Static Output-Feedback H_infinity Control of a Class of Stochastic Hybrid Systems with Wiener Process.

Samir Aberkane, Jean-Christophe Ponsart, Dominique Sauter

Static Output-Feedback $\mathscr{H}_\infty$ Control of a Class of Stochastic Hybrid Systems with Wiener Process

Samir Aberkane, Jean Christophe Ponsart and Dominique Sauter

***Abstract*— This paper deals with static output feedback $\mathscr{H}_\infty$ control of continuous time Active Fault Tolerant Control Systems with Markovian Parameters (AFTCSMP) and state-dependent noise. It adopts a new framework, based on the synthesis of ellipsoidal sets of controllers, introduced in [18], [19]. It is also shown that the obtained results can easily be applied to the problematic of mode-independent static output feedback $\mathscr{H}_\infty$ control of another class of stochastic hybrid systems known as Markovian Jump Linear Systems. Results are formulated as matrix inequalities one of which is nonlinear. A numerical algorithm based on nonconvex optimization is provided and its running is illustrated on classical examples from literature.**

I. INTRODUCTION

As performance requirements increase in advanced technological systems, their associated control systems are becoming more and more complex. At the same time, complicated systems could have various consequences in the event of component failures. Therefore, it is very important to consider the safety and fault tolerance of such systems at the design stage. For these safety-critical systems, Fault Tolerant Control Systems (FTCS) have been developed to meet these essential objectives. FTCS have been a subject of great practical importance, which has attracted a lot of interest for the last three decades. A bibliographical review on reconfigurable fault tolerant control systems can be found in [25].

Active fault tolerant control systems are feedback control systems that reconfigure the control law in real time based on the response from an automatic fault detection and identification (FDI) scheme. The dynamic behaviour of Active Fault Tolerant Control Systems (AFTCS) is governed by stochastic differential equations and can be viewed as a general hybrid system [23]. A major class of hybrid systems is Markovian Jump Linear Systems (MJLS). In MJLS, a single jump process is used to describe the random variations affecting the system parameters. This process is represented by a finite state Markov chain and is called the plant regime mode. The theory of stability, optimal control and $\mathscr{H}_2/\mathscr{H}_\infty$ control, as well as important applications of such systems, can be found in several papers in the current literature, for instance in [5], [6], [7], [9], [10], [11], [13], [14].

To deal with AFTCS, another class of hybrid systems was defined, denoted as AFTCSMP. In this class of hybrid systems, two random processes are defined: the first random process represents system components failures and the second random process represents the FDI process used to reconfigure the control law. This model was proposed by Srichander and Walker [23]. Necessary and sufficient conditions for stochastic stability of AFTCSMP were developed for a single component failure (actuator failures). The problem of stochastic stability of AFTCSMP in the presence of noise, parameter uncertainties, detection errors, detection delays and actuator saturation limits has also been investigated in [15], [16]. Another issue related to the synthesis of fault tolerant control laws was also addressed by [20], [21]. The problem of $\mathscr{H}_\infty$ and robust $\mathscr{H}_\infty$ control was treated in [20], [21] for both continuous and discret time AFTCSMP. The authors showed that the state feedback control problem can be solved in terms of the solutions of a set of coupled Riccati inequalities. The dynamic/static output feedback counterpart was treated by [1], [2], [3] in a convex programming framework. Indeed, the authors provide an LMI characterization of dynamical/static output feedback compensators that stochastically stabilize (robustly stabilize) the AFTCSMP and ensures $\mathscr{H}_\infty$ (robust $\mathscr{H}_\infty$) constraints. In addition, it is important to mention that the design problem in the framework of AFTCSMP remains an open and challenging problematic. This is due, particulary, to the fact that the controller only depends on the FDI process. Generally speaking, there lacks tractable design methods for this stochastic FTC problem. Indeed, in [1], [2], [16], [20], [21], the authors make the assumption that the controller must access both failures and FDI processes. However, this assumption is too restrictive to be applicable in practical FTC systems. In this note, the assumption on the availability of failure processes, for the synthesis purposes, is stressed.

On the other hand, one of the most challenging open problems in control theory is the synthesis of fixed-order or static output feedback controllers that meet desired performances and specifications [24]. Among all variations of this problem, this note is concerned with the problem of static output feedback $\mathscr{H}_\infty$ control of continuous time AFTCSMP with state-dependent noise. This problematic is addressed under a new framework, based on the synthesis of ellipsoidal sets of controllers, introduced in [18], [19]. The problematic resulting from the fact that the controller only depends on the FDI process is shown to be naturally dealt with in this context. It is also shown that the obtained results can easily be applied to the problematic of mode-independent static output feedback $\mathscr{H}_\infty$ control of MJLS. Results are formulated as matrix inequalities one of which is nonlinear. A numerical algorithm based on nonconvex optimization is provided and

The authors are with Université Henri Poincaré, Nancy 1, CRAN – CNRS UMR 7039, BP 239, F-54506 Vandœuvre-lès-Nancy Cedex samir.aberkane@cran.uhp-nancy.fr

its running is illustrated on classical examples from literature. This paper is organized as follows: Section 2 describes the dynamical model of the system with appropriately defined random processes. A brief summary of basic stochastic terms, results and definitions are given in Section 3. Section 4 addresses the internal stochastic stabilization of the AFTCSMP. Sections 5 considers the $\mathscr{H}_\infty$ control problem for the output feedback. In Section 6, a numerical algorithm based on nonconvex optimization is provided and its running is illustrated on classical examples from literature. Finally, a conclusion is given in Section 7.

Notations. The notations in this paper are quite standard. $\mathbb{R}^{m\times n}$ is the set of m-by-n real matrices and $\mathbb{S}^n$ is the subset of symmetric matrices in $\mathbb{R}^{n\times n}$. A' is the transpose of the matrix A. The notation $X \geq Y$ ($X > Y$, respectively), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (positive definite, respectively); $\mathbb{I}$ and $\mathbf{0}$ are identity and zero matrices of appropriate dimensions, respectively; $\mathscr{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure P; $L^2[0,\infty)$ stands for the space of square-integrable vector functions over the interval $[0,\infty)$; $\|\cdot\|$ refers to either the Euclidean vector norm or the matrix norm, which is the operator norm induced by the standard vector norm; $\|\cdot\|_2$ stands for the norm in $L^2[0,\infty)$; while $\|\cdot\|_{\mathscr{E}_2}$ denotes the norm in $L^2((\Omega,\mathscr{F},P),[0,\infty))$; $(\Omega,\mathscr{F},P)$ is a probability space.

II. Dynamical Model of the AFTCSMP with Wiener Process

The dynamical model of the AFTCSMP with Wiener Process, defined in the fundamental probability space $(\Omega,\mathscr{F},P)$, is described by the following differential equations:

$$\varphi:\begin{cases} dx(t)=A(\xi(t))x(t)dt+B(\eta(t))u(y(t),\psi(t),t)dt \\ +E(\xi(t),\eta(t))w(t)dt+\sum_{l=1}^{v}\mathbb{W}_l(\xi(t),\eta(t))x(t)d\varpi_l(t) \\ y(t)=C_2x(t)+D_2(\xi(t),\eta(t))w(t) \\ z(t)=C_1x(t)+D_1(\eta(t))u(y(t),\psi(t),t) \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $u(y(t),\psi(t),t)\in\mathbb{R}^r$ is the system input, $y(t)\in\mathbb{R}^q$ is the system measured output, $z(t)\in\mathbb{R}^p$ is the controlled output, $w(t)\in\mathbb{R}^m$ is the system external disturbance, $\xi(t)$, $\eta(t)$ and $\psi(t)$ represent the plant component failure process, the actuator failure process and the FDI process, respectively. $\xi(t)$, $\eta(t)$ and $\psi(t)$ are separable and mesurable Markov processes with finite state spaces $Z=\{1,2,...,z\}$, $S=\{1,2,...,s\}$ and $R=\{1,2,...,r\}$, respectively. $\varpi(t)=[\varpi_1(t)\dots\varpi_v(t)]'$ is a v-dimensional standard Wiener process on a given probability space $(\Omega,\mathscr{F},P)$, that is assumed to be independent of the Markov processes. The matrices $A(\xi(t))$, $B(\eta(t))$, $E(\xi(t),\eta(t))$, $D_2(\xi(t),\eta(t))$, $D_1(\eta(t))$ and $\mathbb{W}_l(\xi(t),\eta(t))$ are properly dimensioned matrices which depend on random parameters.

In AFTCS, we consider that the control law is only a function of the mesurable FDI process $\psi(t)$. Therefore, we introduce a static output feedback compensator (φ_s) of the form:

$$\varphi_s:\Big\{u(t)=\mathscr{K}(\psi(t))y(t) \tag{2}$$

Applying the controller φ_s to the AFTCSMP φ, we obtain the following closed loop system:

$$\varphi_{cl}:\begin{cases} dx(t)=\bar{A}(\xi(t),\eta(t),\psi(t))x(t)dt+\bar{E}(\xi(t),\eta(t),\psi(t))w(t)dt \\ +\sum_{l=1}^{v}\mathbb{W}_l(\xi(t),\eta(t))x(t)d\varpi_l(t) \\ y(t)=C_2x(t)+D_2(\xi(t),\eta(t))w(t) \\ z(t)=\bar{C}_1(\eta(t),\psi(t))x(t)+\bar{D}_1(\xi(t),\eta(t),\psi(t))w(t) \end{cases} \tag{3}$$

where

$$\begin{bmatrix} \bar{A}(\xi(t),\eta(t),\psi(t)) & \bar{E}(\xi(t),\eta(t),\psi(t)) \\ C_1(\eta(t),\psi(t)) & \bar{D}_1(\xi(t),\eta(t),\psi(t)) \end{bmatrix} = \begin{bmatrix} A(\xi(t)) & E(\xi(t),\eta(t)) \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} B(\eta(t)) \\ D_1(\eta(t)) \end{bmatrix}\mathscr{K}(\psi(t))\begin{bmatrix} C_2 & D_2(\xi(t),\eta(t)) \end{bmatrix}$$

A. The FDI and the Failure Processes

$\xi(t)$, $\eta(t)$ and $\psi(t)$ being homogeneous Markov processes with finite state spaces, we can define the transition probability of the plant components failure process as [16], [23]:

$$\begin{cases} p_{ij}(\Delta t)=\pi_{ij}\Delta t+o(\Delta t) & (i\neq j) \\ p_{ii}(\Delta t)=1-\sum\limits_{i\neq j}\pi_{ij}\Delta t+o(\Delta t) & (i=j) \end{cases}$$

The transition probability of the actuator failure process is given by:

$$\begin{cases} p_{kl}(\Delta t)=\nu_{kl}\Delta t+o(\Delta t) & (k\neq l) \\ p_{kk}(\Delta t)=1-\sum\limits_{k\neq l}\nu_{kl}\Delta t+o(\Delta t) & (k=l) \end{cases}$$

where π_{ij} is the plant components failure rate, and ν_{kl} is the actuator failure rate.

Given that $\xi=k$ and $\eta=l$, the conditional transition probability of the FDI process $\psi(t)$ is:

$$\begin{cases} p_{iv}^{kl}(\Delta t)=\lambda_{iv}^{kl}\Delta t+o(\Delta t) & (i\neq v) \\ p_{ii}^{kl}(\Delta t)=1-\sum\limits_{i\neq v}\lambda_{iv}^{kl}\Delta t+o(\Delta t) & (i=v) \end{cases}$$

Here, λ_{iv}^{kl} represents the transition rate from i to v for the Markov process $\psi(t)$ conditioned on $\xi=k\in Z$ and $\eta=l\in S$. For notational simplicity, we will denote $\bullet(\xi(t))=\bullet_i$ when $\xi(t)=i\in Z$, $\bullet(\eta(t))=\bullet_j$ when $\eta(t)=j\in S$, $\bullet(\xi(t),\eta(t))=\bullet_{ij}$, when $\xi(t)=i\in Z,\eta(t)=j\in S$ and $\bullet(\psi(t))=\bullet_k$ when $\psi(t)=k\in R$. We also denote $\bullet(t)=\bullet_t$ and the initial conditions $\bullet(t_0)=\bullet_0$.

III. Definitions

Without loss of generality, we assume that the equilibrium point, $x=0$, is the solution at which stability properties are examined. Under the assumption that the system (φ) coupled with (φ_s) satisfies the global Lispchitz condition, the solution x_t determines a family of unique continuous stochastic processes, one for each choice of the random variable x_0. The joint process $\{x_t,\xi_t,\eta_t,\psi_t\}$ is a Markov process.

A. Stochastic Stability

System (3) is said to be

(i) stochastically stable (SS) if there exists a finite positive constant $K(x_0,\xi_0,\eta_0,\psi_0)$ such that the following holds for any initial conditions $(x_0,\xi_0,\eta_0,\psi_0)$:

$$\mathscr{E}\left\{\int_0^\infty \| x_t \|^2 \, dt\right\} \le K(x_0,\xi_0,\eta_0,\psi_0) \tag{4}$$

(ii) internally exponentially stable in the mean square sense (IESS) if it is exponentially stable in the mean square sense for $w_t = 0$, i.e. for any ξ_0,η_0,ψ_0 and some $\gamma(\xi_0,\eta_0,\psi_0)$, there exists two numbers $a > 0$ and $b > 0$ such that when $\|x_0\| \le \gamma(\xi_0,\eta_0,\psi_0)$, the following inequality holds $\forall t \ge t_0$ for all solution of (3) with initial condition x_0:

$$\mathscr{E}\{\|x_t\|^2\} \le b\|x_0\|^2 \exp[-a(t-t_0)] \tag{5}$$

The following theorem gives a sufficient condition for internal exponential stability in the mean square sense for the system (φ) coupled with (φ_s).

Theorem 1: *The solution $x = 0$ of the system (φ) coupled with (φ_s) is internally exponentially stable in the mean square for $t \ge t_0$ if there exists a Lyapunov function $\vartheta(x_t,\xi_t,\eta_t,\psi_t,t)$ such that*

$$K_1\|x_t\|^2 \le \vartheta(x_t,\xi_t,\eta_t,\psi_t,t) \le K_2\|x_t\|^2 \tag{6}$$

and

$$\mathscr{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t,t) \le -K_3\|x_t\|^2 \tag{7}$$

for some positive constants K_1, K_2 and K_3, where $\mathscr{L}$ is the weak infinitesimal operator of the joint Markov process $\{x_t,\xi_t,\eta_t,\psi_t\}$.

A necessary condition for internal exponential stability in the mean square for the system (φ) coupled with (φ_s) is given by theorem 2.

Theorem 2: *If the solution $x = 0$ of the system (φ) coupled with (φ_s) is internally exponentially stable in the mean square, then for any given quadratic positive definite function $W(x_t,\xi_t,\eta_t,\psi_t,t)$ in the variables x which is bounded and continuous $\forall t \ge t_0$, $\forall \xi_t \in Z$, $\forall \eta_t \in S$ and $\forall \psi_t \in R$, there exists a quadratic positive definite function $\vartheta(x_t,\xi_t,\eta_t,\psi_t,t)$ in x such that $\mathscr{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t,t) = -W(x_t,\xi_t,\eta_t,\psi_t,t)$.*

Remark 1: The proofs of these theorems follow the same arguments as in [16], [23] for their proposed stochastic Lyapunov functions, so they are not shown in this paper to avoid repetition.

Proposition 1: A necessary and sufficient condition for internal exponential stability in the mean square of the system (3) is that there exist symmetric positive-definite matrices $\mathscr{P}_{ijk}$, $i \in Z$, $j \in S$ and $k \in R$ such that:

$$\tilde{A}'_{ijk}\mathscr{P}_{ijk} + \mathscr{P}_{ijk}\tilde{A}_{ijk} + \sum_{l=1}^{v} \mathbb{W}'_{lij}\mathscr{P}_{ijk}\mathbb{W}_{lij} + \sum_{h\in Z}\pi_{ih}\mathscr{P}_{hjk} + \sum_{l\in S}\nu_{jl}\mathscr{P}_{ilk} + \sum_{v\in R}\lambda_{kv}^{ij}\mathscr{P}_{ijv} = \mathfrak{I}_{ijk} < 0 \tag{8}$$

$\forall i \in Z$, $j \in S$ and $k \in R$, where

$$\tilde{A}_{ijk} = A_i + B_j\mathscr{K}_k C_2 \tag{9}$$

Proof: The proof of this proposition is easily deduced from theorems 1 and 2.

Proposition 2: If the system (3) is internally exponentially stable in the mean square sense, then it is stochastically stable.

Proof: The proof of this proposition follows the same lines as for the proof of proposition 4 in [3].

B. Matrix Ellipsoids

Through this note, a particular set of matrices is used. Due to the notations and by extension of the notion of $\mathbb{R}^n$ ellipsoids, these sets are referred to as matrix ellipsoids of $\mathbb{R}^{(m\times p)}$.

Definition 1 [18], [19]: Given three matrices $\mathbb{X} \in \mathbb{S}^q$, $\mathbb{Y} \in \mathbb{R}^{q\times r}$ and $\mathbb{Z} \in \mathbb{S}^r$, the $\{\mathbb{X},\mathbb{Y},\mathbb{Z}\}$-ellipsoid of $\mathbb{R}^{r\times q}$ is the set of matrices $\mathscr{K}$ satisfying the following matrix inequalities:

$$\mathbb{Z} > 0 \quad \begin{bmatrix} \mathbb{I} & \mathscr{K}' \end{bmatrix} \begin{bmatrix} \mathbb{X} & \mathbb{Y} \\ \star & \mathbb{Z} \end{bmatrix} \begin{bmatrix} \mathbb{I} \\ \mathscr{K} \end{bmatrix} \le 0 \tag{10}$$

By definition, $\mathscr{K}_0 = -\mathbb{Z}^{-1}\mathbb{Y}'$ is the center of the ellipsoid and $R = \mathscr{K}_0'\mathbb{Z}\mathscr{K}_0 - \mathbb{X}$ is the radius. Inequalities (8) can also be written as

$$\mathbb{Z} > 0 \quad (\mathscr{K} - \mathscr{K}_0)'\mathbb{Z}(\mathscr{K} - \mathscr{K}_0) \le R \tag{11}$$

This definition shows that matrix ellipsoids are special cases of matrix sets defined by quadratic matrix inequality. Some properties of these sets are

i) A matrix ellipsoid is a convex set;
ii) the $\{\mathbb{X},\mathbb{Y},\mathbb{Z}\}$-ellipsoid is nonempty iff the radius ($R \ge 0$) is positive semi definite. This property can also be expressed as

$$\mathbb{X} \le \mathbb{Y}\mathbb{Z}^{-1}\mathbb{Y}' \tag{12}$$

IV. Stochastic Stabilization

In this section, we shall address the problem of finding all static compensators (φ_s), as defined in section 2, such that the system (φ) coupled with (φ_s) becomes internally exponentially stochastically stable in the mean square. To this end, we use proposition 1 to get the following necessary and sufficient conditions for the internal exponential stability in the mean square of the system (3).

Proposition 3: System (3) is internally exponentially stabilisable in the mean square by static output-feedback if and only if there exist matrices $\mathscr{P}_{ijk} = \mathscr{P}'_{ijk} > 0$, $\mathbb{X}_k \in \mathbb{S}^q$, $\mathbb{Y}_k \in \mathbb{R}^{q\times r}$ and $\mathbb{Z}_k \in \mathbb{S}^r$ that simultaneously satisfy the following LMI constraints

$$\mathbb{Z}_k > 0 \quad \mathscr{P}_{ijk} > 0 \tag{13}$$

$$\begin{bmatrix} \mathbb{I} & \mathbf{0} \\ A_i & B_j \end{bmatrix}' \begin{bmatrix} \Theta_{ijk} & \mathscr{P}_{ijk} \\ \mathscr{P}_{ijk} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbb{I} & \mathbf{0} \\ A_i & B_j \end{bmatrix} < \begin{bmatrix} C_2 & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{bmatrix}' \begin{bmatrix} \mathbb{X}_k & \mathbb{Y}_k \\ \star & \mathbb{Z}_k \end{bmatrix} \begin{bmatrix} C_2 & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{bmatrix} \tag{14}$$

and the nonlinear inequalities constraints

$$\mathbb{X}_k \le \mathbb{Y}_k\mathbb{Z}_k^{-1}\mathbb{Y}'_k \tag{15}$$

$\forall i \in Z$, $j \in S$ and $k \in R$, where

$$\Theta_{ijk} = \sum_{l=1}^{v} \mathbb{W}'_{lij}\mathscr{P}_{ijk}\mathbb{W}_{lij} + \sum_{h\in Z}\pi_{ih}\mathscr{P}_{hjk} + \sum_{l\in S}\nu_{jl}\mathscr{P}_{ilk} + \sum_{v\in R}\lambda_{kv}^{ij}\mathscr{P}_{ijv} \tag{16}$$

Let $\{\mathscr{P}_{ijk},\mathbb{X}_k,\mathbb{Y}_k,\mathbb{Z}_k\}$ be a solution, then the nonempty $\{\mathbb{X}_k,\mathbb{Y}_k,\mathbb{Z}_k\}$-ellipsoids are sets of stabilizing gains.

Proof: The proof of this proposition is given in [4].

Remark 2: The results developed above can be easily applied to the *mode-independent* static output feedback stochastic stabilization of MJLS. Indeed, let us consider the following closed loop dynamical model

$$\varphi_{cl}:\begin{cases} dx(t)=\bar{A}(\phi_t)x(t)dt+\bar{E}(\phi_t)w(t)dt+\sum_{l=1}^{v}\mathbb{W}_l(\phi_t)x(t)d\varpi_l(t)\\ y(t)=C_2(\phi_t)x(t)+D_2(\phi_t)w(t)\\ z(t)=\bar{C}_1(\phi_t)x(t)+\bar{D}_1(\phi_t)w(t)\end{cases} \tag{17}$$

where

$$\begin{bmatrix} \bar{A}(\phi_t) & \bar{E}(\phi_t)\\ C_1(\phi_t) & D_1(\phi_t)\end{bmatrix}=\begin{bmatrix} A(\phi_t) & E(\phi_t)\\ C_1(\phi_t) & 0\end{bmatrix} + \begin{bmatrix} B(\phi_t)\\ D_1(\phi_t)\end{bmatrix}\mathscr{K}\begin{bmatrix} C_2(\phi_t) & D_2(\phi_t)\end{bmatrix}$$

The process ϕ_t represents a continuous time discret state Markov process with values in a finite set $H=\{1,...,h\}$ with transition probability rate matrix $\Xi=[\Phi]_{i,j=1,...,h}$. In this case, the transition probability for the jump process, ϕ_t, can be defined as:

$$p_{kj}(\Delta t)=\Phi_{kj}\Delta t+\circ(\Delta t)\ \ (k\neq j) \tag{18}$$

with $\sum\limits_{\substack{j\in H\\ j\neq i}}\Phi_{ij}=-\Phi_{ii}=\Phi_i$.

Then, the following corollary can be stated

Corollary 1: System (17) is internally exponentially stabilisable in the mean square by static output-feedback If and only if there exist matrices $\mathscr{P}_i=\mathscr{P}_i'>0$, $\mathbb{X}\in\mathbb{S}^q$, $\mathbb{Y}\in\mathbb{R}^{q\times r}$ and $\mathbb{Z}\in\mathbb{S}^r$ that simultaneously satisfy the following LMI constraints

$$\mathbb{Z}>0\quad \mathscr{P}_i>0 \tag{19}$$

$$\begin{bmatrix} \mathbb{I} & \mathbf{0}\\ A_i & B_i\end{bmatrix}'\begin{bmatrix} \Theta_i & \mathscr{P}_i\\ \mathscr{P}_i & \mathbf{0}\end{bmatrix}\begin{bmatrix} \mathbb{I} & \mathbf{0}\\ A_i & B_i\end{bmatrix} < \begin{bmatrix} C_{2i} & \mathbf{0}\\ \mathbf{0} & \mathbb{I}\end{bmatrix}'\begin{bmatrix} \mathbb{X} & \mathbb{Y}\\ \star & \mathbb{Z}\end{bmatrix}\begin{bmatrix} C_{2i} & \mathbf{0}\\ \mathbf{0} & \mathbb{I}\end{bmatrix} \tag{20}$$

and the nonlinear inequalities constraints

$$\mathbb{X}\leq\mathbb{Y}\mathbb{Z}^{-1}\mathbb{Y}' \tag{21}$$

$\forall i\in H$, where

$$\Theta_i=\sum_{l=1}^{v}\mathbb{W}_{li}'\mathscr{P}_i\mathbb{W}_{li}+\sum_{v\in H}\Phi_{iv}\mathscr{P}_v \tag{22}$$

Let $\{\mathscr{P}_i,\mathbb{X},\mathbb{Y},\mathbb{Z}\}$ be a solution, then the nonempty $\{\mathbb{X},\mathbb{Y},\mathbb{Z}\}$-ellipsoid is a set of stabilizing gains.

V. The $\mathscr{H}_\infty$ Control Problem

Let us consider the system (3) with

$$z(t)=z_\infty(t)=C_{\infty 1}x(t)+D_{\infty 1}(\eta(t))u(y(t),\psi(t),t)$$

$z_\infty(t)$ stands for the controlled output related to $\mathscr{H}_\infty$ performance. In this section, we deal with the design of controllers that stochastically stabilize the closed-loop system and guarantee the disturbance rejection, with a certain level $\gamma_\infty>0$. Mathematically, we are concerned with the characterization of compensators φ_s that stochastically stabilize the system (3) and guarantee the following for all $w\in L^2[0,\infty)$:

$$\|\,z_\infty\,\|_{\mathscr{E}_2}=\mathscr{E}\left\{\int_0^\infty z_{\infty t}'z_{\infty t}dt\right\}^{1/2}<\gamma_\infty\,\|\,w\,\|_2 \tag{23}$$

where $\gamma_\infty>0$ is a prescribed level of disturbance attenuation to be achieved. To this end, we need the auxiliary result given by the following proposition.

Proposition 4: If there exist symmetric positive-definite matrices $\mathscr{P}_{\infty ijk}$, $i\in Z$, $j\in S$ and $k\in R$ such that

$$\begin{bmatrix} \Upsilon_{ijk} & \bar{C}_{1jk}'\bar{D}_{1ijk}+\mathscr{P}_{\infty ijk}\bar{E}_{ijk}\\ \star & \bar{D}_{1ijk}'\bar{D}_{1ijk}-\gamma_\infty^2\mathbb{I}\end{bmatrix}=\Phi_{ijk}<0 \tag{24}$$

where

$$\Upsilon_{ijk}=\tilde{A}_{ijk}'\mathscr{P}_{\infty ijk}+\mathscr{P}_{\infty ijk}\tilde{A}_{ijk}+\sum_{l=1}^{v}\mathbb{W}_{lij}'\mathscr{P}_{\infty ijk}\mathbb{W}_{lij} +\bar{C}_{1jk}'\bar{C}_{1jk}+\sum_{h\in Z}\pi_{ih}\mathscr{P}_{\infty hjk}+\sum_{l\in S}\nu_{jl}\mathscr{P}_{\infty ilk}+\sum_{v\in R}\lambda_{kv}^{ij}\mathscr{P}_{\infty ijv}$$

$\forall i\in Z$, $j\in S$ and $k\in R$.

then the system (3) is stochastically stable and satisfies

$$\|\,z_\infty\,\|_{\mathscr{E}_2}=\mathscr{E}\left\{\int_0^\infty z_{\infty t}'z_{\infty t}dt\right\}^{1/2}<\gamma_\infty\,\|\,w\,\|_2 \tag{25}$$

Proof: See [3].

Using the previous proposition, the following $\mathscr{H}_\infty$ control result can be stated.

Proposition 5: If there exist matrices $\mathscr{P}_{\infty ijk}=\mathscr{P}'_{\infty ijk}>0$, $\mathbb{X}_k\in\mathbb{S}^q$, $\mathbb{Y}_k\in\mathbb{R}^{q\times r}$ and $\mathbb{Z}_k\in\mathbb{S}^r$ that simultaneously satisfy the following LMI constraints

$$\mathbb{Z}_k>0\quad \mathscr{P}_{\infty ijk}>0 \tag{26}$$

$$\mathbb{M}_{1ij}'\begin{bmatrix} \Theta_{ijk} & \mathscr{P}_{\infty ijk}\\ \mathscr{P}_{\infty ijk} & \mathbf{0}\end{bmatrix}\mathbb{M}_{1ij}<\mathbb{M}_2'\begin{bmatrix} -\mathbb{I} & \mathbf{0}\\ \mathbf{0} & \gamma_\infty^2\mathbb{I}\end{bmatrix}\mathbb{M}_2 +\mathbb{M}_3'\begin{bmatrix} \mathbb{X}_k & \mathbb{Y}_k\\ \star & \mathbb{Z}_k\end{bmatrix}\mathbb{M}_3 \tag{27}$$

and the nonlinear inequalities constraints

$$\mathbb{X}_k\leq\mathbb{Y}_k\mathbb{Z}_k^{-1}\mathbb{Y}_k' \tag{28}$$

$\forall i\in Z$, $j\in S$ and $k\in R$, where

$$\mathbb{M}_{1ij}=\begin{bmatrix} \mathbb{I} & \mathbf{0} & \mathbf{0}\\ A_i & E_{ij} & B_j\end{bmatrix},\quad \mathbb{M}_2=\begin{bmatrix} C_{\infty 1} & \mathbf{0} & D_{\infty 1}\\ \mathbf{0} & \mathbb{I} & \mathbf{0}\end{bmatrix},$$
$$\mathbb{M}_3=\begin{bmatrix} C_2 & D_2 & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbb{I}\end{bmatrix}.$$

then the $\{\mathbb{X}_k,\mathbb{Y}_k,\mathbb{Z}_k\}$-ellipsoids are sets of stabilizing gains such that

$$\|\,z_\infty\,\|_{\mathscr{E}_2}=\mathscr{E}\left\{\int_0^\infty z_{\infty t}'z_{\infty t}dt\right\}^{1/2}<\gamma_\infty\,\|\,w\,\|_2 \tag{29}$$

Proof: The proof of this proposition follows the same arguments as for the proof of proposition 3.

Remark 3: As for the internal stochastic stabilization problematic, the *mode-independent* static output feedback $\mathscr{H}_\infty$ control of MJLS can be solved in the same way as for AFTCSMP. This result is illustrated by corollary 2.

Corollary 2: If there exist matrices $\mathscr{P}_{\infty i}=\mathscr{P}'_{\infty i}>0$, $\mathbb{X}\in\mathbb{S}^q$, $\mathbb{Y}\in\mathbb{R}^{q\times r}$ and $\mathbb{Z}\in\mathbb{S}^r$ that simultaneously satisfy the following LMI constraints

$$\mathbb{Z}>0\quad \mathscr{P}_{\infty i}>0 \tag{30}$$

$$\mathbb{M}_{1i}'\begin{bmatrix} \Theta_i & \mathscr{P}_{\infty i}\\ \mathscr{P}_{\infty i} & \mathbf{0}\end{bmatrix}\mathbb{M}_{1i}<\mathbb{M}_{2i}'\begin{bmatrix} -\mathbb{I} & \mathbf{0}\\ \mathbf{0} & \gamma_\infty^2\mathbb{I}\end{bmatrix}\mathbb{M}_{2i}+\mathbb{M}_{3i}'\begin{bmatrix} \mathbb{X} & \mathbb{Y}\\ \star & \mathbb{Z}\end{bmatrix}\mathbb{M}_{3i} \tag{31}$$

and the nonlinear inequalities constraints

$$\mathbb{X} \leq \mathbb{Y}\mathbb{Z}^{-1}\mathbb{Y}' \tag{32}$$

$\forall i \in H$, where

$$\mathbb{M}_{1i} = \begin{bmatrix} \mathbb{I} & \mathbf{0} & \mathbf{0} \\ A_i & E_i & B_i \end{bmatrix}, \quad \mathbb{M}_{2i} = \begin{bmatrix} C_{\infty 1i} & \mathbf{0} & D_{\infty 1i} \\ \mathbf{0} & \mathbb{I} & \mathbf{0} \end{bmatrix},$$
$$\mathbb{M}_{3i} = \begin{bmatrix} C_{2i} & D_{2i} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbb{I} \end{bmatrix}.$$

then the $\{\mathbb{X}, \mathbb{Y}, \mathbb{Z}\}$-ellipsoid is a set of stabilizing gains such that

$$\| z_\infty \|_{\mathscr{E}_2} = \mathscr{E}\left\{ \int_0^\infty z'_{\infty t} z_{\infty t} dt \right\}^{1/2} < \gamma_\infty \| w \|_2 \tag{33}$$

VI. Computational Issues and Examples

A. A Cone Complementary Algorithm

The numerical examples are solved using a first order iterative algorithm. It is based on a cone complementary technique [12], that allows to concentrate the non convex constraint in the criterion of some optimisation problem.

Lemma 1: The problem (26)-(28) is feasible if and only if zero is the global optimum of the optimisation problem

$$\begin{cases} \min \quad \mathrm{tr}(\mathbb{T}\mathbb{S}) \\ \text{s.t.} \quad (26),\ (27) \\ \mathbb{X}_k \leq \widehat{\mathbb{X}}_k \quad \mathscr{S}_k = \begin{bmatrix} \widehat{\mathbb{X}}_k & \mathbb{Y}_k \\ \star & \mathbb{Z}_k \end{bmatrix} \geq 0 \\ \mathscr{T}_{1k} \geq \mathbb{I} \quad \mathscr{T}_k = \begin{bmatrix} \mathscr{T}_{1k} & \mathscr{T}_{2k} \\ \star & \mathscr{T}_{3k} \end{bmatrix} \geq 0 \end{cases} \tag{34}$$

where

$$\mathbb{S} = \mathrm{diag}\{\mathscr{S}_1, \ldots, \mathscr{S}_r\}, \quad \mathbb{T} = \mathrm{diag}\{\mathscr{T}_1, \ldots, \mathscr{T}_r\}$$

Proof: The proof of this Lemma follows the same arguments as in [19]. With the constraints $\mathscr{T}_k \geq 0$ and $\mathscr{S}_k \geq 0$, we have that $\mathbb{T} \geq 0$ and $\mathbb{S} \geq 0$ which induce the following implications

$$\mathrm{tr}(\mathbb{T}\mathbb{S}) = 0 \Rightarrow \mathbb{T}\mathbb{S} = \mathbf{0} \Rightarrow \mathscr{T}_k \mathscr{S}_k = \mathbf{0}, \quad \forall k \in R \tag{35}$$

Therefore, after some manipulations, one gets

$$\widehat{\mathbb{X}}_k = -\mathscr{T}_{1k}^{-1}\mathscr{T}_{2k}\mathbb{Y}'_k = -\mathscr{T}_{1k}^{-1}(-\mathscr{T}_{1k}\mathbb{Y}_k\mathbb{Z}_k^{-1})\mathbb{Y}'_k = \mathbb{Y}_k\mathbb{Z}_k^{-1}\mathbb{Y}'_k$$

Thus the nonlinear constraints is satisfied

$$\mathbb{X}_k \leq \widehat{\mathbb{X}}_k = \mathbb{Y}_k\mathbb{Z}_k^{-1}\mathbb{Y}'_k$$

The converse implication is proved taking $\widehat{\mathbb{X}}_k = \mathbb{Y}_k\mathbb{Z}_k^{-1}\mathbb{Y}'_k$ and $\mathscr{T}_k$ such that $\mathscr{T}_k\mathscr{S}_k = 0$, $\forall k \in R$.

A detailed description of the implementation of this algorithm is given in [4].

B. Numerical Examples

a) Fault Tolerant Control

In this section, the proposed $\mathscr{H}_\infty$ static output feedback control of AFTCSMP is illustrated using a flight control example. Consider the nominal system with

$$A = \begin{bmatrix} -0.0565 & 29.072 & -175.610 & 9.6783 & 1.6022 \\ -0.0601 & -0.7979 & -0.2996 & 0 & 0 \\ 9.218\times10^{-3} & -0.0179 & -0.1339 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.1339 & 0.1339 & 2.0092 \\ 2.3491 & -2.3491 & 0.7703 \\ 0.0444 & -0.0444 & -1.3575 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \\ 0.1 & 0.1 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathbb{W}_1 = 0.1 \times \mathbb{I},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}, C_{\infty 1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, D_{\infty 1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This model is adapted from [17]. It represents the lateral-directional dynamics of McDonnell F-4C Phantom flying at Mach 0.6 at an altitude of 35000 ft. The states x_i, $i = 1, \ldots, 5$ denote the lateral velocity (ft per second), the roll rate (radian per second), yaw rate (radian per second), roll angle (radian) and yaw angle (radian), respectively. The control inputs u_1, u_2 and u_3 correspond to the left aileron, the right aileron and the rudder surface displacement, respectively.

For illustration purposes, we will consider two faulty modes:

i) **Mode 2**: A 50% power loss on the left aileron;
ii) **Mode 3**: Right aileron outage.

From above, we have that $S = \{1,2,3\}$, where the **mode 1** represents the nominal case. The failure process is assumed to have Markovian transition characteristics. The FDI process is also Markovian with three states $R = \{1,2,3\}$.

The actuator failure rates are assumed to be:

$$[\pi_{ij}] = \begin{bmatrix} -0.002 & 0.0010 & 0.0010 \\ 0.0010 & -0.002 & 0.0010 \\ 0.0010 & 0.0010 & -0.002 \end{bmatrix}$$

The FDI conditional transition rates are:

$$[\lambda_{ij}^1] = \begin{bmatrix} -0.02 & 0.01 & 0.01 \\ 1.00 & -1.01 & 0.01 \\ 1.00 & 0.01 & -1.01 \end{bmatrix}, [\lambda_{ij}^2] = \begin{bmatrix} -1.01 & 1.00 & 0.01 \\ 0.01 & -0.02 & 0.01 \\ 0.01 & 1.00 & -1.01 \end{bmatrix},$$
$$[\lambda_{ij}^3] = \begin{bmatrix} -1.01 & 0.01 & 1.00 \\ 0.01 & -1.01 & 1.00 \\ 0.01 & 0.01 & -0.02 \end{bmatrix}.$$

For the above AFTCSMP, several numerical experiments are performed using the cone complementary algorithm. These tests are realised for various specifications on the $\mathscr{H}_\infty$ performance (γ_∞). Here are presented some cases described in Table 1, where `iter` is the number of the algorithms iterations, `time` is the computation time (LMIs solved with LMI toolbox, Matlab 6.5.1), `Tr(`$\mathbb{T}\mathbb{S}$`)` is the value of the optimisation criteria trace ($\mathbb{T}_k\mathbb{S}_k$) at the step when the algorithm stopped, and $\mathscr{K}_{k0}$, $k = 1,2,3$ are the controllers obtained as the centers of the stabilising ellipsoids.

b) Mode-Independent Control of MJLS

We applied the proposed static output feedback $\mathscr{H}_\infty$ control to a VTOL helicopter model adapted from [10]. The dynamics can be written as

$$\begin{cases} dx_t = A(\phi_t)x(t)dt + B(\phi_t)u(y_t,t)dt + Ew_t dt + \mathbb{W}_1 x_t d\varpi_t \\ y_t = C_2 x_t + D_2 w_t \\ z_{\infty t} = C_{\infty 1} x_t + D_{\infty 1} u(y_t,t) \end{cases}$$

where ϕ_t indicates the airspeed. The parameters are given by

$$A(\phi_t) = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & a_{32}(\phi_t) & -0.707 & a_{34}(\phi_t) \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B(\phi_t) = \begin{bmatrix} 0.4422 & 0.1761 \\ b_{21}(\phi_t) & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 0.0468 & 0 \\ 0.0457 & 0.0099 \\ 0.0437 & 0.0011 \\ -0.0218 & 0 \end{bmatrix}, \mathbb{W}_1 = 0.1 * \mathbb{I},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, C_{\infty 1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D_{\infty 1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The behavior of ϕ_t is modelled as a Markov chain with three different states, corresponding to airspeeds of 135 (nominal value), 60, 170, Knots. The values of parameters a_{32}, a_{34}, and b_{32} are shown in Table 2. The transition matrix is given

TABLE I

NUMERICAL EXPERIMENTS

test	γ_∞^2	iter	time(s)	Tr($\mathbb{TS}$)	$\mathscr{K}'_{10}$	$\mathscr{K}'_{20}$	$\mathscr{K}'_{30}$
1	20	3	28.9060	615.7093	$\begin{bmatrix} -0.7589 & 0.9560 & 0.6559 \end{bmatrix}$	$\begin{bmatrix} -0.7007 & 1.1969 & 0.6718 \end{bmatrix}$	$\begin{bmatrix} -1.2359 & -0.1512 & 0.6826 \end{bmatrix}$
2	10	3	28.8750	810.2702	$\begin{bmatrix} -0.7694 & 1.0038 & 0.6897 \end{bmatrix}$	$\begin{bmatrix} -0.7056 & 1.2640 & 0.7075 \end{bmatrix}$	$\begin{bmatrix} -1.2744 & -0.1605 & 0.7129 \end{bmatrix}$
3	5	3	28.0620	4.1924e+003	$\begin{bmatrix} -0.7843 & 1.1103 & 0.7451 \end{bmatrix}$	$\begin{bmatrix} -0.7187 & 1.3857 & 0.7645 \end{bmatrix}$	$\begin{bmatrix} -1.3583 & -0.1397 & 0.7706 \end{bmatrix}$
4	1	10	126.6410	1.4087e+006	fails	fails	fails

TABLE II

PARAMETERS

Airspeed (Knots)	a_{32}	a_{34}	b_{21}
135	0.3681	1.4200	3.5446
60	0.0664	0.1198	0.9775
170	0.5047	2.5460	5.1120

by

$$\Xi = \begin{bmatrix} -0.0907 & 0.0671 & 0.0236 \\ 0.0671 & -0.0671 & 0 \\ 0.0236 & 0 & -0.0236 \end{bmatrix}$$

As for the previous example, several numerical experiments are performed using the cone complementary algorithm. These tests are realised for various specifications on the $\mathscr{H}_\infty$ performance (γ_∞). Here are presented some cases described in Table 3, where $\mathscr{K}_0$ is the controller obtained as the center of the stabilising ellipsoid.

TABLE III

NUMERICAL EXPERIMENTS

test	γ_∞^2	iter	time(s)	Tr($\mathbb{TS}$)	$\mathscr{K}'_0$
1	10	3	2.2810	15.2404	$\begin{bmatrix} 0.4321 & -0.4037 \end{bmatrix}$
2	5	3	1.8750	3.6171	$\begin{bmatrix} 0.2692 & -0.3942 \end{bmatrix}$
3	1	3	4.1250	0.2235	$\begin{bmatrix} 0.2465 & -0.3116 \end{bmatrix}$
4	0.5	11	10.9060	751.6071	fails

VII. CONCLUSION

In this paper, the static output feedback $\mathscr{H}_\infty$ control of continuous time AFTCSMP was considered within a new framework. This last one is based on the synthesis of ellipsoidal sets of controllers and was introduced by [18], [19]. The problematic resulting from the fact that the controller only depends on the FDI process is shown to be naturally dealt with in this context. It was also shown that the obtained results could easily be applied to the problem of mode-independent static output feedback $\mathscr{H}_\infty$ control of Markovian Jump Linear Systems. The numerical resolution of the obtained results was done using a cone complementary algorithm and its running was illustrated on classical examples from literature.

REFERENCES

[1] S. Aberkane, J.C. Ponsart and D. Sauter, Output Feedback Stochastic Stabilization of Active Fault Tolerant Control Systems: LMI Formulation, *16th IFAC World Congress, Prague, Czech Republic*, 2005.

[2] S. Aberkane, D. Sauter and J.C. Ponsart, $\mathscr{H}_\infty$ Stochastic Stabilization of Active Fault Tolerant Control Systems: Convex Approach, *44th IEEE Conference on Decision and Control and European Control Conference ECC 2005, Seville, Spain*, 2005.

[3] S. Aberkane, J.C. Ponsart and D. Sauter, Output Feedback $\mathscr{H}_\infty$ Control of a Class of Stochastic Hybrid Systems with Wiener Process via Convex Analysis, *Submitted*, 2005.

[4] S. Aberkane, J.C. Ponsart, M. Rodrigues and D. Sauter, Ellipsoidal Output-Feedback Sets for Multi-Objective Control of a Class of Stochastic Hybrid Systems with State-Dependent Noise *Submitted*, 2006.

[5] E. K. Boukas, Static Output Feedback Control for Stochastic Hybrid Systems: LMI Approach, *Automatica*, Vol.42, pp.183-188, 2006.

[6] E. K. Boukas, Stabilization of Stochastic Nonlinear Hybrid Systems, *Int. J. Innovative Computing, Information and Control*, Vol.1, No.1, pp.131-141, 2005.

[7] E. K. Boukas, Exponential stabilizability of stochastic systems with Markovian jumping parameters, *Automatica*, Vol.35, pp.1437-1441, 1999.

[8] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Philadelphia: SIAM; 1994.

[9] O. L. V. Costa, J. B. R. do Val and J. C. Geromel, Continuous-time state-feedback $\mathscr{H}_2$-control of Markovian jump linear systems via convex analysis, *Automatica*, Vol 35, 1999, pp 259-268.

[10] D. P. de Farias, J. C. Geromel, J. B. R. do Val and O. L. V. Costa, Output Feedback Control of Markov Jump Linear Systems in Continuous-Time, *IEEE Transactions on Automatic Control*, Vol 45, 2000, pp 944-949.

[11] C. E. de Souza and M. D. Fragoso, $\mathscr{H}_\infty$ Control For Linear Systems With Markovian Jumping Parameters, *Control Theory and Advanced Technology*, Vol 9, No. 2, 1993, pp 457-466.

[12] L. El Ghaoui, F. Oustry and M. AitRami, A Cone Complementary Linearization Algorithm for Static Output-Feedback and related Problems, *IEEE Transactions on Automatic Control*, Vol 42, No. 8, 1997, pp 1171-1176.

[13] Y. Ji and H. J. Chizeck, Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control, *IEEE Transactions on Automatic Control*, Vol 35, 1990, pp 777-788.

[14] Y. Ji and H. J. Chizeck, Jump linear quadratic Gaussian control in continuous time, *IEEE Transactions on Automatic Control*, Vol 37, 1992, pp 1884-1892.

[15] M. Mahmoud, J. Jiang and Y. Zhang, Stochastic Stability Analysis of Active Fault-Tolerant Control Systems in the Presence of Noise, *IEEE Transactions on Automatic Control*, Vol 46, 2001, pp 1810-1815.

[16] M. Mahmoud, J. Jiang and Y. Zhang, Active Fault Tolerant Control Systems: Stochastic Analysis and Synthesis, *Springer*, 2003.

[17] M. Maki, J. Jiang and K. Hagino, A Stability Guaranteed Active Fault Tolerant Control System Against Actuator Failures, *International Journal of Robust and Nonlinear Control*, Vol 14, 2004, pp 1061-1077.

[18] D. Peaucelle, D. Arzelier and R. Bertrand, Ellipsoidal Sets for Static Output-Feedback, *15th IFAC World Congress, Barcelona, Spain*, 2002.

[19] D. Peaucelle and D. Arzelier, Ellipsoidal Sets for Resilient and Robust Static Output-Feedback, *IEEE Transaction on Automatic Control*, Vol 50, No. 6, 2005, pp 899-904.

[20] P. Shi and E. K. Boukas, $\mathscr{H}_\infty$-Control for Markovian Jumping Linear Systems with Parametric Uncertainty, *Journal of Optimization Theory and Applications*, Vol 95, 1997, pp 75-99.

[21] P. Shi, E. K. Boukas, S. K. Nguang and X. Guo, Robust disturbance attenuation for discrete-time active fault tolerant control systems with uncertainties, *Optimal Control Applications and Methods*, Vol 24, 2003, pp 85-101.

[22] R. E. Skelton, T. Iwasaki and K. Grigoriadis, A Unified Algebraic Approach to Linear Control Design, *Taylor and Francis*, 1998.

[23] R. Srichander and B. K. Walker, Stochastic stability analysis for continuous-time fault tolerant control systems, *International Journal of Control*, Vol 57, 1993, pp 433-452.

[24] V. L. Syrmos, C. T. Abdallah, P. Dorato and K. Grigoriadis, Static Output Feedback: A Survey, *Automatica*, Vol 33, No. 2, 1997, pp 125-137.

[25] Y. Zhang and J. Jiang, Bibliographical review on reconfigurable fault-tolerant control systems, *IFAC SAFEPROCESS*, 2003, pp 265-276.