and from the mean value of mesh stiffness ( $4.10^8$  N/m). Each tapered-roller element bearing is modelled by a 10x10 stiffness matrix (Radial stiffnesses  $K_x$  and  $K_y$  are equal to  $10^9$  N/m, axial stiffnesses  $K_z$  are equal to  $10^8$  N/m, angular stiffnesses  $K_{\theta x}$  and  $K_{\theta y}$  are equal to  $10^6$  Nm/rad). The elastic model of the whole gearbox (figure 1) has 1700 elements, 2100 nodes and 11000 degrees of freedom.

Static transmission error is supposed to be the only excitation. The matrix equation which governs forced vibrations of the discretized gearbox can be written as follows :

$$\underline{\underline{M}} \cdot \underline{\underline{\ddot{X}}} + \underline{\underline{C}} \cdot \underline{\underline{\dot{X}}} + \underline{\underline{K}} \cdot \underline{\underline{X}} + \underline{k(t)} \cdot \underline{\underline{D}} \cdot \underline{\underline{X}} = \underline{\underline{E}}(t)$$

M and K are the classical mass and stiffness matrix provided by the finite element method; Matrix D is derived from geometric characteristics of the gear pair;  $k(t)$  is the periodic mesh stiffness and E(t) is an equivalent force vector induced by the static transmission error. A modal analysis of the gearbox can be done from the time-invariant homogeneous counterpart equation. Damping is introduced using a 3 % equivalent viscous damping rate for each mode. Forced responses of gear, shafts and housing are computed simultaneously using a spectral and iterative method [1]. This one provides a direct spectral description of the vibratory response at each degree of freedom.

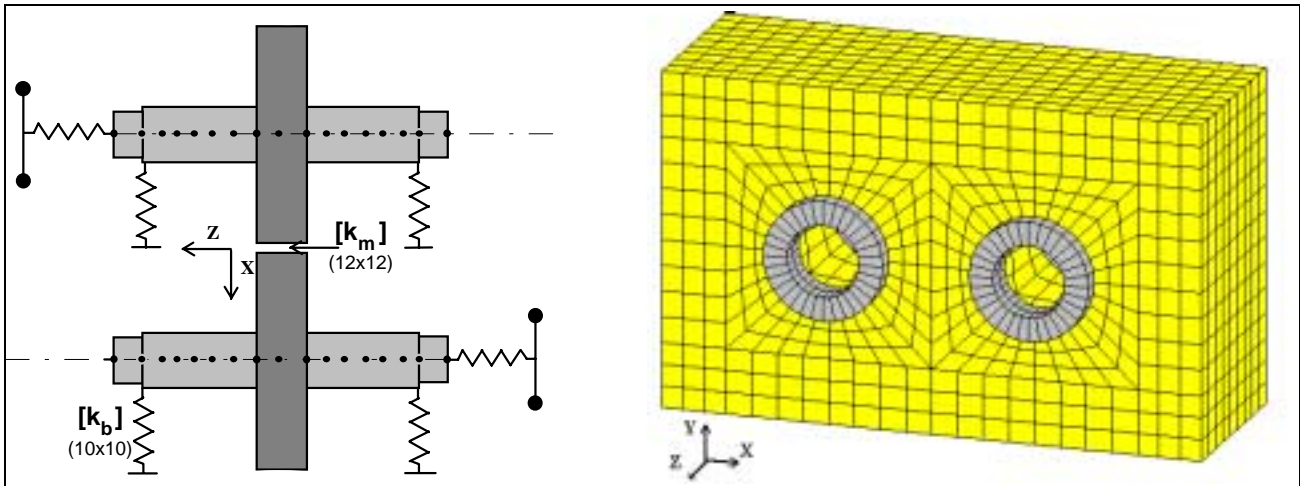


Figure 1 : Model of the gearbox.

## II- MESHING MODES $\Phi_m$ AND BEARING MODES $\Phi_b$

In order to interpret the vibratory response of the gearbox, it is necessary to analyse its natural modes and to identify some particular modes called "meshing" modes ( $\Phi_m$ ) and "bearing" modes ( $\Phi_b$ ). For this, an energy approach is used. For the j-th mode, the local potential energy  $U_m$  associated with the mesh stiffness  $k_m$ , the total potential energy  $U_T$  and the energy rate  $\rho_m$  are defined by :

$$(1) U_m^{(j)} = \frac{1}{2} ({}^t\phi^{(j)}) [k_m] \phi^{(j)} \quad (2) U_T^{(j)} = \frac{1}{2} ({}^t\phi^{(j)}) [K_T] \phi^{(j)} \quad (3) \rho_m^{(j)} = \frac{U_m^{(j)}}{U_T^{(j)}}$$

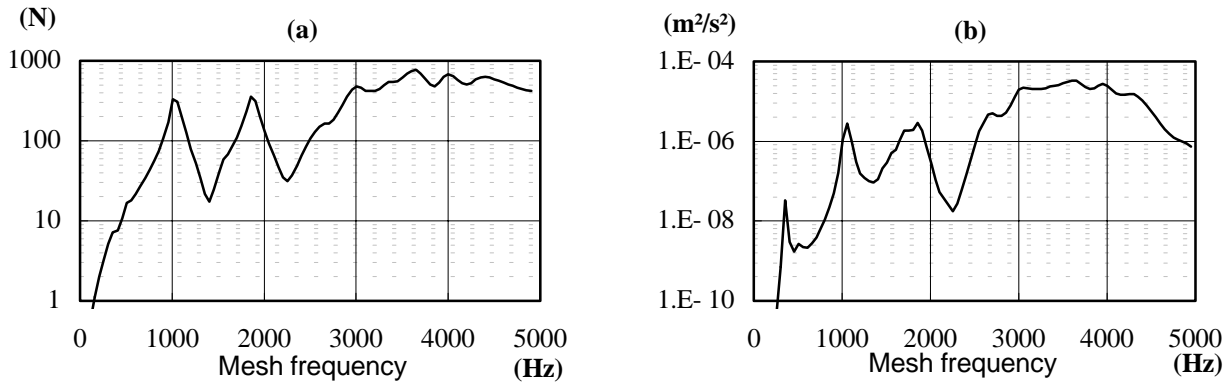
where  $[k_m]$  is the local mesh stiffness matrix and  $[K_T]$  is the global stiffness matrix.

The modes which have the highest  $\rho_m$  energy rates are called the meshing modes ( $\Phi_m$ ).

For each bearing, the  $\rho_b$  energy rates associated with the bearing stiffness matrix can similarly be defined. The modes which have the highest  $\rho_b$  energy rates are called the bearing modes ( $\Phi_b$ ).

### III- DYNAMIC MESH FORCE

Figure 2 displays the root mean square value of the dynamic mesh force against mesh frequency. The critical rotational speeds which correspond to the highest values of dynamic mesh force are associated with resonant excitation of the meshing modes  $\Phi_m$  [2]. For each mode excited by static transmission error, the higher the value of  $\rho_m$ , the higher the dynamic mesh force. We have showed that bending along the shafts, elasticity of bearings and mechanical properties of housing governed critical speeds and dynamic mesh force [2].



**Figure 2** : Root mean square value of the dynamic mesh force **(a)** and time- and space-averaged mean square vibratory velocity of housing **(b)**.

### IV- TIME- AND SPACE-AVERAGED MEAN SQUARE VELOCITY OF THE HOUSING

The noise radiated by the gearbox is :

$$W_{ac}(\omega) = \rho_0 \cdot c_0 \cdot S \cdot \sigma_{rad}(\omega) \cdot \langle v^2(\omega) \rangle_s$$

with :

$$\langle v^2(\omega) \rangle_s = \frac{1}{S} \int_S \left( \frac{1}{T} \int_T v^2(M, \omega, t) dt \right) dS$$

$\langle v^2(\omega) \rangle_s$  is the time- and space-averaged mean square vibratory velocity of the housing;  $\rho_0$  is the air density;  $c_0$  is the sound celerity;  $\sigma_{rad}(\omega)$  is the radiation efficiency. Above a critical frequency, the radiation efficiency is almost constant. Therefore, the noise radiated by the gearbox is proportional to the mean square vibratory velocity of the housing.

As illustrated in figure 2, the evolutions of the vibratory response of housing and of the dynamic mesh force are almost the same : the mean square vibratory velocity of the housing presents resonances (at 1050 Hz and 1900 Hz) which correspond to the excitation of some meshing modes ( $\Phi_m$ ). Furthermore, the rotational speeds leading to the maximum mean square vibratory velocity of the housing (mesh frequencies contained between 3000 and 5000 Hz) are identical to the ones corresponding to the highest dynamic mesh force.

### V- FORCES TRANSMITTED TO THE HOUSING THROUGH THE BEARINGS

In order to analyse the transfer mechanisms between dynamic mesh force and mean square vibratory velocity of the housing, the generalised forces (radial, axial and angular) transmitted to the housing through the bearings have been estimated. The maximum levels of these forces correspond to the excitation of modes which are both meshing mode ( $\Phi_m$ ) and bearing mode ( $\Phi_b$ ).

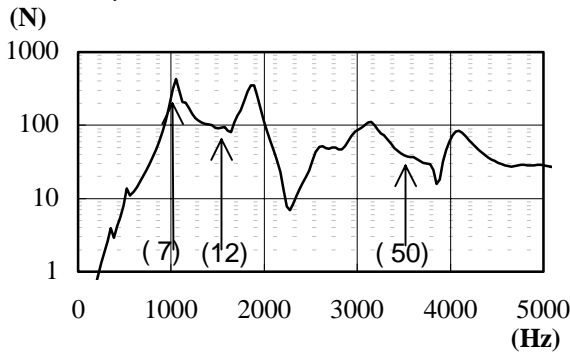
The energy rates associated with the mesh stiffness ( $\rho_m$ ) and with the radial stiffness  $k_y$  of one of the four bearings ( $\rho_y$ ) illustrate this result. Four situations are possible (table 1) :

- the mode is a meshing mode and a bearing mode (example : mode 7),
- the mode is a bearing mode but it is not a meshing mode (example : mode 12),
- the mode is a meshing mode but it is not a bearing mode (example : mode 50),
- the mode is neither a meshing mode, nor a bearing mode.

Figure 3 displays the radial force  $F_y$  transmitted to the housing through the bearing. Only the excitation of the mode (7) leads to a high radial force  $F_y$ . On the contrary, excitation of mode (12) and mode (50) does not correspond to a high radial force. This result can be generalized to the

whole forces transmitted to the housing through the bearings. The higher  $\sqrt{(\rho_m \cdot \rho_b)}$  is, the higher the transmitted forces.

For the gearbox studied, vibratory response of the housing is mainly induced by axial forces  $F_z$  transmitted through the bearings between 0 and 1000 Hz, by radial forces  $F_x$  and  $F_y$  and moments  $M_x$  and  $M_y$  between 1000 and 2000 Hz and by moments beyond 2000 Hz.



Frequency	$\rho_e$	$\rho_y$	$\sqrt{(\rho_e \rho_y)}$
Mode 7: 1050 Hz	9.6 %	4.0 %	6.2 %
Mode 12: 1420 Hz	0 %	16.4 %	0 %
Mode 50: 3440 Hz	11.3 %	0 %	0.2 %

Table 1.

Figure 3 : Radial force  $F_y$  transmitted to the housing through one bearing.

## VI- EFFECT OF BEARINGS STIFFNESSES ON THE VIBRATORY RESPONSE OF THE HOUSING

In order to analyse the effect of bearings on the vibratory response of the housing, their angular stiffnesses have been modified ( $10^5$  Nm/rad instead of  $10^6$  Nm/rad). As illustrated in figure 4, the meshing modes ( $\Phi_m$ ) have changed between 3000 and 4000 Hz and the maximum dynamic mesh force has increased. It confirms the effect of bearing elasticity on the critical rotational speeds [2]. The modification of bearings leads to a diminution of the moments  $M_x$  and  $M_y$  transmitted to the housing through the bearings. Therefore, in spite of the increasing of the dynamic mesh force, it leads to a diminution of the mean square vibratory velocity of the housing between 3000 and 5000 Hz. This frequency range corresponds to the maximum levels of the vibratory response of housing. It also corresponds to the frequency range where the vibratory response of housing was mainly induced by the moments  $M_x$  and  $M_y$  transmitted through the bearings.

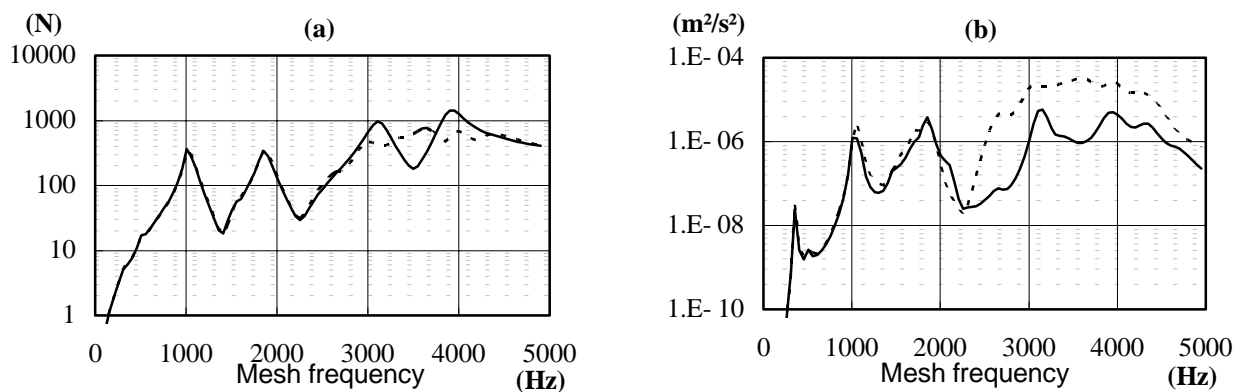


Figure 4 : Root mean square value of the dynamic mesh force (a) and time- and space-averaged mean square vibratory velocity of housing (b).

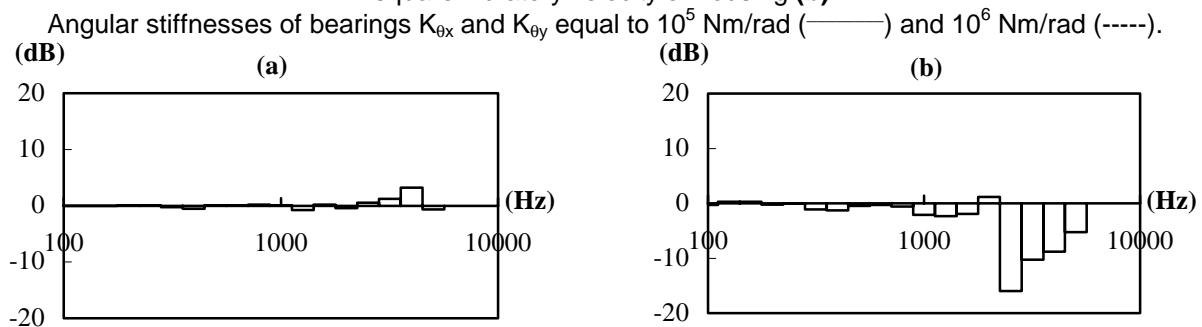


Figure 5 : Evolution of  $L_{Fz}(\omega)$  (a) and  $L_{vz}(\omega)$  (b).

The effect of bearings modification can be evaluated in decibels using the following factors :  
 $L_{v^2}(\omega) = 10 \cdot \log [ \langle \overline{v^2(\omega)} \rangle / \langle \overline{v^2(\omega)} \rangle_0 ]$  (in dB) et  $L_{F^2}(\omega) = 10 \cdot \log [ F^2(\omega) / F_0^2(\omega) ]$  (in dB).

$\langle \overline{v^2(\omega)} \rangle_0$  and  $F_0^2(\omega)$  correspond to the reference configuration ( $10^6$  Nm/rad).  $\langle \overline{v^2(\omega)} \rangle$  and  $F^2(\omega)$  correspond to the new configuration ( $10^5$  Nm/rad). As illustrated in figure 5, the modification of bearings leads to a diminution of the vibratory response of housing equal to nearly 10 dB between 1000 and 5000 Hz.

## VII- EFFECT OF HOUSING DESIGN ON ITS VIBRATORY RESPONSE

Several designs of the housing have been compared, in order to analyse the noise radiated by the gearbox. Thickness has been modified to keep the whole mass of gearbox constant. We have noted that the rounding of gearbox edges or the increasing of bearings mass have no notable effects. However, a reinforcement rib surrounding the housing in the plane containing bearings (figure 6) modifies the gearbox dynamics. First, it changes the frequencies of some meshing modes  $\Phi_m$  (1590 Hz instead of 1040 Hz). This confirms the effect of mechanical properties of housing on critical rotational speeds. Then, adding the rib induces a decreasing of the mean square vibratory velocity of the housing in a large frequency range (-20 dB between 2500 and 5000 Hz) (see figure 7). This diminution adds to the one obtained by modifying the bearings.

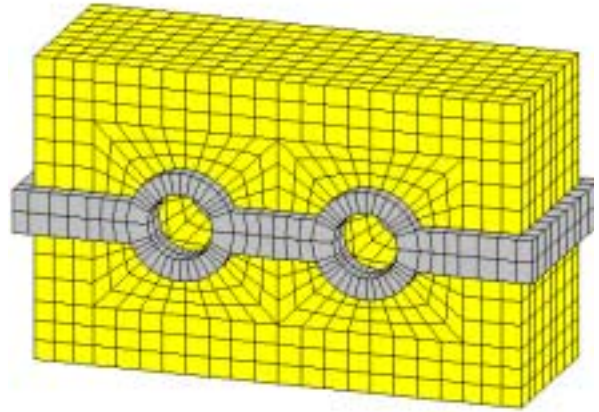


Figure 6 : New design of the housing.

Then, adding the rib induces a decreasing of the mean square vibratory velocity of the housing in a large frequency range (-20 dB between 2500 and 5000 Hz) (see figure 7). This diminution adds to the one obtained by modifying the bearings.

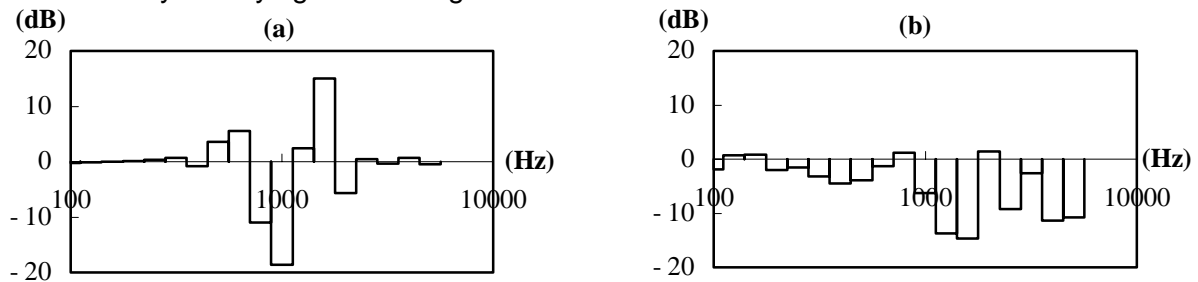


Figure 7 : Evolution of  $L_{F^2}(\omega)$  (a) and  $L_{v^2}(\omega)$  (b).

## CONCLUSION

An integrated model including the whole components of a gearbox has been developed, in order to analyse transfer mechanisms between static transmission error, dynamic mesh force, generalised forces transmitted to the housing through the bearings and mean square vibratory velocity of housing. Resonances of the vibratory response of housing are induced by high forces transmitted through the bearings. These ones correspond to the excitation by static transmission error of both the meshing modes and the bearing modes. Solutions to reduce noise radiated by gearboxes depend on the type of force transmitted to the housing (radial forces, axial forces or angular forces). Finally, we have shown that modifying the bearings type or the housing design can lead to a significant reduction of the vibratory response of housing of gearboxes.

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