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**ROBUST STATIC OUTPUT FEEDBACK \mathcal{H}_∞
CONTROL OF A CLASS OF STOCHASTIC HYBRID
SYSTEMS IN NOISY ENVIRONMENT: LMI
FORMULATION**

S. Aberkane J.C. Ponsart F. Hamelin

*CRAN – CNRS UMR 7039
Université Henri Poincaré, Nancy 1, BP 239,
F-54506 Vandœuvre-lès-Nancy Cedex
Tel.: +33 3 83 68 44 80, Fax: +33 3 83 68 44 62
e-mail: samir.aberkane@cran.uhp-nancy.fr*

Abstract: In this paper, we address the robust static output feedback control problem of continuous time active fault tolerant control system with Markovian parameters (AFTCSMP) subject to structured parameter uncertainties, in noisy environment. We will first derive a necessary and sufficient condition for the robust internal exponential stability in the mean square of the uncertain AFTCSMP in terms of nonlinear matrix inequalities. Then, an LMI relaxation scheme is presented to eliminate the nonlinearities, yielding a new easily tractable sufficient condition. Having obtained this result, we can move on the control problem and write the robust output feedback \mathcal{H}_∞ control problem of the uncertain continuous time AFTCSMP in terms of an LMI optimization problem. The convex approach naturally leads to powerful numerical algorithms to solve these problematic. A numerical example is presented to illustrate the theoretical results.

Keywords: Active Fault Tolerant Control - Stochastic Hybrid systems - Markovian jumping parameters - \mathcal{H}_∞ control - Output Feedback - Structured Uncertainties - Wiener Process - Lyapunov stability - LMI.

1. INTRODUCTION

Many physical systems have variable structures subject to random changes, which may result from abrupt phenomena such as component and interconnection failures, parameters shifting, tracking, and time required to measure some of the variables at different stages. Systems with this character may be modeled as hybrid ones, i.e, the state space of the systems contains both discrete and continuous states. Among this kind of systems, fault tolerant control systems (FTCS) have been a subject of great practical importance, which has attracted a lot of interest for the last three decades (Zhang and Jiang 2003).

Active fault tolerant control systems (AFTCS) can be viewed as stochastic hybrid systems (Srichander and Walker 1993). A major class of stochastic hybrid systems is jump linear systems (JLS). In the last two decades this class of systems has been extensively studied. Theoretical and practical achievements have been reported in the literature. Without any intention of being exhaustive here, we mention (Boukas 2005, Costa *et al.* 1999, de Farias *et al.* 2000, de Souza and Fragoso 1993, Ji and Chizeck 1990, Ji and Chizeck 1992). Other references dealing with this class of systems can be found in the quoted references.

To deal with AFTCS, another class of hybrid systems was defined, denoted as active fault toler-

ant control systems with Markovian parameters (AFTCSMP). This model was proposed by Srichander and Walker (Srichander and Walker 1993). Necessary and sufficient conditions for stochastic stability of AFTCSMP were developed for a single component failure. The problem of stochastic stability of AFTCSMP in the presence of noise, parameter uncertainties, detection errors, detection delays and actuator saturation limits has also been investigated in (Mahmoud *et al.* 2001, Mahmoud *et al.* 2003). Another issue related to the synthesis of fault tolerant control laws was also addressed by (Shi and Boukas 1997, Shi *et al.* 2003). The problem of \mathcal{H}_∞ and robust \mathcal{H}_∞ control was treated in (Shi and Boukas 1997, Shi *et al.* 2003) for both continuous and discrete time AFTCSMP. The authors showed that the state feedback control problem can be solved in terms of the solutions of a set of coupled Riccati inequalities. The dynamic output feedback counterpart was treated by (Aberkane *et al.* 2005a) in a convex programming framework. Indeed, the authors provide an LMI characterization of dynamical compensators that stochastically stabilize (robustly stabilize) the AFTCSMP and ensures \mathcal{H}_∞ (robust \mathcal{H}_∞) constraints. Furthermore, it is important to note that in (Aberkane *et al.* 2005a, Mahmoud *et al.* 2003, Shi and Boukas 1997, Shi *et al.* 2003), the authors make the assumption that the controller must access both failures and FDI processes. However, this assumption is too restrictive to be applicable in practical FTC systems. In this work the assumption on the availability of failures processes, for the synthesis purposes, is stressed which makes the problematic more challenging. In this paper, we will consider the problem of static output feedback robust \mathcal{H}_∞ control of an uncertain AFTCSMP with Wiener process via convex analysis. The first problematic we consider in this paper is the output feedback *stochastic stabilization* of the uncertain AFTCSMP. It is shown that the necessary and sufficient conditions for the *robust internal exponential stability* in the mean square sense can be written in terms of an nonlinear matrix inequality feasibility problem. Then, an LMI relaxation scheme is presented to eliminate the nonlinearities, yielding to a new easily tractable sufficient condition. Having obtained this result, we can move on the control problem and write the output feedback robust \mathcal{H}_∞ control problem of continuous time AFTCSMP in terms of an LMI optimization problem. This paper is organized as follows: section 2 describes the dynamical model of the system with appropriately defined random processes. A brief summary of basic stochastic terms, results and definitions are given in section 3. Section 4 derives the necessary and sufficient conditions for the stochastic exponential stability in the mean square, and the LMI characterization of the static output feedback compensators. Sections 5 considers the robust \mathcal{H}_∞

control problem. Finally, a conclusion is given in section 6.

2. DYNAMICAL MODEL

To describe the class of linear systems with Markovian jumping parameters that we deal with in this paper, let us fix a complete probability space (Ω, \mathcal{F}, P) . This class of systems owns a hybrid state vector. The first component vector is continuous and represents the system states, and the second one is discrete and represents the failure processes affecting the system. The dynamical model of the AFTCSMP with Wiener Process, defined in the fundamental probability space (Ω, \mathcal{F}, P) , is described by the following differential equations:

$$\varphi : \begin{cases} dx(t) = [A(\xi(t)) + \Delta A(\xi(t))]x(t)dt + [B(\eta(t)) + \Delta B(\eta(t))]u(y, \psi, t)dt \\ \quad + E(\xi(t), \eta(t))w(t)dt + \mathcal{W}(\xi(t), \eta(t))x(t)d\varpi(t) \\ y(t) = C_2x(t) + D_2(\xi(t), \eta(t))w(t) \\ z_\infty(t) = C_1x(t) + D_1(\eta(t))u(y(t), \psi(t), t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(y(t), \psi(t), t) \in \mathbb{R}^r$ is the system input, $y(t) \in \mathbb{R}^q$ is the system measured output, $z_\infty(t) \in \mathbb{R}^p$ is the controlled output, $w(t) \in \mathbb{R}^m$ is the system external disturbance, $\xi(t)$, $\eta(t)$ and $\psi(t)$ are separable and measurable Markov processes with finite state spaces $Z = \{1, 2, \dots, z\}$, $S = \{1, 2, \dots, s\}$ and $R = \{1, 2, \dots, r\}$, respectively. $\varpi(t)$ is a standard Wiener process that is assumed to be independent of the Markov processes. $\Delta A(\xi(t))$ and $\Delta B(\eta(t))$ represent system parameter uncertainties. The matrices $A(\xi(t))$, $B(\eta(t))$, $E(\xi(t), \eta(t))$, $D_2(\xi(t), \eta(t))$, $D_1(\eta(t))$ and $\mathcal{W}(\xi(t), \eta(t))$ are properly dimensioned matrices which depend on random parameters. The system disturbance $w(t)$ is assumed to belong to $L^2[0, \infty)$ *i.e.*

$$\|w\|_2 = \left\{ \int_0^\infty w^T(t)w(t)dt \right\}^{1/2} < \infty$$

This implies that the disturbance has finite energy. In AFTCS, we consider that the control law is only a function of the measurable FDI process $\psi(t)$. Therefore, we introduce a static output feedback compensator (φ_s) of the form:

$$\varphi_s : \{u(t) = \mathcal{K}(\psi(t))y(t)\} \quad (2)$$

Applying the controller φ_s to the AFTCSMP φ , we obtain the following closed loop system:

$$\begin{cases} dx(t) = \bar{A}(\xi(t), \eta(t), \psi(t))x(t)dt + \bar{E}(\xi(t), \eta(t), \psi(t))w(t)dt \\ \quad + \bar{\mathcal{W}}(\xi(t), \eta(t), \psi(t))x(t)d\varpi(t) \\ y(t) = C_2x(t) + D_2(\xi(t), \eta(t))w(t) \\ z_\infty(t) = C_1(\eta(t), \psi(t))x(t) + \bar{D}_1(\xi(t), \eta(t), \psi(t))w(t) \end{cases} \quad (3)$$

where

$$\begin{aligned} & \begin{bmatrix} \bar{A}(\xi(t), \eta(t), \psi(t)) & \bar{E}(\xi(t), \eta(t), \psi(t)) \\ C_1(\eta(t), \psi(t)) & \bar{D}_1(\xi(t), \eta(t), \psi(t)) \end{bmatrix} = \\ & \begin{bmatrix} A(\xi(t)) + \Delta A(\xi(t)) & E(\xi(t), \eta(t)) \\ C_1 & 0 \end{bmatrix} \\ & + \begin{bmatrix} B(\eta(t)) + \Delta B(\eta(t)) \\ D_1(\eta(t)) \end{bmatrix} \mathcal{K}(\psi(t)) \begin{bmatrix} C_2 & D_2(\xi(t), \eta(t)) \end{bmatrix} \end{aligned}$$

The FDI and the Failure Processes: $\xi(t)$, $\eta(t)$

and $\psi(t)$ being homogeneous Markov processes with finite state spaces, we can define the transition probability of the plant components failure process as (Mahmoud *et al.* 2003, Srichander and Walker 1993):

$$\begin{cases} p_{ij}(\Delta t) = \pi_{ij}\Delta t + o(\Delta t) & (i \neq j) \\ p_{ii}(\Delta t) = 1 - \sum_{i \neq j} \pi_{ij}\Delta t + o(\Delta t) & (i = j) \end{cases}$$

The transition probability of the actuator failure process is given by:

$$\begin{cases} p_{kl}(\Delta t) = \nu_{kl}\Delta t + o(\Delta t) & (k \neq l) \\ p_{kk}(\Delta t) = 1 - \sum_{k \neq l} \nu_{kl}\Delta t + o(\Delta t) & (k = l) \end{cases}$$

where π_{ij} is the plant components failure rate, and ν_{kl} is the actuator failure rate.

Given that $\xi = k$ and $\eta = l$, the conditional transition probability of the FDI process $\psi(t)$ is:

$$\begin{cases} p_{iv}^{kl}(\Delta t) = \lambda_{iv}^{kl}\Delta t + o(\Delta t) & (i \neq v) \\ p_{ii}^{kl}(\Delta t) = 1 - \sum_{i \neq v} \lambda_{iv}^{kl}\Delta t + o(\Delta t) & (i = v) \end{cases}$$

Here, λ_{iv}^{kl} represents the transition rate from i to v for the Markov process $\psi(t)$ conditioned on $\xi = k \in Z$ and $\eta = l \in S$.

The Model of Parameter Uncertainties: To study the robust stabilization (and the robust \mathcal{H}_∞ control) of the uncertain AFTCSMP, we will assume in this work that the admissible structured parameter uncertainties have a norm bounded uncertainty (NBU) form. In this form, the admissible structured parameter uncertainties are modelled as:

$$\begin{cases} \Delta A(\xi(t)) = D_A(\xi(t))F_A(\xi(t))E_A(\xi(t)) \\ \Delta B(\eta(t)) = D_B(\eta(t))F_B(\eta(t))E_B(\eta(t)) \end{cases} \quad (4)$$

where $D_A(\xi(t)) \in \mathbb{R}^{n \times p_x}$, $F_A(\xi(t)) \in \mathbb{R}^{p_x \times q_x}$, $E_A(\xi(t)) \in \mathbb{R}^{q_x \times n}$, $D_B(\eta(t)) \in \mathbb{R}^{n \times p_u}$, $F_B(\eta(t)) \in \mathbb{R}^{p_u \times q_u}$ and $E_B(\eta(t)) \in \mathbb{R}^{q_u \times m}$. $D_A(\xi(t)) \in \mathbb{R}^{n \times p_x}$, $D_B(\eta(t)) \in \mathbb{R}^{n \times p_u}$, $E_A(\xi(t)) \in \mathbb{R}^{q_x \times n}$, $E_B(\eta(t)) \in \mathbb{R}^{q_u \times m}$ are known constant matrices, $F_A(\xi(t)) \in \mathbb{R}^{p_x \times q_x}$ and $F_B(\eta(t)) \in \mathbb{R}^{p_u \times q_u}$ are Lipschitz measurable matrix functions satisfying the condition

$$\begin{cases} F_A^T(\xi(t))F_A(\xi(t)) \leq I_{p_x} \\ F_B^T(\eta(t))F_B(\eta(t)) \leq I_{p_u}, \forall t \geq 0 \end{cases} \quad (5)$$

3. DEFINITIONS

Under the assumption that the system (φ) coupled with (φ_s) satisfies the global Lipschitz condition, the solution x_t determines a family of unique continuous stochastic processes, one for each choice of the random variable x_0 . The joint process $\{x_t, \xi_t, \eta_t, \psi_t\}$ is a Markov process.

Definition 1: System (3) is said to be

- (i) *robustly stochastically stable (RSS)* if there exists a finite positive constant $K(x_0, \xi_0, \eta_0, \psi_0)$ such that the following holds for any initial conditions $(x_0, \xi_0, \eta_0, \psi_0)$ and for all admissible uncertainties:

$$\mathcal{E} \left\{ \int_0^\infty \|x_t\|^2 dt \right\} \leq K(x_0, \xi_0, \eta_0, \psi_0) \quad (6)$$

- (ii) *internally robustly exponentially stable in the mean square sense* if it is exponentially stable in the mean square sense for $w_t = 0$, i.e. for any ξ_0, η_0, ψ_0 and some $\gamma(\xi_0, \eta_0, \psi_0)$, there exists two numbers $a > 0$ and $b > 0$ such that when $\|x_0\| \leq \gamma(\xi_0, \eta_0, \psi_0)$, the following inequality holds $\forall t \geq t_0$ for all solution of (3) with initial condition x_0 and for all admissible uncertainties:

$$\mathcal{E} \{ \|x_t\|^2 \} \leq b \|x_0\|^2 \exp[-a(t - t_0)] \quad (7)$$

We conclude this section by recalling the following lemma which will be useful for the proof of our main results in the next sections.

Lemma 1 (Reciprocal Projection Lemma) (Apkarian *et al.* 2001): Let P be any given positive-definite matrix. The following statements are equivalent:

- 1) $\Psi + S + S^T < 0$;
- 2) the LMI problem

$$\begin{bmatrix} \Psi + P - (W + W^T) & S^T + W^T \\ S + W & -P \end{bmatrix} < 0$$

is feasible with respect to W .

4. ROBUST STOCHASTIC STABILIZATION

Before introducing the main results of this section, let us first give the following preliminary results which will be useful in the derivation of numerically tractable conditions for robust stability of (3).

Theorem 1: The solution $x = 0$ of the system (φ) coupled with (φ_s) is internally robustly exponentially stable in the mean square for $t \geq t_0$ if there exists a Lyapunov function $\vartheta(x_t, \xi_t, \eta_t, \psi_t, t)$ such that

$$K_1 \|x_t\|^2 \leq \vartheta(x_t, \xi_t, \eta_t, \psi_t, t) \leq K_2 \|x_t\|^2 \quad (8)$$

and

$$\mathcal{L}_{up} \vartheta(x_t, \xi_t, \eta_t, \psi_t, t) \leq -K_3 \|x_t\|^2 \quad (9)$$

for some positive constants K_1, K_2 and K_3 , where \mathcal{L}_{up} is the upper bound of \mathcal{L} obtained for the maximal value of the uncertainties.

Theorem 2: If the solution $x = 0$ of the system (φ) coupled with (φ_s) is internally robustly exponentially stable in the mean square, then, for any given quadratic positive definite function $W(x_t, \xi_t, \eta_t, \psi_t, t)$ in the variables x which is bounded and continuous $\forall t \geq t_0, \forall \xi_t \in Z, \forall \eta_t \in S$ and $\forall \psi_t \in R$, there exists a quadratic positive definite function $\vartheta(x_t, \xi_t, \eta_t, \psi_t, t)$ in x_t that satisfies the conditions to be a stochastic Lyapunov function and is such that $\mathcal{L}_{up} \vartheta(x_t, \xi_t, \eta_t, \psi_t, t) = -W(x_t, \xi_t, \eta_t, \psi_t, t)$.

Remark 1: The proofs of these theorems follow the same arguments as in (Mahmoud *et al.* 2003, Srichander and Walker 1993) for their proposed stochastic Lyapunov functions, so they are not shown in this paper to avoid repetition.

The following proposition gives a necessary and sufficient condition for internal robust exponential stability for the system (3).

Proposition 1: A necessary and sufficient condition for internal robust exponential stability in the mean square of the system (3) is that there exist symmetric positive-definite matrices \mathcal{P}_{ijk} and

positive scalars ε_{A_i} and ε_{B_j} , $i \in Z$, $j \in S$ and $k \in R$ such that:

$$\begin{aligned} & \tilde{\Lambda}_{ijk}^T \mathcal{P}_{ijk} + \mathcal{P}_{ijk} \tilde{\Lambda}_{ijk} + \mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \Theta_{ijk} + \varepsilon_{A_i} \mathcal{P}_{ijk} D_{A_i} D_{A_i}^T \mathcal{P}_{ijk} \\ & + \varepsilon_{B_j} \mathcal{P}_{ijk} D_{B_j} D_{B_j}^T \mathcal{P}_{ijk} + \varepsilon_{A_i}^{-1} E_{A_i}^T E_{A_i} + \varepsilon_{B_j}^{-1} C_2^T \mathcal{K}_k^T E_{B_j}^T E_{B_j} \mathcal{K}_k C_2 < 0 \end{aligned} \quad (10)$$

$\forall i \in Z$, $j \in S$ and $k \in R$, where

$$\Theta_{ijk} = \sum_{\substack{h \in Z \\ h \neq i}} \pi_{ih} \mathcal{P}_{hjk} + \sum_{\substack{l \in S \\ l \neq j}} \nu_{jl} \mathcal{P}_{ilk} + \sum_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \mathcal{P}_{ijv}$$

and

$$\tilde{\Lambda}_{ijk} = A_i + B_j \mathcal{K}_k C_2 - 0.5I \left(\sum_{\substack{h \in Z \\ h \neq i}} \pi_{ih} + \sum_{\substack{l \in S \\ l \neq j}} \nu_{jl} + \sum_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \right) \quad (11)$$

Proof: The proof of this proposition is easily deduced from theorems 1 and 2. \blacksquare

We are now able to present the following proposition which gives a nonlinear matrix inequalities characterization of compensators (φ_s) that internally robustly stabilize the closed-loop system in the mean square sense.

Proposition 2 (Necessary and sufficient condition): System (3) is internally robustly exponentially stable in the mean square if and only if there exist matrices $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, \mathcal{K}_k , $\bar{\Omega}_{ijk}$ and positive scalars ε_{A_i} and ε_{B_j} satisfying the following coupled matrix inequalities

$$\begin{bmatrix} \mathfrak{J}_{ijk} & \tilde{A}_{ijk} + B_j \mathcal{K}_k C_2 + \bar{\Omega}_{ijk} & \mathcal{X}_{ijk} \mathcal{W}_{ij}^T & \mathcal{X}_{ijk} E_{A_i}^T \\ * & -\mathcal{Z}_{ijk} & 0 & 0 \\ * & * & -\mathcal{X}_{ijk} & 0 \\ * & * & * & -\varepsilon_{A_i} \mathbb{I} \\ * & * & * & * \\ & & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) & \\ & & 0 & \\ & & 0 & \\ & & 0 & \\ & & -S_{ijk}(\mathcal{X}_{ijk}) & \end{bmatrix} < 0 \quad (12)$$

where

$$\mathfrak{J}_{ijk} = \mathcal{X}_{ijk} \bar{\mathcal{Z}}_{ijk} \mathcal{X}_{ijk} - \bar{\Omega}_{ijk} \mathcal{X}_{ijk} - \mathcal{X}_{ijk} \bar{\Omega}_{ijk}^T + \varepsilon_{A_i} D_{A_i} D_{A_i}^T + \varepsilon_{B_j} D_{B_j} D_{B_j}^T$$

\mathcal{Z}_{ijk} is any given positive-definite matrices and

$$\begin{cases} \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) = [\mathcal{R}1_{ijk}(\mathcal{X}_{ijk}), \mathcal{R}2_{ijk}(\mathcal{X}_{ijk}), \mathcal{R}3_{ijk}(\mathcal{X}_{ijk})] \\ \mathcal{R}1_{ijk}(\mathcal{X}_{ijk}) = [\alpha_{i1} \mathcal{X}_{ijk}, \dots, \alpha_{i(i-1)} \mathcal{X}_{ijk}, \alpha_{i(i+1)} \mathcal{X}_{ijk}, \dots, \alpha_{iz} \mathcal{X}_{ijk}] \\ \mathcal{R}2_{ijk}(\mathcal{X}_{ijk}) = [\beta_{j1} \mathcal{X}_{ijk}, \dots, \beta_{j(j-1)} \mathcal{X}_{ijk}, \beta_{j(j+1)} \mathcal{X}_{ijk}, \dots, \beta_{js} \mathcal{X}_{ijk}] \\ \mathcal{R}3_{ijk}(\mathcal{X}_{ijk}) = [\gamma_{k1} \mathcal{X}_{ijk}, \dots, \gamma_{k(k-1)} \mathcal{X}_{ijk}, \gamma_{k(k+1)} \mathcal{X}_{ijk}, \dots, \gamma_{kr} \mathcal{X}_{ijk}] \\ \alpha_{il} = \sqrt{\pi_{il}}; \beta_{jl} = \sqrt{\nu_{jl}}; \gamma_{kl} = \sqrt{\lambda_{kl}^{ij}} \\ S_{ijk}(\mathcal{X}_{ijk}) = -\text{diag} [S1_{ijk}(\mathcal{X}_{ijk}), S2_{ijk}(\mathcal{X}_{ijk}), S3_{ijk}(\mathcal{X}_{ijk})] \\ S1_{ijk}(\mathcal{X}_{ijk}) = [\mathcal{X}_{1jk}, \dots, \mathcal{X}_{(i-1)jk}, \mathcal{X}_{(i+1)jk}, \dots, \mathcal{X}_{zjk}] \\ S2_{ijk}(\mathcal{X}_{ijk}) = [\mathcal{X}_{1k}, \dots, \mathcal{X}_{(j-1)k}, \mathcal{X}_{(j+1)k}, \dots, \mathcal{X}_{sk}] \\ S3_{ijk}(\mathcal{X}_{ijk}) = [\mathcal{X}_{ij1}, \dots, \mathcal{X}_{ij(k-1)}, \mathcal{X}_{ij(k+1)}, \dots, \mathcal{X}_{ijr}] \\ \tilde{A}_{ijk} = A_i - 0.5I \left(\sum_{\substack{h \in Z \\ h \neq i}} \pi_{ih} + \sum_{\substack{l \in S \\ l \neq j}} \nu_{jl} + \sum_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \right) \\ \bar{\mathcal{Z}}_{ijk} = \mathcal{Z}_{ijk} + \varepsilon_{B_j}^{-1} C_2^T \mathcal{K}_k^T E_{B_j}^T E_{B_j} \mathcal{K}_k C_2 \end{cases}$$

Then, when (12) is feasible, the stabilizing output feedback control law is given by

$$u_k(t) = \mathcal{K}_k y(t)$$

Proof: Let us consider the matrix inequalities given by (10). The use of the reciprocal projection lemma with

$$\Psi = \mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \Theta_{ijk} + \varepsilon_{A_i} \mathcal{P}_{ijk} D_{A_i} D_{A_i}^T \mathcal{P}_{ijk} + \varepsilon_{B_j} \mathcal{P}_{ijk} D_{B_j} D_{B_j}^T \mathcal{P}_{ijk} + \varepsilon_{A_i}^{-1} E_{A_i}^T E_{A_i} + \varepsilon_{B_j}^{-1} C_2^T \mathcal{K}_k^T E_{B_j}^T E_{B_j} \mathcal{K}_k C_2 \quad (13)$$

and $S = \tilde{\Lambda}_{ijk}^T \mathcal{P}_{ijk}$ yields

$$\begin{bmatrix} \Psi_{ijk} + \mathcal{Z}_{ijk} - (\Omega_{ijk} + \Omega_{ijk}^T) S_{ijk}^T + \Omega_{ijk}^T \\ * \\ -\mathcal{Z}_{ijk} \end{bmatrix} < 0 \quad (14)$$

where \mathcal{Z}_{ijk} is any given positive-definite matrix, $\forall i \in Z$, $j \in S$ and $k \in R$.

Let us define $\mathcal{X}_{ijk} = \mathcal{P}_{ijk}^{-1}$ and $\bar{\Omega}_{ijk} = \mathcal{X}_{ijk} \Omega_{ijk}$, then by the congruence transformation

$$\begin{bmatrix} \mathcal{X}_{ijk} & 0 \\ 0 & \mathbb{I} \end{bmatrix}$$

and with a Schur complement operation with respect to the term

$$\mathcal{X}_{ijk} \left(\mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \Theta_{ijk} + \varepsilon_{A_i}^{-1} E_{A_i}^T E_{A_i} \right) \mathcal{X}_{ijk}$$

the inequality (14) in turn becomes

$$\begin{bmatrix} \mathfrak{J}_{ijk} & \tilde{A}_{ijk} + B_j \mathcal{K}_k C_2 + \bar{\Omega}_{ijk} & \mathcal{X}_{ijk} \mathcal{W}_{ij}^T & \mathcal{X}_{ijk} E_{A_i}^T \\ * & -\mathcal{Z}_{ijk} & 0 & 0 \\ * & * & -\mathcal{X}_{ijk} & 0 \\ * & * & * & -\varepsilon_{A_i} \mathbb{I} \\ * & * & * & * \\ & & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) & \\ & & 0 & \\ & & 0 & \\ & & 0 & \\ & & -S_{ijk}(\mathcal{X}_{ijk}) & \end{bmatrix} < 0 \quad (15)$$

Hence the proof is complete. \blacksquare

Remark 2: The condition of proposition 2 is nonlinear in the unknown variables \mathcal{X}_{ijk} and $\bar{\Omega}_{ijk}$ and hence not easily tractable by convex optimization techniques. However, the degree of freedom introduced by the variables \mathcal{Z}_{ijk} can be adequately used to provide an easily tractable sufficient condition for the stochastic stability in terms of an LMI feasibility problem. This is illustrated by the following proposition.

Proposition 3 (Sufficient condition): If there exist matrices $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, \mathcal{K}_k , $\bar{\Omega}_{ijk}$ and positive scalars ε_{A_i} and ε_{B_j} satisfying the following LMI

$$\begin{bmatrix} F_{ijk} & (\mu_{ijk} \bar{\mathcal{Y}}_{ijk} - \bar{\Omega}_{ijk}) & \mathcal{X}_{ijk} \mathcal{W}_{ij}^T & \mathcal{X}_{ijk} E_{A_i}^T & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) \\ * & -\mu_{ijk} \mathbb{I} & 0 & 0 & 0 \\ * & * & -\mathcal{X}_{ijk} & 0 & 0 \\ * & * & * & -\varepsilon_{A_i} \mathbb{I} & 0 \\ * & * & * & * & -S_{ijk} \\ * & * & * & * & * \\ & & \tilde{A}_{ijk} + B_j \mathcal{K}_k C_2 & 0 & \\ & & 0 & 0 & \\ & & 0 & 0 & \\ & & 0 & 0 & \\ & & 0 & 0 & \\ & & -\mu_{ijk} \mathbb{I} & C_2^T \mathcal{K}_k^T E_{B_j}^T & \\ & & * & -\varepsilon_{B_j} \mathbb{I} & \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} \mu_{ijk} \mathbb{I} & C_2^T \mathcal{K}_k^T E_{B_j}^T \\ * & \varepsilon_{B_j} \mathbb{I} \end{bmatrix} > 0 \quad (17)$$

where

$$F_{ijk} = \mu_{ijk} \mathbb{I} - \bar{\Omega}_{ijk} - \bar{\Omega}_{ijk}^T + \varepsilon_{A_i} D_{A_i} D_{A_i}^T + \varepsilon_{B_j} D_{B_j} D_{B_j}^T$$

and μ_{ijk} are arbitrary positive scalars. These free scalar parameters are introduced to reduce the conservatism of the sufficient condition for exponential stochastic stability. Then the system (φ) coupled with (φ_s) is internally robustly exponentially stable in the mean square.

Proof: Since, according to proposition 2, \mathcal{Z}_{ijk} can be any positive-definite matrix, we let $\mathcal{Z}_{ijk} = \mu_{ijk} \mathbb{I} - \varepsilon_{B_j}^{-1} C_2^T \mathcal{K}_k^T E_{B_j}^T E_{B_j} \mathcal{K}_k C_2$, $\forall i \in Z$, $j \in S$ and

$k \in R$. Then, using the Schur complement operation with respect to the term

$$\bar{\mathcal{X}}_{ijk} \bar{\mathcal{Z}}_{ijk} \bar{\mathcal{X}}_{ijk} - \bar{\Omega}_{ijk}^T \bar{\mathcal{X}}_{ijk} - \bar{\mathcal{X}}_{ijk} \bar{\Omega}_{ijk}$$

the inequality (12) can be further be written as

$$\begin{bmatrix} \Upsilon_{ijk} & (\mu_{ijk} \bar{\mathcal{Y}}_{ijk} - \bar{\Omega}_{ijk}) & \mathcal{X}_{ijk} \mathcal{W}_{ij}^T & \mathcal{X}_{ijk} E_{A_i}^T & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) \\ * & -\mu_{ijk} \mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\mathcal{X}_{ijk} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\varepsilon_{A_i} \mathbb{I} & \mathbf{0} \\ * & * & * & * & -\mathcal{S}_{ijk} \\ * & * & * & * & * \\ & \bar{A}_{ijk} + B_j \mathcal{K}_k C_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & -\mu_{ijk} \mathbb{I} & C_2^T \mathcal{K}_k^T E_{B_j}^T & \mathbf{0} & \mathbf{0} \\ * & * & -\varepsilon_{B_j} \mathbb{I} & * & * \end{bmatrix} < 0 \quad (18)$$

where

$$\Upsilon_{ijk} = -\mu_{ijk}^{-1} \bar{\Omega}_{ijk} \bar{\Omega}_{ijk}^T + \varepsilon_{A_i} D_{A_i} D_{A_i}^T + \varepsilon_{B_j} D_{B_j} D_{B_j}^T$$

then, using the relation

$$-\mu_{ijk}^{-1} \bar{\Omega}_{ijk} \bar{\Omega}_{ijk}^T \leq -\bar{\Omega}_{ijk} - \bar{\Omega}_{ijk} + \mu_{ijk} \mathbb{I}$$

yields the matrix inequality (16). Hence the proof is complete. \blacksquare

5. ROBUST \mathcal{H}_∞ CONTROL

In this section, we are concerned with the LMI characterization of compensators φ_s that robustly stochastically stabilize the system (3) and guarantee the following for all $w \in L^2[0, \infty)$, and for all admissible uncertainties:

$$\|z_\infty\|_{\mathcal{E}_2} = \mathcal{E} \left\{ \int_0^\infty z_{\infty t}^T z_{\infty t} dt \right\}^{1/2} < \mu \left[\|w\|_2^2 + a(x_0, \xi_0, \eta_0, \psi_0) \right]^{1/2} \quad (19)$$

where $\mu > 0$ is a prescribed level of disturbance attenuation to be achieved and $a(x_0, \xi_0, \eta_0, \psi_0)$ is a constant that depends on the initial conditions $(x_0, \xi_0, \eta_0, \psi_0)$. To this end, we need the auxiliary result given by the following proposition.

Proposition 4: If the system (3) is internally robustly exponentially stable in the mean square sense, then it is robustly stochastically stable.

Proof: The proof of this proposition follows the same arguments as for the proof of proposition 5.1 in (Aberkane *et al.* 2005b). \blacksquare

We are now able to state the sufficient condition for robust stochastic stability and \mathcal{H}_∞ performance for the certain AFTCSMP driven by a static output feedback controller.

Proposition 5: If there exist symmetric positive-definite matrices \mathcal{P}_{ijk} and positive scalars ε_{A_i} and ε_{B_j} , $i \in Z$, $j \in S$ and $k \in R$ such that

$$\begin{bmatrix} \Delta_{ijk} + \bar{C}_{1jk}^T \bar{C}_{1jk} & \bar{C}_{1jk}^T \bar{D}_{1ijk} + \mathcal{P}_{ijk} \bar{E}_{ijk} \\ * & \bar{D}_{1ijk}^T \bar{D}_{1ijk} - \mu^2 \mathbb{I} \end{bmatrix} = \Phi_{ijk} < 0 \quad (20)$$

$\forall i \in Z, j \in S$ and $k \in R$.

then the system (3) is robustly stochastically stable and satisfies

$$\|z_\infty\|_{\mathcal{E}_2} < \left[\mu^2 \|w\|_2^2 + x_0^T \mathcal{P}(\xi_0, \eta_0, \psi_0) x_0 \right]^{1/2}. \quad (21)$$

for all admissible uncertainties.

Proof The proof of this proposition follows the same arguments as for the proof of proposition 5.2 in (Aberkane *et al.* 2005b). \blacksquare

The \mathcal{H}_∞ constraints (19) can be rephrased in LMI form. This is illustrated by proposition 6, which gives an LMI characterization of static output feedback compensators (φ_s) that robustly stochastically stabilize the AFTCSMP and ensures (19).

Proposition 6: If there exist matrices $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, $\bar{\Omega}_{ijk}$, \mathcal{K}_k and positive scalars ε_{A_i} and ε_{B_j} such that the following LMIs are feasible

$$\begin{bmatrix} \Upsilon_{ijk} & (\mu_{ijk} \bar{\mathcal{X}}_{ijk} - \bar{\Omega}_{ijk}) & \begin{bmatrix} \bar{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk}) \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} & \begin{bmatrix} \mathcal{X}_{ijk} E_{A_i}^T \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ * & -\mu_{ijk} \mathbb{I} & \mathbf{0} & \mathbf{0} \\ * & * & -\bar{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} \\ * & * & * & -\varepsilon_{A_i} \mathbb{I} \\ * & * & * & * \\ * & * & * & * \\ & \Theta_{ijk} + \bar{\Omega}_{ijk} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & -\mu_{ijk} \mathbb{I} & \Pi_{ijk}^T \begin{bmatrix} C_2^T \mathcal{K}_k^T E_{B_j}^T \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} & \mathbf{0} \\ * & -\mathbb{I} & \mathbf{0} & \mathbf{0} \\ * & * & -\varepsilon_{B_j} \mathbb{I} & * \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} \mu_{ijk} \mathbb{I} & \Pi_{ijk}^T \begin{bmatrix} C_2^T \mathcal{K}_k^T E_{B_j}^T \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \\ * & \mathbb{I} \\ * & * & \varepsilon_{B_j} \mathbb{I} \end{bmatrix} > 0 \quad (23)$$

where

$$\Upsilon_{ijk} = \mu_{ijk} \mathbb{I} - \bar{\Omega}_{ijk} - \bar{\Omega}_{ijk}^T + \begin{bmatrix} \varepsilon_{A_i} D_{A_i} D_{A_i}^T + \varepsilon_{B_j} D_{B_j} D_{B_j}^T & \mathbf{0} \\ \mathbf{0} & -\mu^2 \mathbb{I} \end{bmatrix}$$

$$\begin{cases} \Pi_{ijk} = \begin{bmatrix} C_1 & \mathbf{0} \end{bmatrix} + D_{1j} \mathcal{K}_k \begin{bmatrix} C_2 & D_{2jk} \end{bmatrix} \\ \Theta_{ijk} = \begin{bmatrix} \bar{A}_{ijk} & E_{ij} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} B_j \\ \mathbf{0} \end{bmatrix} \mathcal{K}_k \begin{bmatrix} C_2 & D_{2ij} \end{bmatrix} \\ \bar{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk}) = \begin{bmatrix} \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) & \mathcal{X}_{ijk} \mathcal{W}_{ij} \\ \mathcal{S}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} \end{bmatrix} \\ \bar{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) = \begin{bmatrix} \mathcal{S}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} \\ \mathbf{0} & \mathcal{X}_{ijk} \end{bmatrix} \\ \bar{\mathcal{X}}_{ijk} = \begin{bmatrix} \mathcal{X}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{bmatrix} \end{cases} \quad (24)$$

and μ_{ijk} are positive scalars $\forall i \in Z, j \in S$ and $k \in R$. Then the system (3) is robustly stochastically stable and satisfies

$$\|z_\infty\|_{\mathcal{E}_2} < \left[\mu^2 \|w\|_2^2 + x_0^T \mathcal{P}(\xi_0, \eta_0, \psi_0) x_0 \right]^{1/2}. \quad (25)$$

for all admissible uncertainties. Then, the corresponding output feedback control law is given by

$$u_k(t) = \mathcal{K}_k y(t)$$

Proof: The proof of this proposition follows the same arguments as for the proof of proposition 3. \blacksquare

Remark 4 If the synthesis objective is to compute the controller that ensure the robust stochastic stability and at the same time guarantee the minimum disturbance rejection level, then this controller can be obtained by solving the following optimization problem:

$$\mathcal{O}_r : \begin{cases} \delta > 0, \mathcal{V}_{ijk} = \mathcal{V}_{ijk}^T > 0, \mathcal{K}_k, \bar{\Omega}_{ijk}, \varepsilon_{A_i}, \varepsilon_{B_j} \\ \text{s.t. : (22), (23)} \end{cases} \delta$$

Where the LMIs in the constraints are obtained by replacing μ^2 by δ . This leads to the following Corollary:

Corollary 1 Let $\delta > 0$, $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, \mathcal{K}_k , and $\bar{\Omega}_{ijk}$, ε_{A_i} , ε_{B_j} be the solution of the optimization problem \mathcal{O}_r . Then, the controller (2)

robustly stochastically stabilizes the AFTCSMP we are considering and moreover the closed loop system satisfies the disturbance rejection of level $\sqrt{\delta}$.

Numerical example: To illustrate the theoretical results presented above, let us consider a system with one possible fault in the plant components, *i.e.* $S = \{1, 2\}$. The failure process is assumed to have Markovian transition characteristics. The FDI process is also Markovian with two states $R = \{1, 2\}$. The following numerical values are used

$$A = \begin{cases} A_1 = \begin{bmatrix} 0.9749 & -0.3257 & 0.2333 \\ -2.3779 & -2.0122 & 0.6464 \\ -1.0923 & 1.5677 & -1.129 \end{bmatrix} & \text{Healthy} \\ A_2 = \begin{bmatrix} 0.1970 & 0.7925 & -1.1087 \\ 1.6969 & 0.6034 & 2.1442 \\ 0.7260 & -0.0584 & -1.3528 \end{bmatrix} & \text{Faulty} \end{cases}; B = \begin{bmatrix} 1 & 1 \\ 0.25 & 4 \\ 0 & 0.5 \end{bmatrix};$$

$$E = \begin{cases} E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \\ E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \end{cases}; W = \begin{cases} W_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \\ W_2 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \end{cases};$$

$$D_2 = \begin{cases} D_{21} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \\ D_{22} = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.6 \end{bmatrix} \end{cases}; \begin{cases} C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ C_2 = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \end{cases}; D_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};$$

$$\begin{cases} D_{A_1} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix} \\ D_{A_2} = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix} \end{cases} \begin{cases} E_{A_1} = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.5 \end{bmatrix} \\ E_{A_2} = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.5 \end{bmatrix} \end{cases}$$

$$\begin{cases} D_B = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.1 \end{bmatrix} \\ E_B = \begin{bmatrix} 0.2 & 0.5 \end{bmatrix} \end{cases}$$

$$[\pi_{ij}] = \begin{bmatrix} -0.005 & 0.005 \\ 0.01 & -0.01 \end{bmatrix}; \lambda_{ij}^1 = \begin{bmatrix} -0.1 & 0.1 \\ 0.9 & -0.9 \end{bmatrix}; \lambda_{ij}^2 = \begin{bmatrix} -1 & 1 \\ 0.1 & -0.1 \end{bmatrix}.$$

Then, solving the optimization problem \mathcal{O}_r , we get the following parameters

$$\mu = 1.2715, \varepsilon_{A_1} = 10.6058, \varepsilon_{A_2} = 0.2610, \varepsilon_B = 0.4523;$$

$$\mathcal{K}_1 = \begin{bmatrix} -4.3111 & 1.4865 \\ 0.4302 & -1.0211 \end{bmatrix}; \mathcal{K}_2 = \begin{bmatrix} -4.3903 & 1.6799 \\ 0.3570 & -0.9245 \end{bmatrix};$$

6. CONCLUSION

This paper has introduced an LMI approach to the robust static output feedback \mathcal{H}_∞ control for linear uncertain continuous time AFTCSMP in noisy environment. We have derived some linear matrix inequalities whose solutions indicate the achievability of the desired control problem; *i.e.* we have shown that the robust \mathcal{H}_∞ control problematic can be recast as a convex optimization problem under constraints of LMIs which can be solved effectively using the recently developed LMI tool.

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