Output Feedback H_infinity Control of a Class of Stochastic Hybrid Systems with Wiener Process via Convex Analysis

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Output Feedback $\mathcal{H}_\infty$ Control of a Class of Stochastic Hybrid Systems with Wiener Process via Convex Analysis

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3rd April 2006

This paper deals with the problem of $\mathcal{H}_\infty$ control, via static output feedback, of continuous time Active Fault Tolerant Control Systems with Markovian Parameters (AFTCSMP) with Wiener process. The above problematic is addressed under a convex programming approach. Indeed, the fundamental tool in the analysis is an LMI (*Linear Matrix Inequalities*) characterization of output feedback compensators that *stochastically stabilize* the closed loop system and ensure $\mathcal{H}_\infty$ performances. A numerical example is presented to illustrate the theoretical results.

Keywords: Active Fault Tolerant Control - Stochastic Hybrid systems - Markovian jumping parameters - $\mathcal{H}_\infty$ control - Output Feedback - Lyapunov stability - LMI.

1 Introduction

Fault tolerant control systems (FTCS) have been developed in order to achieve high levels of reliability and performance in situations where the controlled system can have potentially damaging effects on the environment if failures of its components take place. FTCS have been a subject of great practical importance, which have attracted a lot of interest for the last three decades. A bibliographical review on reconfigurable fault tolerant control systems can be found in [24].
Active fault tolerant control systems are feedback control systems that reconfigure the control law in real time based on the response from an automatic fault detection and identification (FDI) scheme. The dynamic behaviour of this class of systems is governed by stochastic differential equations and can be viewed as a general stochastic hybrid system [23]. In the literature, there are two major classes of stochastic hybrid systems: jump linear systems (JLS) and active fault tolerant control systems with Markovian parameters (AFTCSMP). In JLS, a single jump process is used to describe the random variations affecting the system parameters. This process is represented by a finite state Markov chain and is called the plant regime mode. The theory of stability, optimal control and $\mathcal{H}_2/\mathcal{H}_\infty$ control, as well as important applications of such systems, can be found in several papers in the current literature, for instance in [5, 6, 7, 9, 10, 11, 14, 15, 20].
The AFTCSMP model allows to study the impact of time delays and errors in the decisions of the FDI process. That is, it does not assume instantaneous and perfect failure detection. In this class of hybrid systems, two random processes are defined: the first random process represents system components failures and the second random process represents the FDI process used to reconfigure the control law. This model was proposed by Srichander and Walker [23]. Necessary and sufficient conditions for stochastic stability of AFTCSMP were developed for a single component failure (actuator failures). The problem of stochastic stability of AFTCSMP in the presence of noise, parameter

uncertainties, detection errors, detection delays and actuator saturation limits has also been investigated in [16, 18, 19]. Another issue related to the synthesis of fault tolerant control laws was also addressed by [17, 21, 22]. In [17], the authors designed an optimal control law for AFTCSMP using the matrix minimum principle to minimize an equivalent deterministic cost function. The problem of $\mathcal{H}_\infty$ and robust $\mathcal{H}_\infty$ control was treated in [21, 22] for both continuous and discrete time AFTCSMP. The authors showed that the state feedback control problem can be solved in terms of the solutions of a set of coupled Riccati inequalities. The dynamic output feedback counterpart was treated by [1, 2] in a convex programming framework. Indeed, the authors provide an LMI characterization of dynamical compensators that stochastically stabilize (robustly stabilize) the AFTCSMP and ensure $\mathcal{H}_\infty$ (robust $\mathcal{H}_\infty$) constraints. Nevertheless, in [1, 2, 19, 21, 22], the authors make the assumption that the controllers must access both failures and FDI processes. However, in practice, the different random failures affecting the system are not directly measurable but rather can only be monitored by the FDI scheme. This fact makes these controllers inapplicable in practical AFTCSMP. In addition, it is important to mention that the design problem in the framework of AFTCSMP is more complicated when comparing with JLS formulation. This is due, particulary, to the fact that the controller only depends on the FDI process *i.e.* the number of controllers to be designed is less than the total number of the closed loop systems modes by combining both failure an FDI processes. The design problem involves searching feasible solutions of a problem where there are more constraints than variables to be solved. In this work, the assumption on the availability of failure processes, for the synthesis purposes, is stressed.

In this paper, we are concerned with the problem of static output feedback $\mathcal{H}_\infty$ control of an AFTCSMP with Wiener process via convex analysis, which has shown to be a powerful tool to derive numerical algorithms for several important control problems. The first problematic we consider in this paper is the output feedback *stochastic stabilization* of the AFTCSMP with Wiener process subject to multiple failure processes. It is shown that the necessary and sufficient conditions for the *internal exponential stability* in the mean square sense can be written in terms of an nonlinear matrix inequality feasibility problem. Then, an LMI relaxation scheme is presented to eliminate the nonlinearities, yielding to a new easily tractable sufficient condition. Having obtained this result, we can move on the control problem and write the output feedback $\mathcal{H}_\infty$ control problem of continuous time AFTCSMP in terms of an LMI optimization problem. The convex approach naturally leads to powerful numerical algorithms to solve these problematic.

This paper is organized as follows: section 2 describes the dynamical model of the system with appropriately defined random processes. A brief summary of basic stochastic terms, results and definitions are given in section 3. Section 4 derives the necessary and sufficient conditions for the stochastic exponential stability in the mean square sense, and the LMI characterization of the static output feedback compensators. Section 5 considers the $\mathcal{H}_\infty$ control problem for the output feedback system via LMI optimization problems. In Section 6, numerical examples are given to illustrate the effectiveness of the theoretical results. Finally, a conclusion is given in section 7.

Notations. The notations in this paper are quite standard. The notation $X \geq Y$ ($X > Y$, respectively), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (positive definite, respectively); $\mathbb{I}$ and $\mathbf{0}$ are identity and zero matrices of appropriate dimensions, respectively; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure P; $L^2[0,\infty)$ stands for the space of square-integrable vector functions over the interval $[0,\infty)$; $\|\cdot\|$ refers to either the Euclidean vector norm or the matrix norm, which is the operator norm induced by the standard vector norm; $\|\cdot\|_2$ stands for the norm in $L^2[0,\infty)$; while $\|\cdot\|_{\mathcal{E}_2}$ denotes the norm in $L^2((\Omega,\mathcal{F},P),[0,\infty))$; $(\Omega,\mathcal{F},P)$ is a probability space.

2 Dynamical Model of the AFTCSMP with Wiener Process

Consider general FTCS as shown in Fig. 1. The dynamical model of the AFTCSMP with Wiener process, defined in a fundamental probability space $(\Omega,\mathcal{F},P)$, is described by the following differential equations:

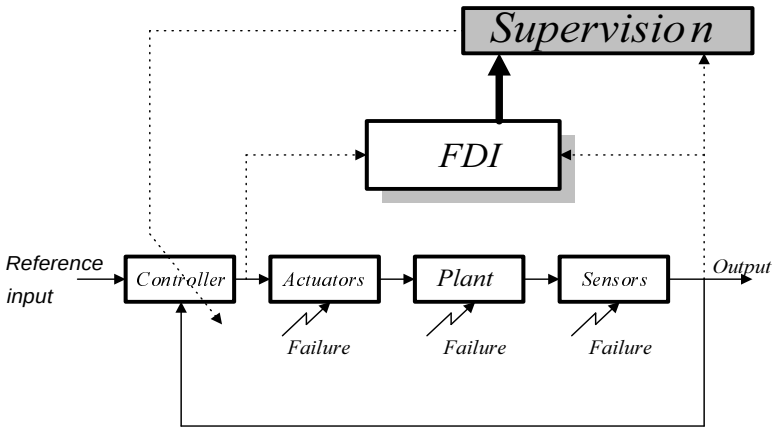

Figure 1: General schematic diagram for fault tolerant control systems

$$\varphi : \begin{cases} dx(t) = A(\xi(t))x(t)dt + B(\eta(t))u(y(t),\psi(t),t)dt + E(\xi(t),\eta(t))w(t)dt + \mathcal{W}(\xi(t),\eta(t))x(t)d\varpi(t) \\ y(t) = C_2 x(t) + D_2(\xi(t),\eta(t))w(t) \\ z_\infty(t) = C_1 x(t) + D_1(\eta(t))u(y(t),\psi(t),t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(y(t),\psi(t),t) \in \mathbb{R}^r$ is the system input, $y(t) \in \mathbb{R}^q$ is the system measured output, $z_\infty(t) \in \mathbb{R}^p$ is the controlled output, $w(t) \in \mathbb{R}^m$ is the system external disturbance, $\xi(t)$, $\eta(t)$ and $\psi(t)$ represent the plant component failure process, the actuator failure process and the FDI process, respectively. $\xi(t)$, $\eta(t)$ and $\psi(t)$ are separable and mesurable Markov processes with finite state spaces $Z = \{1,2,...,z\}$, $S = \{1,2,...,s\}$ and $R = \{1,2,...,r\}$, respectively. $\varpi(t)$ is a standard Wiener process that is assumed to be independent of the Markov processes. The matrices $A(\xi(t))$, $B(\eta(t))$, $E(\xi(t),\eta(t))$, $D_2(\xi(t),\eta(t))$, $D_1(\eta(t))$ and $\mathcal{W}(\xi(t),\eta(t))$ are properly dimensioned matrices which depend on random parameters. The system disturbance $w(t)$ is assumed to belong to $L^2[0,\infty)$ *i.e.*

$$\| w \|_2 = \left\{ \int_0^\infty w^T(t)w(t)dt \right\}^{1/2} < \infty$$

This implies that the disturbance has finite energy.
In AFTCS, we consider that the control law is only a function of the mesurable FDI process $\psi(t)$. Therefore, we introduce a static output feedback compensator (φ_s) of the form:

$$\varphi_s : \Big\{ u(t) = \mathcal{K}(\psi(t))y(t) \tag{2}$$

Applying the controller φ_s to the AFTCSMP φ, we obtain the following closed loop system:

$$\varphi_{cl} : \begin{cases} dx(t) = \bar{A}(\xi(t),\eta(t),\psi(t))x(t)dt + \bar{E}(\xi(t),\eta(t),\psi(t))w(t)dt + \mathcal{W}(\xi(t),\eta(t))x(t)d\varpi(t) \\ y(t) = C_2 x(t) + D_2(\xi(t),\eta(t))w(t) \\ z_\infty(t) = \bar{C}_1(\eta(t),\psi(t))x(t) + \bar{D}_1(\xi(t),\eta(t),\psi(t))w(t) \end{cases} \tag{3}$$

where

$$\begin{bmatrix} \bar{A}(\xi(t),\eta(t),\psi(t)) & \bar{E}(\xi(t),\eta(t),\psi(t)) \\ \bar{C}_1(\eta(t),\psi(t)) & \bar{D}_1(\xi(t),\eta(t),\psi(t)) \end{bmatrix} = \begin{bmatrix} A(\xi(t)) & E(\xi(t),\eta(t)) \\ C_1 & 0 \end{bmatrix} + \begin{bmatrix} B(\eta(t)) \\ D_1(\eta(t)) \end{bmatrix} \mathcal{K}(\psi(t)) \begin{bmatrix} C_2 & D_2(\xi(t),\eta(t)) \end{bmatrix}$$

For notational simplicity, we will denote $A(\xi(t)) = A_i$ when $\xi(t) = i \in Z$, $B(\eta(t)) = B_j$ and $D_1(\eta(t)) = D_{1j}$ when $\eta(t) = j \in S$, $E(\xi(t),\eta(t)) = E_{ij}$, $D_2(\xi(t),\eta(t)) = D_{2ij}$ and $\mathcal{W}(\xi(t),\eta(t)) = \mathcal{W}_{ij}$ when $\xi(t) = i \in Z, \eta(t) = j \in S$ and $\mathcal{K}(\psi(t)) = \mathcal{K}_k$ when $\psi(t) = k \in R$. We also denote $x(t) = x_t$, $y(t) = y_t$, $z_\infty(t) = z_{\infty t}$, $w(t) = w_t$, $\xi(t) = \xi_t$, $\eta(t) = \eta_t$, $\psi(t) = \psi_t$ and the initial conditions $x(t_0) = x_0$, $\xi(t_0) = \xi_0$, $\eta(t_0) = \eta_0$ and $\psi(t_0) = \psi_0$.

The FDI and the Failure Processes:

ξ_t, η_t and ψ_t being homogeneous Markov processes with finite state spaces, we can define the transition probability of the plant components failure process as [19, 23]:

$$\begin{cases} p_{ij}(\Delta t) = \pi_{ij}\Delta t + o(\Delta t) \\ p_{ii}(\Delta t) = 1 - \sum\limits_{i \neq j} \pi_{ij}\Delta t + o(\Delta t) \end{cases} ;\ i, j \in Z$$

The transition probability of the actuator failure process is given by:

$$\begin{cases} p_{kl}(\Delta t) = \nu_{kl}\Delta t + o(\Delta t) \\ p_{kk}(\Delta t) = 1 - \sum\limits_{k \neq l} \nu_{kl}\Delta t + o(\Delta t) \end{cases} ;\ k, l \in S$$

where π_{ij} is the plant components failure rate, and ν_{kl} is the actuator failure rate. Δt is the infinitesimal transition time interval and $o(\Delta t)$ is composed of infinitesimal terms of order higher than that of Δt.
Given that $\xi = k$ and $\eta = l$, the conditional transition probability of the FDI process $\psi(t)$ is:

$$\begin{cases} p_{iv}^{kl}(\Delta t) = \lambda_{iv}^{kl}\Delta t + o(\Delta t) \\ p_{ii}^{kl}(\Delta t) = 1 - \sum\limits_{i \neq v} \lambda_{iv}^{kl}\Delta t + o(\Delta t) \end{cases} ;\ i, v \in R$$

Here, λ_{iv}^{kl} represents the transition rate from i to v for the Markov process $\psi(t)$ conditioned on $\xi = k \in Z$ and $\eta = l \in S$. Depending on the values of i, $v \in R$, $k \in Z$ and $l \in S$, various interpretations, such as rate of false detection and isolation, rate of correct detection and isolation, false alarm recovery rate, etc, can be given to λ_{iv}^{kl} [19, 23].

3 Basic Definitions and Results

In this section, we will first give some basic definitions related to stochastic stability notions and then we will summarize some results about exponential stability in the mean square sense of the AFTCSMP with Wiener process. Without loss of generality, we assume that the equilibrium point, $x = 0$, is the solution at which stability properties are examined.
Under the assumption that the system (φ) coupled with (φ_s) satisfies the global Lipchitz condition, the solution x_t determines a family of unique continuous stochastic processes, one for each choice of the random variable x_0. The joint process $\{x_t, \xi_t, \eta_t, \psi_t\}$ is a Markov process.

3.1 Stochastic Lyapunov Function

A fundamental tool in the analysis of the stability of stochastic systems is the stochastic Lyapunov function which is used to describe the stability behavior without explicit solution of the differential equation.
Definition 1 [23]: *The random function $\vartheta(x_t, \eta_t, \xi_t, \psi_t, t)$ of the joint Markov process $\{x_t, \eta_t, \xi_t, \psi_t\}$ qualifies as a stochastic Lyapunov function candidate if the following conditions hold for some fixed $\varepsilon < \infty$:*

a) *The function $\vartheta(x_t, \eta_t, \xi_t, \psi_t, t)$ is positive definite and continuous in x_t and t in the open set $O_\varepsilon = \{x_t : \vartheta(x_t, \eta_t, \xi_t, \psi_t, t) < \varepsilon\}$ $\forall \eta_t \in S, \forall \xi_t \in Z, \forall \psi_t \in R$ and $\forall t \geq t_0$, and $\vartheta(x_t, \eta_t, \xi_t, \psi_t, t) = 0$ only if $x_t = 0$. (The function $\vartheta(x_t, \eta_t, \xi_t, \psi_t, t)$ is said to be positive definite if $\vartheta(x_t, \eta_t, \xi_t, \psi_t, t) \geq W(x_t)$ $\forall \eta_t \in S, \forall \xi_t \in Z, \forall \psi_t \in R$ and $\forall t \geq t_0$, where $W(x_t)$ is positive definite in the sense of Lyapunov).*

b) *The joint Markov process $\{x_t, \eta_t, \xi_t, \psi_t\}$ is defined until $t = \tau_\varepsilon$ where $\tau_\varepsilon = \inf\{t : x_t \notin O_\varepsilon\}$. If $x_t \in O_\varepsilon \quad \forall t < \infty$, then $\tau_\varepsilon = \infty$.*

c) *The function $\vartheta(x_t, \eta_t, \xi_t, \psi_t, t)$ is in the domain of $\mathcal{L}$ where $\mathcal{L}$ is the weak infinitesimal operator of the joint Markov process $\{x_{\tau_t}, \eta_{\tau_t}, \xi_{\tau_t}, \psi_{\tau_t}\}$ and $\tau_t = \min(t, \tau_\varepsilon)$.*

The definition of the weak infinitesimal operator is given as follows:
Definition 2 [23]: *A bounded function $f(\zeta)$ is said to be in the domain of the weak infinitesimal operator $\mathcal{L}$ of the random process $\zeta(t)$ if the limit*

$$\lim_{\tau \to 0} \frac{\mathcal{E}\{f(\zeta(t+\tau))|\zeta(t)\} - f(\zeta(t))}{\tau} = \mathcal{L}f(\zeta) = h(\zeta)$$

exists pointwise in $\mathbb{R}$ and satisfies,

$$\lim_{\tau \to 0} \mathcal{E}\{h(\zeta(t+\tau))|\zeta(t)\} = h(\zeta(t))$$

If we generalize definition 2 to time varying functions $f(\zeta, t)$, then we have

$$\mathcal{L}f(\zeta, t) = \frac{\partial}{\partial t} f(\zeta, t) + h(\zeta, t)$$

In general, $\mathcal{L}f(\zeta)$ is interpreted as the average time rate of change of the process $f(\zeta)$ at time t given that $\zeta(t) = \zeta$.

3.2 Stochastic Stability

System (3) is said to be

(i) stochastically stable (SS) if there exists a finite positive constant $K(x_0, \xi_0, \eta_0, \psi_0)$ such that the following holds for any initial conditions $(x_0, \xi_0, \eta_0, \psi_0)$:

$$\mathcal{E}\left\{\int_0^\infty \| x_t \|^2 \, dt\right\} \leq K(x_0, \xi_0, \eta_0, \psi_0) \tag{4}$$

(ii) internally exponentially stable in the mean square sense if it is exponentially stable in the mean square sense for $w_t = 0$, i.e. for any ξ_0, η_0, ψ_0 and some $\gamma(\xi_0, \eta_0, \psi_0)$, there exists two numbers $a > 0$ and $b > 0$ such that when $\|x_0\| \leq \gamma(\xi_0, \eta_0, \psi_0)$, the following inequality holds $\forall t \geq t_0$ for all solution of (3) with initial condition x_0:

$$\mathcal{E}\left\{\|x_t\|^2\right\} \leq b\|x_0\|^2 \exp\left[-a(t - t_0)\right] \tag{5}$$

The following theorem gives a sufficient condition for internal exponential stability in the mean square sense for the system (φ) coupled with (φ_s).
Theorem 1: *The solution $x = 0$ of the system (φ) coupled with (φ_s) is internally exponentially stable in the mean square for $t \geq t_0$ if there exists a Lyapunov function $\vartheta(x_t, \xi_t, \eta_t, \psi_t, t)$ such that*

$$K_1\|x_t\|^2 \leq \vartheta(x_t, \xi_t, \eta_t, \psi_t, t) \leq K_2\|x_t\|^2 \tag{6}$$

and

$$\mathcal{L}\vartheta(x_t, \xi_t, \eta_t, \psi_t, t) \leq -K_3\|x_t\|^2 \tag{7}$$

for some positive constants K_1, K_2 and K_3.
A necessary condition for internal exponential stability in the mean square for the system (φ) coupled with (φ_s) is given by theorem 2.
Theorem 2: *If the solution $x = 0$ of the system (φ) coupled with (φ_s) is internally exponentially stable in the mean square, then for any given quadratic positive definite function $W(x_t, \xi_t, \eta_t, \psi_t, t)$ in the variables x which is bounded and continuous $\forall t \geq t_0$, $\forall \xi_t \in Z$, $\forall \eta_t \in S$ and $\forall \psi_t \in R$, there exists a quadratic positive definite function $\vartheta(x_t, \xi_t, \eta_t, \psi_t, t)$ in x such that $\mathcal{L}\vartheta(x_t, \xi_t, \eta_t, \psi_t, t) =$*

$-W(x_t, \xi_t, \eta_t, \psi_t, t)$.

Remark 1: The proofs of these theorems follow the same arguments as in [19, 23] for their proposed stochastic Lyapunov functions, so they are not shown in this paper to avoid repetition.

We conclude this section by recalling the following lemmas which will be useful for the proof of our main results in the next sections.

Lemma 1 [19]: Let G, M, N be real matrices of appropriate dimensions. Then, for any $\gamma > 0$, and for all the functional matrices satisfying $M^T(t)M(t) \leq \mathbb{I}$, we have:

$$2x^T PGM(t)Nx \leq \gamma x^T PGG^T Px + (1/\gamma)x^T N^T Nx$$

Lemma 2 (*Schur complement*) [8]: Given constant matrices M, L, Q of appropriate dimensions where M and Q are symmetric, then $Q > 0$ and $M + L^T Q^{-1} L < 0$ if and only if:

$$\begin{bmatrix} M & L^T \\ L & -Q \end{bmatrix} < 0$$

Lemma 3 (*Reciprocal Projection Lemma*) [4]: Let P be any given positive-definite matrix. The following statements are equivalent:

1) $\Psi + S + S^T < 0$;

2) the LMI problem
$$\begin{bmatrix} \Psi + P - (W + W^T) & S^T + W^T \\ S + W & -P \end{bmatrix} < 0$$
is feasible with respect to W.

4 Stochastic Stabilization

In this section, we will first derive a necessary and sufficient condition for the internal exponential stability in the mean square of the system (3). The synthesis condition thus obtained is formulated as coupled matrix inequalities feasibility problem which is non linear in the design variables and hence not easily tractable by convex optimization techniques. Then, an LMI relaxation scheme is presented to eliminate the nonlinearities, yielding a new easily tractable sufficient condition.
The following proposition gives a necessary and sufficient condition for internal exponential stability in the mean square sense for the system (3).
Proposition 1: A necessary and sufficient condition for internal exponential stability in the mean square of the system (3) is that there exists symmetric positive-definite matrices $\mathcal{P}_{ijk}$, $i \in Z$, $j \in S$ and $k \in R$ such that:

$$\tilde{\Lambda}_{ijk}^T \mathcal{P}_{ijk} + \mathcal{P}_{ijk}\tilde{\Lambda}_{ijk} + \mathcal{W}_{ij}^T \mathcal{P}_{ijk}\mathcal{W}_{ij} + \sum_{\substack{h \in Z \\ h \neq i}} \pi_{ih}\mathcal{P}_{hjk} + \sum_{\substack{l \in S \\ l \neq j}} \nu_{jl}\mathcal{P}_{ilk} + \sum_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij}\mathcal{P}_{ijv} < 0 \tag{8}$$

$\forall i \in Z$, $j \in S$ and $k \in R$, where

$$\tilde{\Lambda}_{ijk} = A_i + B_j \mathcal{K}_k C_2 - 0.5I \left(\sum_{\substack{h \in Z \\ h \neq i}} \pi_{ih} + \sum_{\substack{l \in S \\ l \neq j}} \nu_{jl} + \sum_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \right) \tag{9}$$

□

Proof: The proof of this proposition is easily deduced from theorems 1 and 2. ■

We are now able to present the following proposition which gives a nonlinear matrix inequalities characterization of compensators (φ_s) that internally stabilize the closed-loop system in the mean square sense.

Proposition 2 (Necessary and sufficient condition): System (3) is exponentially stable in the mean square If and only if there exists matrices $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, $\mathcal{K}_k$, and $\overline{\Omega}_{ijk}$ satisfying the following coupled matrix inequalities

$$\begin{bmatrix} \mathcal{X}_{ijk}\mathcal{Z}_{ijk}\mathcal{X}_{ijk} - \overline{\Omega}_{ijk}\mathcal{X}_{ijk} - \mathcal{X}_{ijk}\overline{\Omega}_{ijk}^T & \tilde{A}_{ijk} + B_j\mathcal{K}_kC_2 + \overline{\Omega}_{ijk} & \mathcal{X}_{ijk}\mathcal{W}_{ij}^T & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) \\ \star & -\mathcal{Z}_{ijk} & \mathbf{0} & \mathbf{0} \\ \star & \star & -\mathcal{X}_{ijk} & \mathbf{0} \\ \star & \star & \star & -\mathcal{S}_{ijk}(\mathcal{X}_{ijk}) \end{bmatrix} < 0 \quad (10)$$

where $\mathcal{Z}_{ijk}$ are any given positive-definite matrices and

$$\begin{cases} \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) = [\mathcal{R}1_{ijk}(\mathcal{X}_{ijk}), \mathcal{R}2_{ijk}(\mathcal{X}_{ijk}), \mathcal{R}3_{ijk}(\mathcal{X}_{ijk})] \\ \mathcal{R}1_{ijk}(\mathcal{X}_{ijk}) = \left[\alpha_{i1}\mathcal{X}_{ijk}, ...\alpha_{i(i-1)}\mathcal{X}_{ijk}, \alpha_{i(i+1)}\mathcal{X}_{ijk}, ..., \alpha_{iz}\mathcal{X}_{ijk}\right] \\ \mathcal{R}2_{ijk}(\mathcal{X}_{ijk}) = \left[\beta_{j1}\mathcal{X}_{ijk}, ...\beta_{j(j-1)}\mathcal{X}_{ijk}, \beta_{j(j+1)}\mathcal{X}_{ijk}, ..., \beta_{js}\mathcal{X}_{ijk}\right] \\ \mathcal{R}3_{ijk}(\mathcal{X}_{ijk}) = \left[\gamma_{k1}\mathcal{X}_{ijk}, ...\gamma_{k(k-1)}\mathcal{X}_{ijk}, \gamma_{k(k+1)}\mathcal{X}_{ijk}, ..., \gamma_{kr}\mathcal{X}_{ijk}\right] \\ \alpha_{il} = \sqrt{\pi_{il}}; \beta_{jl} = \sqrt{\nu_{jl}}; \gamma_{kl} = \sqrt{\lambda_{kl}^{ij}} \\ \mathcal{S}_{ijk}(\mathcal{X}_{ijk}) = -\mathrm{diag}\left[\mathcal{S}1_{ijk}(\mathcal{X}_{ijk}), \mathcal{S}2_{ijk}(\mathcal{X}_{ijk}), \mathcal{S}3_{ijk}(\mathcal{X}_{ijk})\right] \\ \mathcal{S}1_{ijk}(\mathcal{X}_{ijk}) = \left[\mathcal{X}_{1jk}, ..., \mathcal{X}_{(i-1)jk}, \mathcal{X}_{(i+1)jk}, ..., \mathcal{X}_{zjk}\right] \\ \mathcal{S}2_{ijk}(\mathcal{X}_{ijk}) = \left[\mathcal{X}_{i1k}, ..., \mathcal{X}_{i(j-1)k}, \mathcal{X}_{i(j+1)k}, ..., \mathcal{X}_{isk}\right] \\ \mathcal{S}3_{ijk}(\mathcal{X}_{ijk}) = \left[\mathcal{X}_{ij1}, ..., \mathcal{X}_{ij(k-1)}, \mathcal{X}_{ij(k+1)}, ..., \mathcal{X}_{ijr}\right] \\ \tilde{A}_{ijk} = A_i - 0.5\mathbb{I}\left(\sum\limits_{\substack{h\in Z \\ h\neq i}} \pi_{ih} + \sum\limits_{\substack{l\in S \\ l\neq j}} \nu_{jl} + \sum\limits_{\substack{v\in R \\ v\neq k}} \lambda_{kv}^{ij}\right) \end{cases}$$

Then, when (10) is feasible, the stabilizing output feedback control law is given by

$$u_k(t) = \mathcal{K}_k y(t)$$

□

Proof: Let us consider the matrix inequalities given by (8). The use of the reciprocal projection lemma with $\Psi = \left(\mathcal{W}_{ij}^T\mathcal{P}_{ijk}\mathcal{W}_{ij} + \sum\limits_{\substack{h\in Z \\ h\neq i}} \pi_{ih}\mathcal{P}_{hjk} + \sum\limits_{\substack{l\in S \\ l\neq j}} \nu_{jl}\mathcal{P}_{ilk} + \sum\limits_{\substack{v\in R \\ v\neq k}} \lambda_{kv}^{ij}\mathcal{P}_{ijv}\right)$, and $S = \tilde{\Lambda}_{ijk}^T\mathcal{P}_{ijk}$ yields

$$\begin{bmatrix} \mathcal{W}_{ij}^T\mathcal{P}_{ijk}\mathcal{W}_{ij} + \sum\limits_{\substack{h\in Z \\ h\neq i}} \pi_{ih}\mathcal{P}_{hjk} + \sum\limits_{\substack{l\in S \\ l\neq j}} \nu_{jl}\mathcal{P}_{ilk} + \sum\limits_{\substack{v\in R \\ v\neq k}} \lambda_{kv}^{ij}\mathcal{P}_{ijv} + \mathcal{Z}_{ijk} - (\Omega_{ijk} + \Omega_{ijk}^T) & \mathcal{P}_{ijk}\tilde{\Lambda}_{ijk} + \Omega_{ijk} \\ \star & -\mathcal{Z}_{ijk} \end{bmatrix} < 0 \quad (11)$$

where $\mathcal{Z}_{ijk}$ are any given positive-definite matrices, $\forall i \in Z$, $j \in S$ and $k \in R$.
Let us define $\mathcal{X}_{ijk} = \mathcal{P}_{ijk}^{-1}$ and $\overline{\Omega}_{ijk} = \mathcal{X}_{ijk}\Omega_{ijk}$, then by the congruence transformation

$$\begin{bmatrix} \mathcal{X}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{bmatrix}$$

and with a Schur complement operation with respect to the term

$$\mathcal{X}_{ijk}\left(\mathcal{W}_{ij}^T\mathcal{P}_{ijk}\mathcal{W}_{ij} + \sum_{\substack{h\in Z \\ h\neq i}} \pi_{ih}\mathcal{P}_{hjk} + \sum_{\substack{l\in S \\ l\neq j}} \nu_{jl}\mathcal{P}_{ilk} + \sum_{\substack{v\in R \\ v\neq k}} \lambda_{kv}^{ij}\mathcal{P}_{ijv}\right)\mathcal{X}_{ijk}$$

the inequality (11) in turn becomes

$$\begin{bmatrix} \mathcal{X}_{ijk}\mathcal{Z}_{ijk}\mathcal{X}_{ijk} - \overline{\Omega}_{ijk}\mathcal{X}_{ijk} - \mathcal{X}_{ijk}\overline{\Omega}_{ijk}^T & \tilde{\Lambda}_{ijk} + \overline{\Omega}_{ijk} & \mathcal{X}_{ijk}\mathcal{W}_{ij}^T & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) \\ \star & -\mathcal{Z}_{ijk} & \mathbf{0} & \mathbf{0} \\ \star & \star & -\mathcal{X}_{ijk} & \mathbf{0} \\ \star & \star & \star & -\mathcal{S}_{ijk}(\mathcal{X}_{ijk}) \end{bmatrix} < 0 \qquad (12)$$

then, the proof is complete. ■

Remark 2: The condition of proposition 2 is nonlinear in the unknown variables $\mathcal{X}_{ijk}$ and $\overline{\Omega}_{ijk}$ and hence not easily tractable by convex optimization techniques. However, the degree of freedom introduced by the variables $\mathcal{Z}_{ijk}$ can be adequately used to provide an easily tractable sufficient condition for the stochastic stability in terms of an LMI feasibility problem. This is illustrated by the following proposition.

Proposition 3 (Sufficient condition): If there exists matrices $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, $\mathcal{K}_k$, and $\overline{\Omega}_{ijk}$ satisfying the following LMI

$$\begin{bmatrix} \mu_{ijk}\mathbb{I} - \overline{\Omega}_{ijk} - \overline{\Omega}_{ijk}^T & (\mu_{ijk}\mathcal{X}_{ijk} - \overline{\Omega}_{ijk}) & \tilde{A}_{ijk} + B_j\mathcal{K}_kC_2 + \overline{\Omega}_{ijk} & \mathcal{X}_{ijk}\mathcal{W}_{ij}^T & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) \\ \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -\mathcal{X}_{ijk} & \mathbf{0} \\ \star & \star & \star & \star & -\mathcal{S}_{ijk}(\mathcal{X}_{ijk}) \end{bmatrix} < 0 \qquad (13)$$

where μ_{ijk} are arbitrary positive scalars, then the system (φ) coupled with (φ_s) is internally exponentially stable in the mean square. □

Remark 3: The free scalar parameters μ_{ijk} are introduced to reduce the conservatism of the sufficient condition for exponential stochastic stability in the mean square sense.

Proof: Since, according to proposition 2, $\mathcal{Z}_{ijk}$ can be any positive-definite matrix, we let $\mathcal{Z}_{ijk} = \mu_{ijk}\mathbb{I}$ $\forall i \in Z$, $j \in S$ and $k \in R$. Then, using the Schur complement operation with respect to the term

$$\mathcal{X}_{ijk}\mathcal{Z}_{ijk}\mathcal{X}_{ijk} - \overline{\Omega}_{ijk}^T\mathcal{X}_{ijk} - \mathcal{X}_{ijk}\overline{\Omega}_{ijk}$$

the inequality (10) can be further be written as

$$\begin{bmatrix} -\mu_{ijk}^{-1}\overline{\Omega}_{ijk}\overline{\Omega}_{ijk}^T & (\mu_{ijk}\mathcal{X}_{ijk} - \overline{\Omega}_{ijk}) & \tilde{A}_{ijk} + B_j\mathcal{K}_kC_2 + \overline{\Omega}_{ijk} & \mathcal{X}_{ijk}\mathcal{W}_{ij}^T & \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) \\ \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -\mathcal{X}_{ijk} & \mathbf{0} \\ \star & \star & \star & \star & -\mathcal{S}_{ijk}(\mathcal{X}_{ijk}) \end{bmatrix} < 0 \qquad (14)$$

then, using the relation

$$-\mu_{ijk}^{-1}\overline{\Omega}_{ijk}\overline{\Omega}_{ijk}^T \leq -\overline{\Omega}_{ijk}^T - \overline{\Omega}_{ijk} + \mu_{ijk}\mathbb{I}$$

yields the matrix inequality (13). Hence the proof is complete. ■

5 $\mathcal{H}_\infty$ Control

In this section, we deal with the design of controllers that stochastically stabilize the closed-loop system and guarantee the disturbance rejection, with a certain level $\mu > 0$. This problematic is addressed under a convex optimization framework. Mathematically, we are concerned with the LMI

characterization of compensators φ_s that stochastically stabilize the system (3) and guarantee the following for all $w \in L^2[0,\infty)$:

$$\| z_\infty \|_{\mathcal{E}_2} = \mathcal{E}\left\{\int_0^\infty z_{\infty t}^T z_{\infty t} dt\right\}^{1/2} < \mu \left[\| w \|_2^2 + a(x_0, \xi_0, \eta_0, \psi_0)\right]^{1/2} \tag{15}$$

where $\mu > 0$ is a prescribed level of disturbance attenuation to be achieved and $a(x_0, \xi_0, \eta_0, \psi_0)$ is a constant that depends on the initial conditions $(x_0, \xi_0, \eta_0, \psi_0)$. To this end, we need the auxiliary result given by the following proposition.

Proposition 4: If the system (3) is internally exponentially stable in the mean square sense, then it is stochastically stable. □

Proof: Since the system (3) is internally exponentially stable in the mean square sense, it follows from proposition 1 that there exist symmetric positive-definite matrices $\mathcal{P}_{ijk}$, $i \in Z$, $j \in S$ and $k \in R$ such that

$$\tilde{\Lambda}_{ijk}^T \mathcal{P}_{ijk} + \mathcal{P}_{ijk}\tilde{\Lambda}_{ijk} + \mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \sum_{\substack{h\in Z \\ h\neq i}} \pi_{ih}\mathcal{P}_{hjk} + \sum_{\substack{l\in S \\ l\neq j}} \nu_{jl}\mathcal{P}_{ilk} + \sum_{\substack{v\in R \\ v\neq k}} \lambda_{kv}^{ij}\mathcal{P}_{ijv} = \Xi_{ijk} < 0$$

$\forall i \in Z$, $j \in S$ and $k \in R$.
Note that it is easy to show that there exists $\alpha > 0$, such that

$$\Xi_{ijk} + \alpha \mathcal{P}_{ijk}^2 < 0.$$

$\forall i \in Z$, $j \in S$ and $k \in R$.
Let us consider the following quadratic stochastic Lyapunov function

$$\vartheta(x_t, \xi_t, \eta_t, \psi_t) = x_t^T \mathcal{P}(\xi_t, \eta_t, \psi_t) x_t$$

then

$$\mathcal{L}\vartheta(x_t, \xi_t, \eta_t, \psi_t) = x_t^T \left\{\Xi(\xi_t, \eta_t, \psi_t)\right\} x_t + 2x_t^T \mathcal{P}(\xi_t, \eta_t, \psi_t)\bar{E}(\xi_t, \eta_t, \psi_t) w_t. \tag{16}$$

Using lemma 1, it follows from (16) that

$$\mathcal{L}\vartheta(x_t, \xi_t, \eta_t, \psi_t) \leq -x_t^T \Gamma(\xi_t, \eta_t, \psi_t) x_t + \alpha^{-1} w_t^T \bar{E}(\xi_t, \eta_t, \psi_t)^T \bar{E}(\xi_t, \eta_t, \psi_t) w_t$$

where

$$\Gamma(\xi_t, \eta_t, \psi_t) = -\Xi(\xi_t, \eta_t, \psi_t) - \alpha \mathcal{P}^2(\xi_t, \eta_t, \psi_t)$$

From Dynkin's formula [12], we have

$$\begin{aligned}
\mathcal{E}\left\{\vartheta(x_T, \xi_T, \eta_T, \psi_T)\right\} - \vartheta(x_0, \xi_0, \eta_0, \psi_0) &= \mathcal{E}\left\{\int_0^T \mathcal{L}\vartheta(x_\tau, \xi_\tau, \eta_\tau, \psi_\tau) d\tau\right\} \\
&\leq -\mathcal{E}\left\{\int_0^T x_\tau^T \Gamma(\xi_\tau, \eta_\tau, \psi_\tau) x_\tau d\tau\right\} \\
&+ \alpha^{-1}\mathcal{E}\left\{\int_0^T w_\tau^T \bar{E}(\xi_\tau, \eta_\tau, \psi_\tau)^T \bar{E}(\xi_\tau, \eta_\tau, \psi_\tau) w_\tau d\tau\right\} \\
&\leq -\mathcal{E}\left\{\int_0^T \lambda_{\min}\Gamma(\xi_\tau, \eta_\tau, \psi_\tau) x_\tau^T x_\tau d\tau\right\} \\
&+ \alpha^{-1}\mathcal{E}\left\{\int_0^T \lambda_{\max}(\bar{E}(\xi_\tau, \eta_\tau, \psi_\tau)^T \bar{E}(\xi_\tau, \eta_\tau, \psi_\tau)) w_\tau^T w_\tau d\tau\right\}
\end{aligned} \tag{17}$$

From (17), we get

$$\lim_{T\to\infty}\left\{\mathcal{E}\left\{x_T^T\mathcal{P}(\xi_T,\eta_T,\psi_T)x_T\right\}+\min_{i,j,k}\left\{\lambda_{\min}\Gamma(i,j,k)\right\}\mathcal{E}\left\{\int_0^T x_\tau^T x_\tau d\tau\right\}\right\}$$
$$\leq\left\{x_0^T\mathcal{P}(\xi_0,\eta_0,\psi_0)x_0\right\}+\alpha^{-1}\max_{i,j,k}\left\{\lambda_{\max}\bar{E}^T(i,j,k)\bar{E}(i,j,k)\right\}\mathcal{E}\left\{\int_0^\infty w_\tau^T w_\tau d\tau\right\} \tag{18}$$

From (18), and knowing that $\mathcal{E}\left\{x_t^T\mathcal{P}(\xi_t,\eta_t,\psi_t)x_t\right\}\geq 0$ and $w(\cdot)\in L^2[0,\infty)$, then the system (3) is stochastically stable. Hence the proof is complete. ■

We are now able to state the sufficient condition for stochastic stability and $\mathcal{H}_\infty$ performance for the certain AFTCSMP driven by a static output feedback controller.

Proposition 5: If there exists symmetric positive-definite matrices $\mathcal{P}_{ijk}$, $i\in Z$, $j\in S$ and $k\in R$ such that

$$\left[\begin{array}{cc}\Upsilon_{ijk} & \bar{C}_{1jk}^T\bar{D}_{1ijk}+\mathcal{P}_{ijk}\bar{E}_{ijk}\\ \star & \bar{D}_{1ijk}^T\bar{D}_{1ijk}-\mu^2\mathbb{I}\end{array}\right]=\Phi_{ijk}<0 \tag{19}$$

where

$$\Upsilon_{ijk}=\tilde{\Lambda}_{ijk}^T\mathcal{P}_{ijk}+\mathcal{P}_{ijk}\tilde{\Lambda}_{ijk}+\mathcal{W}_{ij}^T\mathcal{P}_{ijk}\mathcal{W}_{ij}+\bar{C}_{1jk}^T\bar{C}_{1jk}+\sum_{\substack{h\in Z\\ h\neq i}}\pi_{ih}\mathcal{P}_{hjk}+\sum_{\substack{l\in S\\ l\neq j}}\nu_{jl}\mathcal{P}_{ilk}+\sum_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}\mathcal{P}_{ijv}$$

$\forall i\in Z$, $j\in S$ and $k\in R$.
then the system (3) is stochastically stable and satisfies

$$\parallel z_\infty\parallel_{\mathcal{E}_2}<\left[\mu^2\parallel w\parallel_2^2+x_0^T\mathcal{P}(\xi_0,\eta_0,\psi_0)x_0\right]^{1/2}. \tag{20}$$

□

Proof: From (19), we get the following inequality

$$\tilde{\Lambda}_{ijk}^T\mathcal{P}_{ijk}+\mathcal{P}_{ijk}\tilde{\Lambda}_{ijk}+\mathcal{W}_{ij}^T\mathcal{P}_{ijk}\mathcal{W}_{ij}+\bar{C}_{1jk}^T\bar{C}_{1jk}+\sum_{\substack{h\in Z\\ h\neq i}}\pi_{ih}\mathcal{P}_{hjk}+\sum_{\substack{l\in S\\ l\neq j}}\nu_{jl}\mathcal{P}_{ilk}+\sum_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}\mathcal{P}_{ijv}<0$$

which implies the following since $\bar{C}_{1jk}^T\bar{C}_{1jk}\geq 0$

$$\tilde{\Lambda}_{ijk}^T\mathcal{P}_{ijk}+\mathcal{P}_{ijk}\tilde{\Lambda}_{ijk}+\mathcal{W}_{ij}^T\mathcal{P}_{ijk}\mathcal{W}_{ij}+\sum_{\substack{h\in Z\\ h\neq i}}\pi_{ih}\mathcal{P}_{hjk}+\sum_{\substack{l\in S\\ l\neq j}}\nu_{jl}\mathcal{P}_{ilk}+\sum_{\substack{v\in R\\ v\neq k}}\lambda_{kv}^{ij}\mathcal{P}_{ijv}=\Xi_{ijk}<0 \tag{21}$$

it follows from (21) and proposition 1 that the system (3) is internally exponentially stable in the mean square sense. Using proposition 4, it follows that (3) is stochastically stable.
Let us now prove that (20) is verified. We begin by defining the following function

$$\mathcal{J}_T=\mathcal{E}\left\{\int_0^T\left(z_{\infty t}^T z_{\infty t}-\mu^2 w_t^T w_t\right)dt\right\} \tag{22}$$

Then, to prove (20), it suffices to establish that

$$\mathcal{J}_\infty\leq x_0^T P(\xi_0,\eta_0,\psi_0)x_0$$

Let a quadratic stochastic Lyapunov function

$$\vartheta(x_t,\xi_t,\eta_t,\psi_t)=x_t^T\mathcal{P}(\xi_t,\eta_t,\psi_t)x_t \tag{23}$$

then

$$\mathcal{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t) = x_t^T\left\{\Xi(\xi_t,\eta_t,\psi_t)\right\}x_t + 2x_t^T\mathcal{P}(\xi_t,\eta_t,\psi_t)\bar{E}(\xi_t,\eta_t,\psi_t)w_t. \tag{24}$$

and

$$\begin{aligned} z_{\infty t}^T z_{\infty t} - \mu^2 w_t^T w_t &= [\bar{C}_1(\eta_t,\psi_t)x_t + \bar{D}_1(\xi_t,\eta_t,\psi_t)w_t]^T[\bar{C}_1(\eta_t,\psi_t)x_t + \bar{D}_1(\xi_t,\eta_t,\psi_t)w_t] - \mu^2 w_t^T w_t \\ &= x_t^T\bar{C}_1(\eta_t,\psi_t)^T\bar{C}_1(\eta_t,\psi_t)x_t + x_t^T\bar{C}_1(\eta_t,\psi_t)^T\bar{D}_1(\xi_t,\eta_t,\psi_t)w_t \\ &+ w_t^T\bar{D}_1(\xi_t,\eta_t,\psi_t)^T\bar{C}_1(\eta_t,\psi_t)x_t + w_t^T\bar{D}_1(\xi_t,\eta_t,\psi_t)^T\bar{D}_1(\xi_t,\eta_t,\psi_t)w_t - \mu^2 w_t^T w_t \end{aligned} \tag{25}$$

which implies the following equality

$$z_{\infty t}^T z_{\infty t} - \mu^2 w_t^T w_t + \mathcal{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t) = \chi_t^T\Phi(\xi_t,\eta_t,\psi_t)\chi_t \tag{26}$$

where $\chi_t = \begin{bmatrix} x_t \\ w_t \end{bmatrix}$.

adding and subtracting $\mathcal{E}\left\{\int_0^T \mathcal{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t)dt\right\}$ to (22), we get

$$\mathcal{J}_T = \mathcal{E}\left\{\int_0^T \left(z_{\infty t}^T z_{\infty t} - \mu^2 w_t^T w_t + \mathcal{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t)\right)dt\right\} - \mathcal{E}\left\{\int_0^T \left(\mathcal{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t)\right)dt\right\} \tag{27}$$

From Dynkin's formula, we have

$$\mathcal{E}\left\{\vartheta(x_T,\xi_T,\eta_T,\psi_T)\right\} - \vartheta(x_0,\xi_0,\eta_0,\psi_0) = \mathcal{E}\left\{\int_0^T \mathcal{L}\vartheta(x_t,\xi_t,\eta_t,\psi_t)dt\right\} \tag{28}$$

then we get

$$\mathcal{J}_T = \mathcal{E}\left\{\int_0^T \chi_t^T\Phi(\xi_t,\eta_t,\psi_t)\chi_t dt\right\} - \mathcal{E}\left\{\vartheta(x_T,\xi_T,\eta_T,\psi_T)\right\} + \vartheta(x_0,\xi_0,\eta_0,\psi_0) \tag{29}$$

Since $\Phi(\xi_t,\eta_t,\psi_t) < 0$ and $\mathcal{E}\left\{\vartheta(x_T,\xi_T,\eta_T,\psi_T)\right\} \geq 0$, it follows from (29) that

$$\mathcal{J}_T \leq \vartheta(x_0,\xi_0,\eta_0,\psi_0)$$

which yields $\mathcal{J}_\infty \leq x_0^T\mathcal{P}(\xi_0,\eta_0,\psi_0)x_0$. Hence the proof is complete. ■

The $\mathcal{H}_\infty$ constraints (20) can be rephrased in LMI form. This is illustrated by proposition 6, which gives an LMI characterization of static output feedback compensators (φ_s) that stochastically stabilize the AFTCSMP and ensure (20).

Proposition 6: If there exists matrices $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, $\overline{\Omega}_{ijk}$, $\mathcal{K}_k$ such that the following LMIs are feasible

$$\begin{bmatrix} \mu_{ijk}\mathbb{I} - \overline{\Omega}_{ijk} - \overline{\Omega}_{ijk}^T + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mu^2\mathbb{I} \end{bmatrix} & (\mu_{ijk}\overline{\mathcal{X}}_{ijk} - \overline{\Omega}_{ijk}) & \begin{bmatrix} \overline{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk}) \\ \mathbf{0} \end{bmatrix} & \Theta_{ijk} + \overline{\Omega}_{ijk} & \mathbf{0} \\ \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & \star & -\overline{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -\mu_{ijk}\mathbb{I} & \Pi_{ijk}^T \\ \star & \star & \star & \star & -\mathbb{I} \end{bmatrix} < 0 \tag{30}$$

$$\begin{bmatrix} \mu_{ijk}\mathbb{I} & \Pi_{ijk}^T \\ \star & \mathbb{I} \end{bmatrix} > 0 \tag{31}$$

where

$$\begin{cases} \Pi_{ijk} = \left[\begin{array}{cc} C_1 & \mathbf{0} \end{array} \right] + D_{1j}\mathcal{K}_k \left[\begin{array}{cc} C_2 & D_{2jk} \end{array} \right] \\ \Theta_{ijk} = \left[\begin{array}{cc} \tilde{A}_{ijk} & E_{ij} \\ \mathbf{0} & \mathbf{0} \end{array} \right] + \left[\begin{array}{c} B_j \\ \mathbf{0} \end{array} \right] \mathcal{K}_k \left[\begin{array}{cc} C2 & D_{2ij} \end{array} \right] \\ \overline{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk}) = \left[\begin{array}{cc} \mathcal{R}_{ijk}(\mathcal{X}_{ijk}) & \mathcal{X}_{ijk}\mathcal{W}_{ij} \end{array} \right] \\ \overline{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) = \left[\begin{array}{cc} \mathcal{S}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} \\ \mathbf{0} & \mathcal{X}_{ijk} \end{array} \right] \\ \overline{\mathcal{X}}_{ijk} = \left[\begin{array}{cc} \mathcal{X}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{array} \right] \end{cases} \tag{32}$$

and μ_{ijk} are positive scalars $\forall i \in Z$, $j \in S$ and $k \in R$. Then the system (3) is stochastically stable and satisfies

$$\| z_\infty \|_{\mathcal{E}_2} < \left[\mu^2 \| w \|_2^2 + x_0^T \mathcal{P}(\xi_0, \eta_0, \psi_0) x_0 \right]^{1/2}. \tag{33}$$

Then, the stabilizing output feedback control law is given by

$$u_k(t) = \mathcal{K}_k y(t)$$

□

Proof: The matrix inequalities (19) can be equivalently written as follows

$$\begin{aligned} \Phi_{ijk} = & \left[\begin{array}{cc} \tilde{\Lambda}_{ijk}^T & \mathbf{0} \\ \bar{E}_{ijk}^T & \mathbf{0} \end{array} \right] \left[\begin{array}{cc} \mathcal{P}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{array} \right] + \left[\begin{array}{cc} \mathcal{P}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{array} \right] \left[\begin{array}{cc} \tilde{\Lambda}_{ijk} & \bar{E}_{ijk} \\ \mathbf{0} & \mathbf{0} \end{array} \right] + \left[\begin{array}{c} \bar{C}_{1jk}^T \\ \bar{D}_{1ijk}^T \end{array} \right] \left[\begin{array}{cc} \bar{C}_{1jk} & \bar{D}_{1ijk} \end{array} \right] \\ & + \left[\begin{array}{cc} \mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \sum\limits_{\substack{h \in Z \\ h \neq i}} \pi_{ih} \mathcal{P}_{hjk} + \sum\limits_{\substack{l \in S \\ l \neq j}} \nu_{jl} \mathcal{P}_{ilk} + \sum\limits_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \mathcal{P}_{ijv} & \mathbf{0} \\ \mathbf{0} & -\mu^2 \mathbb{I} \end{array} \right] < 0 \end{aligned} \tag{34}$$

The use of the reciprocal projection lemma with

$$\Psi_{ijk} = \left[\begin{array}{cc} \mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \sum\limits_{\substack{h \in Z \\ h \neq i}} \pi_{ih} \mathcal{P}_{hjk} + \sum\limits_{\substack{l \in S \\ l \neq j}} \nu_{jl} \mathcal{P}_{ilk} + \sum\limits_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \mathcal{P}_{ijv} & \mathbf{0} \\ \mathbf{0} & -\mu^2 \mathbb{I} \end{array} \right] + \left[\begin{array}{c} \bar{C}_{1jk}^T \\ \bar{D}_{1ijk}^T \end{array} \right] \left[\begin{array}{cc} \bar{C}_{1jk} & \bar{D}_{1ijk} \end{array} \right] \tag{35}$$

and

$$S_{ijk} = \left[\begin{array}{cc} \tilde{\Lambda}_{ijk}^T & \mathbf{0} \\ \bar{E}_{ijk}^T & \mathbf{0} \end{array} \right] \left[\begin{array}{cc} \mathcal{P}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{array} \right] \tag{36}$$

yields

$$\left[\begin{array}{cc} \Psi_{ijk} + \mathcal{Z}_{ijk} - (\Omega_{ijk} + \Omega_{ijk}^T) & S_{ijk}^T + \Omega_{ijk} \\ \star & -\mathcal{Z}_{ijk} \end{array} \right] < 0 \tag{37}$$

where $\mathcal{Z}_{ijk}$ are any given positive-definite matrices, $\forall i \in Z$, $j \in S$ and $k \in R$.
Let us define $\mathcal{X}_{ijk} = \mathcal{P}_{ijk}^{-1}$ and $\overline{\Omega}_{ijk} = \overline{\mathcal{X}}_{ijk} \Omega_{ijk}$, then by the congruence transformation

$$\left[\begin{array}{cc} \overline{\mathcal{X}}_{ijk} & \mathbf{0} \\ \mathbf{0} & \mathbb{I} \end{array} \right] \tag{38}$$

and with a Schur complement operation with respect to the term

$$\overline{\mathcal{X}}_{ijk} \left[\begin{array}{cc} \mathcal{W}_{ij}^T \mathcal{P}_{ijk} \mathcal{W}_{ij} + \sum\limits_{\substack{h \in Z \\ h \neq i}} \pi_{ih} \mathcal{P}_{hjk} + \sum\limits_{\substack{l \in S \\ l \neq j}} \nu_{jl} \mathcal{P}_{ilk} + \sum\limits_{\substack{v \in R \\ v \neq k}} \lambda_{kv}^{ij} \mathcal{P}_{ijv} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right] \overline{\mathcal{X}}_{ijk}$$

the inequality (37) in turn becomes

$$\left[\begin{array}{ccc} \bar{\mathcal{X}}_{ijk}\left(\mathcal{Z}_{ijk}+\left[\begin{array}{c}\bar{C}_{1jk}^T\\ \bar{D}_{1ijk}^T\end{array}\right]\left[\begin{array}{cc}\bar{C}_{1jk} & \bar{D}_{1ijk}\end{array}\right]\right)\bar{\mathcal{X}}_{ijk}-\bar{\Omega}_{ijk}\bar{\mathcal{X}}_{ijk}-\bar{\mathcal{X}}_{ijk}\Omega_{ijk}^T+\left[\begin{array}{cc}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mu^2\mathbb{I}\end{array}\right] & \left[\begin{array}{c}\overline{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk})\\ \mathbf{0}\end{array}\right] & \Theta_{ijk}+\bar{\Omega}_{ijk}\\ \star & -\overline{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0}\\ \star & \star & -\mathcal{Z}_{ijk}\end{array}\right]<0 \quad (39)$$

Since $\mathcal{Z}_{ijk}$ can be any positive-definite matrix we let

$$\mathcal{Z}_{ijk}=\left(\mu_{ijk}\mathbb{I}-\left[\begin{array}{c}\bar{C}_{1jk}^T\\ \bar{D}_{1ijk}^T\end{array}\right]\left[\begin{array}{cc}\bar{C}_{1jk} & \bar{D}_{1ijk}\end{array}\right]\right)>0$$

or equivalently

$$\left[\begin{array}{cc}\mu_{ijk}\mathbb{I} & \left[\begin{array}{c}\bar{C}_{1jk}^T\\ \bar{D}_{1ijk}^T\end{array}\right]\\ \star & \mathbb{I}\end{array}\right]>0 \quad (40)$$

then using the Schur complement with respect to the term

$$\bar{\mathcal{X}}_{ijk}\left(\mathcal{Z}_{ijk}+\left[\begin{array}{c}\bar{C}_{1jk}^T\\ \bar{D}_{1ijk}^T\end{array}\right]\left[\begin{array}{cc}\bar{C}_{1jk} & \bar{D}_{1ijk}\end{array}\right]\right)\bar{\mathcal{X}}_{ijk}-\bar{\Omega}_{ijk}\bar{\mathcal{X}}_{ijk}-\bar{\mathcal{X}}_{ijk}\Omega_{ijk}^T \quad (41)$$

the inequality (39) can be further be written as

$$\left[\begin{array}{ccccc}\left[\begin{array}{cc}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mu^2\mathbb{I}\end{array}\right]-\mu_{ijk}^{-1}\bar{\Omega}_{ijk}\bar{\Omega}_{ijk}^T & (\mu_{ijk}\overline{\mathcal{X}}_{ijk}-\overline{\Omega}_{ijk}) & \left[\begin{array}{c}\overline{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk})\\ \mathbf{0}\end{array}\right] & \Theta_{ijk}+\overline{\Omega}_{ijk} & \mathbf{0}\\ \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \star & \star & -\overline{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} & \mathbf{0}\\ \star & \star & \star & -\mu_{ijk}\mathbb{I} & \Pi_{ijk}^T\\ \star & \star & \star & \star & -\mathbb{I}\end{array}\right]<0 \quad (42)$$

then, using the relation

$$-\mu_{ijk}^{-1}\overline{\Omega}_{ijk}\overline{\Omega}_{ijk}^T\leq-\overline{\Omega}_{ijk}^T-\overline{\Omega}_{ijk}+\mu_{ijk}\mathbb{I}$$

yields the matrix inequality (30). Hence the proof is complete. ■

From practical point of view, the controller that stochastically stabilizes the AFTCSMP and at the same time guarantees the minimum disturbance rejection is of great interest. This controller can be obtained by solving the following optimization problem:

$$\mathcal{O}:\left\{\begin{array}{l}\min\limits_{\delta>0,\ \mathcal{X}_{ijk}=\mathcal{X}_{ijk}^T>0,\ \mathcal{K}_k,\ \overline{\Omega}_{ijk}}\ \delta\\ \text{s.t}:\\ \left[\begin{array}{ccccc}\mu_{ijk}\mathbb{I}-\overline{\Omega}_{ijk}-\overline{\Omega}_{ijk}^T+\left[\begin{array}{cc}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\delta\mathbb{I}\end{array}\right] & (\mu_{ijk}\overline{\mathcal{X}}_{ijk}-\overline{\Omega}_{ijk}) & \left[\begin{array}{c}\overline{\mathcal{R}}_{ijk}(\mathcal{X}_{ijk})\\ \mathbf{0}\end{array}\right] & \Theta_{ijk}+\overline{\Omega}_{ijk} & \mathbf{0}\\ \star & -\mu_{ijk}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \star & \star & -\overline{\mathcal{S}}_{ijk}(\mathcal{X}_{ijk}) & \mathbf{0} & \mathbf{0}\\ \star & \star & \star & -\mu_{ijk}\mathbb{I} & \Pi_{ijk}^T\\ \star & \star & \star & \star & -\mathbb{I}\end{array}\right]<0\\ \left[\begin{array}{cc}\mu_{ijk}\mathbb{I} & \Pi_{ijk}^T\\ \star & \mathbb{I}\end{array}\right]>0\end{array}\right.$$

where the LMIs in the constraints are obtained from (30)-(31) by replacing μ^2 by δ. This leads to the following Corollary:

Corollary 1: Let $\delta > 0$, $\mathcal{X}_{ijk} = \mathcal{X}_{ijk}^T > 0$, $\mathcal{K}_k$, and $\overline{\Omega}_{ijk}$ be the solution of the optimization problem $\mathcal{O}$. Then, the controller (2) stochastically stabilizes the AFTCSMP we are considering and moreover the closed loop system satisfies the disturbance rejection of level $\sqrt{\delta}$. $\Diamond$

Remark 4: The optimization problem $\mathcal{O}$ represents a minimization of a linear objective under LMI constraints. It can be easily solved using the function `mincx` implemented on the LMI control toolbox for use with MATLAB [13].

Remark 5: For practical applications, the above derived theoretical results could be used as follows: first, compute **off-line**, using the results in Proposition 3 (stochastic stability) or Corollary 1 ($\mathcal{H}_\infty$ control), the controller gains in nominal and faulty situations. Second, and providing the FDI processus states, the supervision mechanism will switch, **on-line**, to the corresponding controller.

6 Numerical Examples

a) Actuator Failures Case

In this section, the proposed $\mathcal{H}_\infty$ static output feedback control of AFTCSMP is illustrated using a flight control example. Consider the nominal system with:

$$A = \begin{bmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{bmatrix},\ B = \begin{bmatrix} -0.194 & -0.0359 \\ -19.29 & -3.803 \end{bmatrix},\ E = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix},\ \mathcal{W} = 0.5 \times \mathbb{I},$$

$$C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},\ C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ D_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

This model is adapted from [3]. It represents the decoupled linearized longitudinal dynamics of F-18 aircraft flying at Mach 0.7 at an altitude of 14000 ft. The state vector $x_t \in \mathbb{R}^2$ is composed by the following:

x_1: angle of attack;

x_2: pitch rate;

and the components of command vector are:

u_1: symmetric elevator position;

u_2: symmetric pitch thrust velocity nozzle position.

For illustration purposes, we will consider two faulty modes:

i) **Mode 2**: A 50% power loss on the second actuator;

ii) **Mode 3**: First actuator outage and a 50% power loss on the second actuator.

From above, we have that $S = \{1, 2, 3\}$, where the **mode 1** represents the nominal case. The failure process is assumed to have Markovian transition characteristics. The FDI process is also Markovian with three states $R = \{1, 2, 3\}$.
The actuator failure rates are assumed to be:

$$[\nu_{ij}] = \begin{bmatrix} -0.002 & 0.0010 & 0.0010 \\ 0.0010 & -0.002 & 0.0010 \\ 0.0010 & 0.0010 & -0.002 \end{bmatrix}$$

The FDI conditional transition rates are:

$$[\lambda_{ij}^1] = \begin{bmatrix} -0.02 & 0.01 & 0.01 \\ 1.00 & -1.01 & 0.01 \\ 1.00 & 0.01 & -1.01 \end{bmatrix},\ [\lambda_{ij}^2] = \begin{bmatrix} -1.01 & 1.00 & 0.01 \\ 0.01 & -0.02 & 0.01 \\ 0.01 & 1.00 & -1.01 \end{bmatrix},\ [\lambda_{ij}^3] = \begin{bmatrix} -1.01 & 0.01 & 1.00 \\ 0.01 & -1.01 & 1.00 \\ 0.01 & 0.01 & -0.02 \end{bmatrix}.$$

For the above AFTCSMP, and solving the optimization problem $\mathcal{O}$, we obtain the following $\mathcal{H}_\infty$ performance from w_t to $z_{\infty t}$: $\mu = 1.4247$.
The corresponding controllers are given as follows:

$$\mathcal{K}_1 = \left[\begin{array}{cc} -0.0010 & 25.1064 \end{array} \right]^T, \ \mathcal{K}_2 = \left[\begin{array}{cc} 0.8996 & 24.9051 \end{array} \right]^T, \ \mathcal{K}_3 = \left[\begin{array}{cc} -0.8854 & 33.4214 \end{array} \right]^T.$$

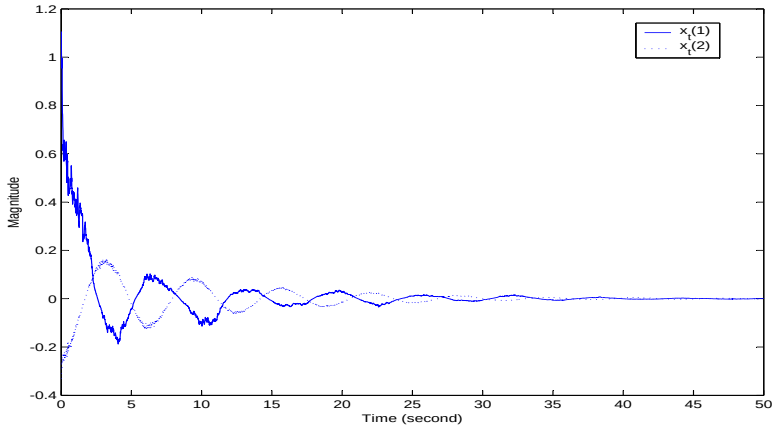

Figure 2: State variables evolution: single sample path simulation

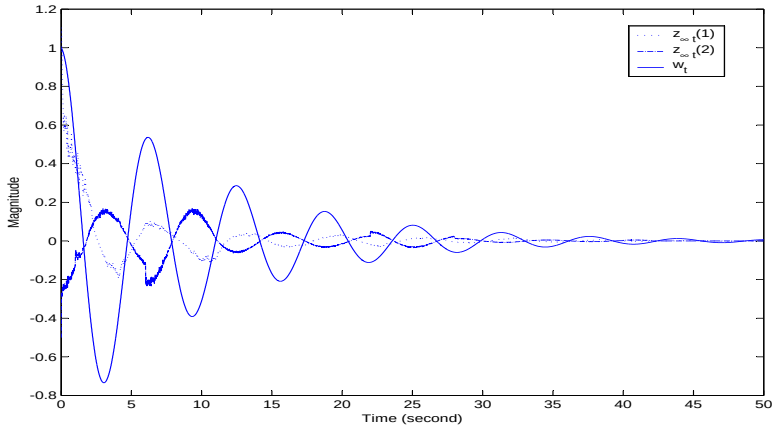

Figure 3: Evolution of the variables $z_{\infty t}$: single sample path simulation

The state trajectories of the closed loop system resulting from the obtained controllers are shown in Figure 2. These trajectories represent a single sample path simulation corresponding to a realization of the failure process η_t and the FDI process ψ_t. Figure 3 represents the evolution of the controlled outputs $z_{\infty t}$. It can be seen that the closed-loop system is stochastically stable and that the disturbance attenuation is achieved.

b) Plant Component Failure case

Let us consider the following academical example:

$$A_1 = \left[\begin{array}{ccc} 0.9749 & -0.3257 & 0.2333 \\ -2.3779 & -2.0122 & 0.6464 \\ -1.0923 & 1.5677 & -1.129 \end{array} \right], \ B = \left[\begin{array}{cc} 1 & 1 \\ 0.25 & 2 \\ 0 & 0.5 \end{array} \right], \ E = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0.1 \\ 0.2 & 0 \end{array} \right], \ \mathcal{W} = 0.5 \times \mathbb{I},$$

$$C_2 = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array} \right], \ D_2 = \left[\begin{array}{cc} 0.5 & 0 \\ 0 & 0.5 \end{array} \right], \ C_1 = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right], \ D_1 = \left[\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right].$$

We will consider that this system is subject to one possible plant component failure that modify the dynamics as follows:

$$A_2 = \begin{bmatrix} 0.1970 & 0.7925 & -1.1087 \\ 1.6969 & 0.6034 & 2.1442 \\ 0.7260 & -0.0584 & -1.3528 \end{bmatrix}.$$

From above, we have that $Z = \{1, 2\}$, where the **mode 1** represents the nominal case and **mode 2** the faulty one. The failure process is assumed to have Markovian transition characteristics. The FDI process is also Markovian with two states $R = \{1, 2\}$.
The plant component failure rates are assumed to be:

$$[\pi_{ij}] = \begin{bmatrix} -0.0050 & 0.0050 \\ 0.0100 & -0.0100 \end{bmatrix}$$

The FDI conditional transition rates are:

$$[\lambda^1_{ij}] = \begin{bmatrix} -0.1000 & 0.1000 \\ 0.9000 & -0.9000 \end{bmatrix}, \; [\lambda^2_{ij}] = \begin{bmatrix} -1.000 & 1.0000 \\ 0.1000 & -0.1000 \end{bmatrix}.$$

As for the previous example, and solving the optimization problem $\mathcal{O}$, we obtain the following $\mathcal{H}_\infty$ performance from w_t to $z_{\infty t}$: $\mu = 1.3595$.
The corresponding controllers are given as follows:

$$\mathcal{K}_1 = \begin{bmatrix} -5.4724 & -0.2437 \\ 0.4205 & -1.0532 \end{bmatrix}, \; \mathcal{K}_2 = \begin{bmatrix} -5.6677 & -1.0610 \\ 0.3593 & -1.0551 \end{bmatrix}.$$

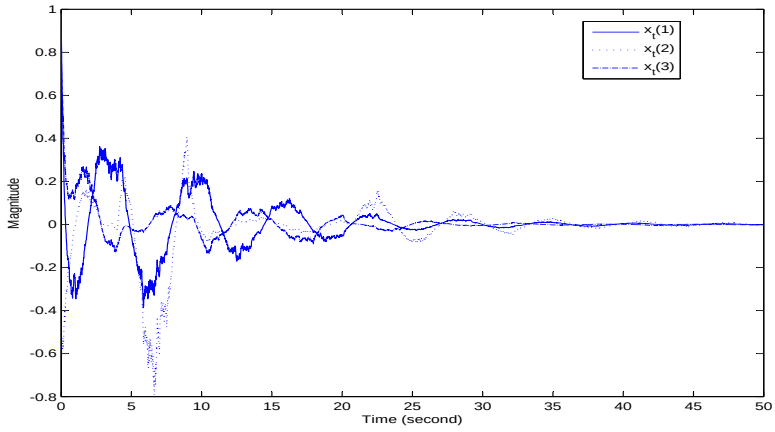

Figure 4: State variables evolution: single sample path simulation

The state trajectories of the closed loop system resulting from the obtained controllers are shown in Figure 4. These trajectories represent, as for the previous example, a single sample path simulation corresponding to a realization of the failure process η_t and the FDI process ψ_t. Figure 5 represents the evolution of the controlled outputs $z_{\infty t}$. It can be seen that the closed-loop stochastic stability and $\mathcal{H}_\infty$ performance requirements are achieved.

7 Conclusion

In this paper, we have first considered the problematic of static output feedback stochastic stabilization of an AFTCSMP. It was shown that the necessary and sufficient conditions for the *internal exponential stability* in the mean square sense can be written in terms of a nonlinear matrix inequality feasibility problem. Then, an LMI relaxation scheme was presented to eliminate the nonlinearities, yielding to a new easily tractable sufficient condition. Having obtained these results, we have moved on the control

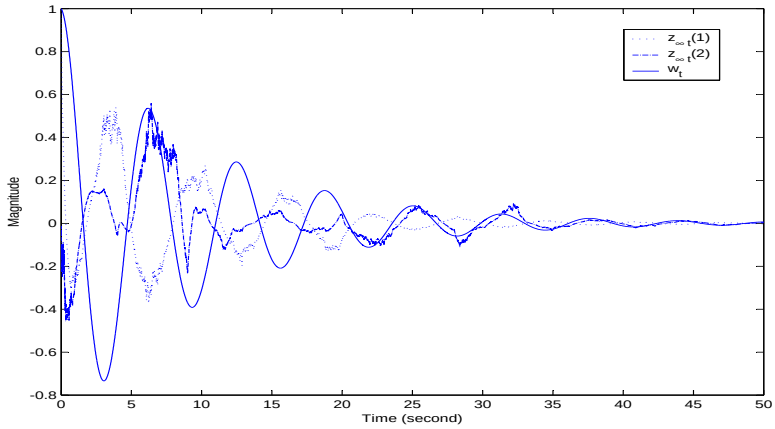

Figure 5: Evolution of the variables $z_{\infty t}$: single sample path simulation

problem. Indeed, we have introduced an LMI approach to the $\mathcal{H}_\infty$ control for linear continuous time AFTCSMP with Wiener process under a static output feedback control. We have derived some linear matrix inequalities whose solutions indicate the achievability of the desired control problem; *i.e.* we have shown that the $\mathcal{H}_\infty$ control problematic can be recast as a convex optimization problem under constraints of LMIs which can be solved effectively using the recently developed LMI tool. The effectiveness of the developed method was illustrated on simulation examples. However, it is important to note that the obtained LMI conditions are only sufficient. Our future works will be dedicated to treat the above problematic in a nonconvex optimization framework which will lead to necessary and sufficient synthesis conditions. Significant results are already obtained using a new framework based on the synthesis of ellipsoidal sets of controllers.

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