Input Independent Chaos Synchronization of Switched Systems

Gilles Millérioux, Jamal Daafouz

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Abstract

Input independent chaos synchronization of switched systems is addressed. The synchronization is carried out by designing an observer which involves a switched matrix gain. A special structure of this gain enables both an input independence of the state reconstruction error equation and a global convergence of this equation. Input independence is ensured by a suitable partial pole placement while the global convergence is based on the poly-quadratic stability concept. Those theoretical results would be interesting for possible applications in communications.

I. Introduction

For a large decade, since the pioneering works of [1], synchronization of chaotic motions has attracted a growing attention from the control theory point of view. One of the most popular approach for achieving the synchronization is based upon the framework of nonlinear observers. Indeed, synchronization problem has been shown [2] to be strongly related to the construction of the full state vector of the chaotic system from which only a transmitted signal consisting of a vector of smaller dimension is available. Distinct methodologies for the design of an observer have been examined. For relative recent results, the reader can refer to the "dynamic inversion" in the sense of [3], the concept of absolute stability [4], the observer design for chaotic systems with Generalized Hamiltonian Forms [5], the polytopic observers design based upon Linear Matrix Inequalities [6]. An attempt to mention all related papers on chaos synchronization would result in a prohibitively long list. It might be worthwhile to consult all devoted special issues and references therein appeared for that purpose [7][8][9][10].

Synchronization problems has become popular because of its possible use in communication. A huge number of papers has been published from this perspective, but security and performances of the proposed algorithms is a current research topic. Recent works and a serious overview can be found in [11] and references therein. What distinguishes the pure synchronization problem from the chaos-based communication problem is the fact that in the latter situation, an input,

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playing the role of the information to be masked, is embedded in the chaos generator, namely the *drive*. And yet, the input must be recovered at the so-called *response* side despite of the fact that it is not transmitted. To face this situation, two methods can be found in the existing literature. The problem is likely to be tackled from the identification point of view. The information is interpreted as a time-varying parameter, not available beforehand [12] and which must be identified. The second approach is based on nonlinear adaptive methods [13]. Both methods suffers from the fact that the parameter must be slowly time-varying to avoid too long convergence transients.

In this paper, a novel observer-based synchronization method is presented. It is called input independent global synchronization (IIGS) and enables to remove such a constraint for a special *drive-response* setup. The method first involves a partial pole placement technique which makes the equation governing the state reconstruction error to become input independent. Partial pole placement has been widely investigated for linear systems. It consists of assigning a set of given eigenvalues while preserving stability by a suitable choice concerning the rest of the spectrum. In [14], a technique using dynamic output-feedback is presented. In [15], the problem is tackled while guaranteeing stability under LQ criteria. On the other hand, to date, very few papers, to our knowledge, have focus on partial pole for piecewise linear systems. Secondly, a special Lyapunov approach ensures the global convergence of the state reconstruction error. It is shown how this approach, resorting to poly-quadratic stability [16][17], turns the design of the gain matrices of the observer into a resolution of a set of Linear Matrix Inequalities.

The layout of the paper is as follows. Section II introduces the required formalism to establish the input independent state reconstruction problem as a special observer design. It is also devoted to some preliminary results required for the forthcoming sections. It particularly includes the poly-quadratic stability notion and the required structure of the observer gain so that the matrix governing the state reconstruction error gets a null space. Indeed, it will be shown that an input independent convergence property requires such a null space. In section III, the main result of the paper providing the design procedure to ensure input independent global convergence and the recovering of the original information is established. The switched matrix gain of the observer is derived from the solution of a set of Linear Matrix Inequalities. Finally, in order to assess the IIGS, an illustrative example borrowed from a communication context is given in section IV.

Notations

Throughout the paper, $\mathbf{1}$ (*resp.* $\mathbf{0}$) is an identity (*resp.* null) matrix of appropriate size. For a matrix X, X^T stands for the transpose of X, $\mathcal{N}(X)$ is the null space of X. When being symmetric, $X > 0$ indicates that X is positive definite and the symbol $(\bullet)^T$ denotes each of its symmetric block.

II. Problem statement and preliminary results

Consider the discrete time system :

$$\begin{aligned} x_{k+1} &= A_i \cdot (x_k + M_i u_k) \quad + \quad E_i \\ y_k &= C_i \cdot (x_k + M_i u_k) \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^m$, $u_k \in \mathbb{R}^m$ are respectively the state vector, the output and the external input representing the information to be masked. The state space $x_k \in \mathbb{R}^n$ is partitioned into P distinct regions R_i with $\bigcup_{i=1}^{i=P} R_i \subseteq \mathbb{R}^n$. A_i, M_i, E_i and C_i are constant matrices assigned, with a one-to-one correspondence, to the region R_i visited by x_k at the discrete time k. The matrices C_i are of rank m, A_i and E_i govern the dynamics such that a chaotic motion is exhibited. Finally, M_i characterizes the way to mix the input u_k with the chaotic signal x_k.

Remark 1. *It is pointed out that for switched systems, chaos behaviors depend not only on the local dynamics but also on the rule which orchestrates the switches. One of the most popular piecewise linear systems are the Chua's circuit in the continuous time case, or the Tent Map, the Uniform Markov Map, the Logistic Map in the discrete time case.*

Consider the discrete time system described by :

$$\begin{aligned} \hat{x}_{k+1} &= A_i \hat{x}_k \quad + \quad E_i \quad + \quad L_i(y_k - \hat{y}_k) \\ \hat{y}_k &= C_i \hat{x}_k \end{aligned} \tag{2}$$

This system acts as an observer with matrix gain L_i. The gain depends on the index i since switched systems are considered. Throughout the paper, according to usual chaos synchronization terminology, the system (1) will be called *drive* and the system (2) will be refer to as *response*.

Problem is to design a *drive-response* system such that $\hat{x}_k$ tends asymptotically towards x_k for any u_k, not available at the *response* side, and for any initial state $\hat{x}_0$. It amounts to find suitable matrices pairs (M_i, L_i) for $i = 1, \ldots, P$ such that :

$$\lim_{k \to \infty} \|x_k - \hat{x}_k\| = 0 \quad \forall \hat{x}_0 \quad \text{and} \quad \forall u_k \tag{3}$$

When (3) is verified, the *drive-response* system will be referred to as Input Independent Global Synchronized (IIGS). Before dealing with the main result of the paper, preliminary results are presented in next section.

A. Poly-quadratic stability

Consider the dynamical discrete time system which depends in a polytopic way on a vector ξ_k :

$$z_{k+1} = \mathcal{A}(\xi_k) z_k \tag{4}$$

The structure of the dynamical matrix $\mathcal{A}$ gets the following polytopic description :

$$\mathcal{A}(\xi_k) = \sum_{p=1}^{P} \xi_k^p A_p, \qquad \xi_k^p \geq 0, \qquad \sum_{p=1}^{P} \xi_k^p = 1 \tag{5}$$

Following reasonings developed in [16], we consider Parameter-Dependent Lyapunov Functions (PDLF) of the form:

$$V(z_k, \xi_k) \; = \; z_k^T \mathcal{P}(\xi_k) z_k \tag{6}$$

with

$$\mathcal{P}(\xi_k) \; = \; \sum_{p=1}^{P} \xi_k^p P_p \tag{7}$$

the various P_p, $p = 1, ..., P$, are symmetric positive definite matrices of appropriate order. The poly-quadratic stability concept is recalled in the following definition.

Definition 1. *System (4) is said to be poly-quadratically stable if there exists a Positive Definite Parameter Dependent Quadratic Lyapunov Function (6) whose difference along the solution of (4) satisfies*

$$\mathcal{L} = V(z_{k+1}, \xi_{k+1}) - V(z_k, \xi_k) \leq -\alpha_0(\|z_k\|) \tag{8}$$

with α_0 a κ_∞ function[1].

Poly-quadratic stability is sufficient for asymptotic stability. Assessing the poly-quadratic stability of a polytopic model is equivalent to find P Positive Definite matrices P_p, $p = 1, ...P$ such that :

$$-\mathcal{P}(\xi_k) \; + \; \mathcal{A}^T(\xi_k) \mathcal{P}_+(\xi_k) \mathcal{A}(\xi_k) \; < \; 0 \quad \forall k \tag{9}$$

with :

$$\left\{ \begin{array}{l} \mathcal{P}(\xi_k) \; = \; \sum_{p=1}^{P} \xi_k^p P_p \\ \qquad\qquad\qquad\qquad \forall k \\ \mathcal{P}_+(\xi_k) \; = \; \sum_{p=1}^{P} \xi_{k+1}^p P_p \end{array} \right. \tag{10}$$

[1] A function $\alpha : [0, \infty) \to [0, \infty)$ is a κ_∞ function if it is continuous, strictly increasing, zero at zero and unbounded ($\alpha(s) \to \infty$ as $s \to \infty$).

The following Theorem gives a necessary and sufficient condition for a time varying polytopic system such as (4) to be poly-quadratically stable.

Theorem 1. *System (4) is poly-quadratically stable if and only if there exist symmetric positive definite matrices S_i and matrices G_i of appropriate dimensions such that*

$$\begin{bmatrix} G_i + G_i^T - S_i & (\bullet)^T \\ A_i G_i & S_j \end{bmatrix} > \mathbf{0} \tag{11}$$

$\forall (i,j) \in \{1,\dots,P\} \times \{1,\dots,P\}$. *In this case, the time varying PDLF is given by (6) with*

$$\mathcal{P}(\xi_k) = \sum_{p=1}^{P} \xi_k^p S_p^{-1}.$$

Proof see [16]. □

Poly-quadratic stability is less conservative than quadratic stability which corresponds to the special choice of the Lyapunov function, $V(z_k, \xi_k) = z_k^T \mathcal{P}(\xi_k) z_k$ with $\mathcal{P}(\xi_k) = P^*$, a constant matrix, and thus corresponds to the particular case $G_i = S_i = S^*$, a constant matrix too. As far as piecewise linear systems are concerned, Theorem 1 holds by defining the quantity ξ_k as an "indicator vector" given by the following definition :

Definition 2. *An indicator vector associated to a switched system is a vector $\xi_k = (\xi_k^1, \dots, \xi_k^P)^T$ whose components are defined as :*

$$\xi_k^p = \begin{cases} 1 \text{ if } z_k \text{ visits } R_p \\ 0 \text{ otherwise} \end{cases}$$

B. Stability under piecewise linear similarity transform

Consider the autonomous discrete time system :

$$\eta_{k+1} = \Phi_i \eta_k \tag{12}$$

where Φ_i belongs to a finite set of P matrices. Let introduce a full rank piecewise linear transform T_i and consider the recursion :

$$\tilde{\eta}_{k+1} = T_i^{-1} \Phi_i T_i \tilde{\eta}_k \tag{13}$$

where the succession of indexes i and the discrete times k corresponding to switches are the same as the ones of (12). It can be stated that, the convergence towards zero of the solution $\tilde{\eta}^*$ of (13) is equivalent the convergence towards zero of the solution η^* of (12).

Indeed, (13) can be rewritten :

$$T_i\tilde{\eta}_{k+1} = \Phi_i T_i \tilde{\eta}_k \tag{14}$$

which is equivalent to (12) and means that the samples η_k^* and $\tilde{\eta}_k^*$ of the respective discrete trajectories η^* and $\tilde{\eta}^*$ verify $\eta_k^* = T_i\tilde{\eta}_k^*$. T_i being of full rank, since $\tilde{\eta}^*$ converges towards zero, it does the same for η^*.

C. Input independence property

In this subsection, a relation between M_i and L_i is stated in order to ensure the convergence of the system governing the state reconstruction error $\epsilon_k = x_k - \hat{x}_k$. This equation is obtained by subtracting (1) and (2) :

$$\epsilon_{k+1} = (A_i - L_iC_i)\epsilon_k + (A_i - L_iC_i)M_iu_k \tag{15}$$

Proposition 1 *For $i = 1, \ldots, P$, assume that L_i satisfies $\mathcal{N}(A_i - L_iC_i) \neq \emptyset$. Denote N_i the matrix whose columns span the null space of $A_i - L_iC_i$. If $M_i = N_i$, thus the convergence is input independent, that is*

$$\epsilon_{k+1} = (A_i - L_iC_i)\epsilon_k \tag{16}$$

Proof The proof is the immediate consequence of the definition of the null space. Indeed, for $i = 1, \ldots, P$, $(A_i - L_iC_i)M_i = (A_i - L_iC_i)N_i = \mathbf{0}$. Hence, (15) turns into (16). □

III. Input independent global synchronization

This section is devoted to the design of the matrices L_i and M_i such that the *drive-response* system is both input independent as well as global synchronized.

A. Input independence and partial pole placement

In this subsection, a special form of the gains L_i which guarantees that $A_i - L_iC_i$ has a non empty null space, required for input independence property, is presented. It is shown that the gains achieve a partial pole placement with zero

eigenvalues.

Consider the following canonical equation :

$$\tilde{\epsilon}_{k+1} = (\tilde{A}_i - \tilde{L}_i\tilde{C}_i)\tilde{\epsilon}_k \tag{17}$$

where, for $i = 1, \ldots, P$, $\tilde{A}_i = (T_i^c)^{-1}A_iT_i^c$, $\tilde{L}_i = (T_i^c)^{-1}L_i$ and $\tilde{C}_i = C_iT_i^c$ with T_i^c the piecewise linear transform verifying :

$$C_iT_i^c = [\mathbf{0} \quad \mathbf{1}] = \tilde{C}_i \tag{18}$$

Here, $\mathbf{0}$ is a $m \times (n-m)$ null matrix and $\mathbf{1}$ is the identity matrix of dimension m. T_i^c always exists since $rank(C_i) = m$.

Proposition 2 *For $i = 1, \ldots, P$, the gain $\tilde{L}_i = \tilde{A}_i\tilde{K}_i$ with $\tilde{K}_i = [\tilde{K}_i^1 \quad \mathbf{1}]^T$ ensures $\mathcal{N}(A_i - L_iC_i) \neq \emptyset$*

Proof The proof is constructive in the sense that it provides the dimensionality of the null space. Consider the partitioned matrix $\tilde{A}_i$ with blocks of appropriate sizes :

$$\tilde{A}_i = \begin{bmatrix} \tilde{A}_i^{11} & \tilde{A}_i^{12} \\ \tilde{A}_i^{21} & \tilde{A}_i^{22} \end{bmatrix}$$

By direct calculation, one obtains :

$$\tilde{A}_i - \tilde{L}_i\tilde{C}_i = \begin{bmatrix} \tilde{A}_i^{11} & -\tilde{A}_i^{11}\tilde{K}_i^1 \\ \tilde{A}_i^{21} & -\tilde{A}_i^{21}\tilde{K}_i^1 \end{bmatrix} \tag{19}$$

Now, take the piecewise linear transform :

$$T_i^L = \begin{bmatrix} \mathbf{1} & -\tilde{K}_i^1 \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Premultiplying (19) by the inverse of T_i^L and postmultiplying by T_i^L, we obtain :

$$(T_i^L)^{-1}(\tilde{A}_i - \tilde{L}_i\tilde{C}_i)T_i^L = \begin{bmatrix} \tilde{A}_i^{11} - \tilde{K}_i^1\tilde{A}_i^{21} & \mathbf{0} \\ \tilde{A}_i^{21} & \mathbf{0} \end{bmatrix} \tag{20}$$

Consequently, $\tilde{A}_i - \tilde{L}_i\tilde{C}_i$ and so $A_i - L_iC_i$ has at least m zero eigenvalues. This completes the proof. □

The gains L_i are retrieved by $L_i = T_i^c\tilde{L}_i$. The constraint on the structure of $\tilde{L}_i$ ensures (15) to be input independent by applying Proposition 1.

Now, the block matrix $\tilde{K}_i^1$ of $\tilde{L}_i$ must be computed in order to obtain the global convergence of (16) by a suitable

assignment of the $n-m$ remaining eigenvalues. Combined with the input independence property, (15) will have the IIGS property.

B. IIGS conditions

In this subsection, the result ensuring the IIGS property is stated.

Let some matrices $\tilde{G}_i$ get the form :

$$\tilde{G}_i = \begin{bmatrix} \tilde{G}_i^{11} & \mathbf{0} \\ \mathbf{0} & \tilde{G}_i^{22} \end{bmatrix}$$

Besides, consider the partitions of some matrices $\tilde{S}_i$:

$$\tilde{S}_i = \begin{bmatrix} \tilde{S}_i^{11} & \tilde{S}_i^{12} \\ \tilde{S}_i^{21} & \tilde{S}_i^{22} \end{bmatrix}$$

Theorem 2. *The drive-response system (1-2) is IIGS if there exist symmetric positive definite matrices $\tilde{S}_i$, $\tilde{G}_i$ partitioned as above and matrices $\tilde{F}_i$ of appropriate dimensions such that*

$$\begin{bmatrix} \tilde{G}_i^{11} + (\tilde{G}_i^{11})^T - \tilde{S}_i^{11} & (\bullet)^T & (\bullet)^T & (\bullet)^T \\ -\tilde{S}_i^{21} & \tilde{G}_i^{22} + (\tilde{G}_i^{22})^T - \tilde{S}_i^{22} & (\bullet)^T & (\bullet)^T \\ (\tilde{A}_i^{11})^T G_i^{11} - (\tilde{A}_i^{21})^T \tilde{F}_i & (\tilde{A}_i^{21})^T \tilde{G}_i^{22} & \tilde{S}_j^{11} & (\bullet)^T \\ \mathbf{0} & \mathbf{0} & \tilde{S}_j^{21} & \tilde{S}_j^{22} \end{bmatrix} > 0 \qquad (21)$$

$\forall (i,j) \in \{1,\dots,P\} \times \{1,\dots,P\}$.

The gains $\tilde{K}_i^1$ satisfy $\tilde{K}_i^1 = (\tilde{G}_i^{11})^{-T} \tilde{F}_i^T$.

Proof Let $\tilde{A}_i$ be the matrices obtained from A_i under the piecewise linear similarity transform fulfilling (18). Furthermore, let $\tilde{L}_i$ be the gain matrices of which structure is defined as in Proposition 2. In this proposition, it has been shown in subsection III-A that such gains make the equation (16) of the error to be input independent. Besides, global convergence of (16) is equivalent to global convergence of (17) according to the result of section II-B. And yet, global convergence of (17) is also equivalent to global convergence of the dual system :

$$\tilde{\nu}_{k+1} = (\tilde{A}_i - \tilde{L}_i \tilde{C}_i)^T \tilde{\nu}_k \qquad (22)$$

Substitute A_i by $(\tilde{A}_i - \tilde{L}_i \tilde{C}_i)^T$, G_i by $\tilde{G}_i$ and S_i by $\tilde{S}_i$ in (11). Finally, making use to the change of variable $\tilde{F}_i = (\tilde{K}_i^1)^T \tilde{G}_i^{11}$, (21) is obtained. The Matrix Inequalities of Theorem 2 ensure (22) and so (16) to be poly-quadratic stable, which is sufficient for global asymptotical stability. Combined with the input independence property, (3) is fulfilled □

Remark 2. *It can be easily shown that the special diagonal structure of the matrices $\tilde{G}_i$ causes the problem to be solvable as a set of Linear Matrix Inequalities while a non diagonal structure would have lead to Bilinear Matrix Inequalities constraints. Consequently, such a matrix allows not only the linearity of the constraints (and so the convexity of the problem) but also prevents a restriction on the structure of $\tilde{S}_i$ involved in the PDLF, which would have rendered the problem to be more conservative. Indeed, let us recall that quadratic stability is a special case of poly-quadratic stability when taking $\tilde{G}_i = \tilde{S}_i = \tilde{S}^*$.*

Remark 3. *The global stability is ensured since all the switches are taken into account, regardless of the rule which orchestrates them. As a consequence, the theorem holds regardless of the chaotic motion exhibited by the drive. If the detailed study of the chaotic dynamics reveals that some specific switches cannot occur, the conservatism can be reduced by removing the corresponding pairs (i, j) in (21).*

C. General Design Procedure

A general procedure for designing an input independent global synchronization of a *drive-response* of the form (1-2), that is for computing the pairs (M_i, L_i), given A_i, E_i and C_i, consists in the following points.

i) perform T_i^c such that $C_i T_i^c = [\mathbf{0} \quad \mathbf{1}]$

ii) compute $\tilde{K}_i^1 = (\tilde{G}_i^{11})^{-T} \tilde{F}_i^T$ with $\tilde{G}_i^{11}$ and $\tilde{F}_i$ solutions of (21)

iii) compute successively $\tilde{L}_i = \tilde{A}_i \tilde{K}_i$ with $\tilde{K}_i = [\tilde{K}_i^1 \quad \mathbf{1}]^T$ and $L_i = T_i^c \tilde{L}_i$

iv) set $M_i = N_i$ with N_i the matrix whose columns span the null space of $A_i - L_i C_i$

It is recalled that IIGS allows the synchronization regardless to u_k. Now, the following proposition provides a way to recover u_k since it is not directly available on the *response* part.

Proposition 4 *Provided that $rank(C_i N_i) = m$, the quantity $\hat{u}_k$ defined as :*

$$\hat{u}_k = (C_i N_i)^{-1}(y_k - C_i \hat{x}_k) \tag{23}$$

will converge towards u_k.

Proof From (23) and the output equation of (1) with $M_i = N_i$, one obtains :

$$u_k - \hat{u}_k = -(C_i N_i)^{-1} C_i (x_k - \hat{x}_k) \tag{24}$$

Thus, global convergence of $\hat{x}_k$ towards x_k implies global convergence of $\hat{u}_k$ towards u_k. □

IV. Illustrative examples

To illustrate the results above, consider the map of which state space representation of the *drive* and the *response* is respectively of the form (1) and (2) with:

- $x_k = [x_k^1 \ x_k^2 \ x_k^3]^T$ and $\hat{x}_k = [\hat{x}_k^1 \ \hat{x}_k^2 \ \hat{x}_k^3]^T$
- $A_i = \begin{bmatrix} 0 & 0.4 & 1 \\ h_i & 0.4 & 0 \\ -0.8 & 0 & 0.9 \end{bmatrix}$, $h_1 = -1.12$, $h_2 = 1$
- $E_1 = [0 \ \ 0 \ \ 0]^T$ and $E_2 = [0 \quad -12.72 \ \ 0]^T$
- the two regions respectively associated to A_1 and A_2 are R_1 and R_2. R_1 is the set $\{x_k | x_k^1 < 6\}$ and R_2 is the set $\{x_k | x_k^1 \geq 6\}$
- the output matrix is chosen to be a constant matrix : $C = [0 \ 0 \ 1]$

This map exhibits a chaotic motion. This complex motion is used in order to mask an information u_k through an embedding corresponding to (2). Let us apply the design procedure of subsection III-C to ensure the recovering of u_k.

step *i)*

This step is useless since $T_i^c = \mathbf{1}$ as the output matrix C is already in the canonical form.

step *ii)*

Since T_i^c equals the identity matrix, $\tilde{A}_i = A_i$. The set of Linear Matrix Inequalities (21) are solved by using a LMI tool (see for instance [18]). The resulting unknown matrices involved in the gains computation are :

- $\tilde{G}_1^{11} = \tilde{G}_2^{11} = 0.6938$
- $\tilde{F}_1 = [-0.0514 \ \ 1.2837]$
- $\tilde{F}_2 = [0.0459 \quad -1.1462]$

The corresponding gains $\tilde{K}_i^1 = (\tilde{G}_i^{11})^{-T}\tilde{F}_i^T$ are $\tilde{K_1^1} = [-0.0741 \quad 1.8503]^T$ and $\tilde{K_2^1} = [0.0661 \quad -1.6520]^T$.

step *iii)*

The gains of the observer $L_i = \tilde{L}_i = \tilde{A}_i\tilde{K}_i$ are computed which yields $L_1 = [1.7401 \;\; 0.8231 \;\; 0.9593]^T$ and $L_2 = [0.3392 \;\; -0.5947 \;\; 0.8471]^T$.

step *iv)*

Both vectors spanning the one-dimensional null space of $A_i - L_iC_i$ ($m = 1$) associated to each region R_i are computed. We get $N_1 = [0.0352 \quad -0.8792 \quad -0.4752]^T$ and $N_2 = [0.0342 \quad -0.8550 \quad 0.5175]^T$. Then, we set $M_i = N_i$.

To assess IIGS, based upon the resulting pairs (M_i, L_i), the proposed *drive-response* setup has been designed such that the sinusoidal signal depicted on Fig 1A is masked by x_k of the chaotic map and recovered after synchronization.

Results are reported on Fig 1BCD. Before the discrete time $k = 60$, the gains of the observer are set to zero such that the *drive* and the *response* are not synchronized. On Fig 1B, it can be seen that before $k = 60$, the recovered signal is completely wrong. After $k = 60$ and a synchronization transient, the information u_k is successfully retrieved. Fig 1C and Fig 1D correspond respectively to the state reconstruction error $\|x_k - \hat{x}_k\|$ and the difference $u_k - \hat{u}_k$ between the recovered information and the initial one. Clearly, both errors converge towards zero.

Fig. 1. A : information u_k. B : recovered information $\hat{u}_k$. C : state reconstruction error $\|x_k - \hat{x}_k\|$. D : $u_k - \hat{u}_k$

V. Conclusion

Input independent global chaos synchronization of switched systems has been examined. The synchronization is achieved based upon a *response* system getting an observer structure. First, a suitable partial pole placement obtained by the choice of a special structure of the observer gains ensures a partial zero eigenvalues assignment and turns the equation governing the error of reconstruction to be independent from the input. Secondly, poly-quadratic stability, more general than quadratic stability, ensures the global convergence while preventing conservatism. The *drive-response* system is referred to as Input Independent Global Synchronized. A preliminary practical implementation of the scheme described in this note has shown the possible interest of the IIGS concept in a secured communication context [19] even though it deserves further works to obtain a viable encryption scheme. From a theoretical point of view, it is

worth pointing out that the problem treated here differs from the unknown input observer approach for piecewise linear systems investigated in [20][21] where the involved systems have relative degree equal to one.

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