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Inventory Management : Forecast Based Approach vs. Standard Approach

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Abstract

In this paper, we provide a literature review on inventory management policies. Two approaches are distinguished : the standard inventory management approach and the advance demand information based approach.

We focus on the advance demand information based approach. In particular, we study a pure single-stage and single-item inventory system where demand is given in the form of forecasts. Two forecast based inventory management policies are proposed, namely : the (r_k,Q) which is a dynamic reorder point policy and the (T,S_k) which is a dynamic order-up-to policy. These policies are compared to the standard (r,Q) and (T,S) policies. We also show that in certain cases the two forecast based inventory management policies and the standard inventory management policies are equivalent.

Further, a new safety parameter, called safety quantity, is introduced and compared to the classical safety stock parameter. A practical approach is proposed to compute this safety quantity.

Keywords

Inventory Management, Forecasts, Safety Stock, Safety Quantity.

1. Introduction

Nowadays, companies evolve in an industrial environment with an increasing competition. Moreover, customer demand is strongly influenced by several economic factors which make it more and more uncertain. To face that, with the development of the information systems, companies make efforts to anticipate the customer demand and to have relevant information, at the right time, in the right place of the supply chain. They try to direct, in parallel, their efforts towards the improvement of their forecasting models, in order to include them in their inventory management models so as to enhance the performance of their system. It should be said that an effective inventory management within the supply chain is the key of customer satisfaction and cost reduction.

In this work, we try to incorporate more realistic assumptions about the structure of the customer demand in inventory management models. New inventory management policies based on advance demand information are presented. More precisely, we suppose that the demand information is given in the form of uncertain forecasts. Then, the objective is to compare these policies to standard inventory management policies where no advance demand information is given, and to show the value of using forecasts in the inventory system, especially where the demand is non-stationary.

This paper is structured as follows : in section 2, we present a literature review on inventory management policies showing the different approaches used in most of the

existent studies. In section 3, we study more in details, a pure single-stage and single-item inventory system. We recall some results of the two basic standard inventory management policies, namely : the (r,Q) and the (T,S) policies. Next, we investigate two new forecast based inventory management policies, namely : the (r_k,Q) and the (T,S_k) policies, and we compare them to the (r,Q) and the (T,S) policies. We summarize the comparative study results in section 4. Some conclusions are given in section 5.

2. Literature Review

The literature dealing with inventory management policies is very rich and has grown fast during the last years. Below, we classify these policies into two approaches according to the type of demand information. In the first approach, the policies suppose that there is no advance demand information and the decisions are made in real time using the inventory depletion. We call this approach "Standard inventory management approach". The second approach includes all the inventory management policies that assume the existence of advance demand information as firm orders or forecasts.

In the standard inventory management approach, several policies were developed since the 30's. Most of the models assumes supply systems with exogenous lead-times. We cite, for example, the (r,Q) policy called reorder point policy, and the (T,S) policy called order-up-to-level policy. Several other variations of these policies have been developed : the (r,S) policy, which combines the two preceding policies by using, at the same time, a reorder point r and a replenishment level S. The (T,r,S) policy which is a combination of the (r,S) and (T,S) policies. For more details on these policies, see the work of Arrow et al. [1], Zipkin [14], and Silver and Peterson [13].

Gross and Harris [8] and Buzacott and Shanthikumar [4] consider supply systems with endogenous lead-times due to congestion effects. They study the Base Stock policy through a detailed analysis based on queuing theory. Note that these works are between the border of inventory management systems and production/inventory management systems.

During the last years, with the development of information technology, the literature on inventory systems has been oriented towards inventory management with advance demand information. The advance demand information can be given in the form of firm orders or forecasts.

Inventory management based on firm orders has occupied a significant place in the literature. Buzacott and Shanthikumar [4] study several models of inventory systems where orders are announced a fixed L units of time in advance of their due date. Karaesmen et al. [11,12] consider the same system and show the value of the advance demand information on the system's performance. They also study the structure of optimal releasing timing and inventory control in a discrete-time single-stage system and they give a near-optimal policy called BSADI (Base Stock With Advance Demand Information). Gallego and Ozer [6] investigate inventory systems with periodic review where advance demand information is in the form of demand placed in a period t, but not due until a future period $t+L$, and the demand lead-time L is typically fixed. Hariharan and Zipkin [9] present a thorough study on the benefits of customer order information for continuous-time inventory system. Their analysis reveals that advance demand information is a substitute for supply lead-times and can reduce safety stock levels and costs significantly when used effectively. A rich literature review on advance demand information based inventory systems is given by Karaesmen et al. [11].

There is also a body of related literature dealing with future demand information in the form of demand forecast. Heath and Jackson [10] and Graves [7] study mono-stage and multi-stage inventory systems managed by an Adaptive Base Stock policy in the presence of forecasts, and they used the MMFE model (Martingal Model of Forecast Evolution) for the updates of the forecasts vector. They also study the influence of the forecast techniques on the computation of the parameters of the system. Also relevant is the work of Chen et al. [5] which quantified the Bullwhip effect in a supply chain when the inventory management is done in the presence of uncertain forecasts. More recently, we propose a new forecast based inventory management approach [2,3]. We introduce the concept of forecast uncertainty and we show the impact of the forecast uncertainty model on the inventory management system.

3. Inventory Management : Forecast Based Policies vs. Standard Policies

In this section, we study a single-stage and single-item non-capacitated inventory system where the inventory replenishment requires a lead-time L (Figure 1).

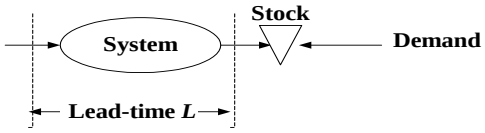

Figure 1. A pure inventory system model

We begin by giving an outline of the two basic standard inventory management policies : the (r,Q) and the (T,S) policies. Next, we present more in details the two forecast based inventory management policies proposed in [3], namely : the (r_k,Q) and the (T,S_k) policies. We describe these policies and the various parameters which characterize them. Finally, we compare these new policies to the standard ones.

3.1 Standard Inventory Management Policies : The (r,Q) and (T,S) Policies

In the standard inventory management approach, most of the models investigated in the literature assumes a stationary demand with a continuous or a periodic review of the system. The demand is modeled using a known probability distribution over a given horizon. Some studies propose the use of specific distributions for some categories of items based on the ABC classification [12]. In this approach, the decisions are made in real time taking into account the average and the variability of the demand

The basic policy which involves a continuous review is the (r,Q) policy. In this policy, a fixed quantity Q is ordered whenever the inventory position drops to the reorder point r or below (Figure 2). The quantity ordered is received after L units of time.

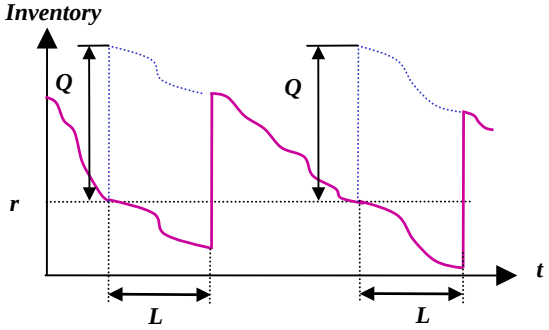

Figure 2. The (r,Q) policy

The basic policy which involves a periodic review is the (*T*,*S*) policy. In this policy, the control procedure is such that every *T* units of time (that is, at each review instant), a variable quantity (Q_k) is ordered to raise the inventory position to the level *S* (Figure 3). The quantity ordered is received after *L* units of time.

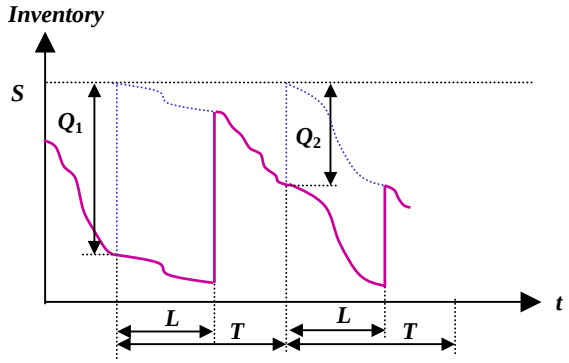

Figure 3. The (*T*,*S*) policy

In practice, the quantity *Q* and the review period *T* are computed using the Wilson's formula. The optimal values balance the inventory holding costs and the order costs. The reorder point *r* and the replenishment level *S*, are computed to cover the expected demand and the variability of the demand during the protection interval.

The protection interval (*PI*) corresponds, in the (*r*,*Q*) policy, to the lead-time *L*, whereas in the (*T*,*S*) policy, it is equal to the lead-time *L* and a review period *T*. Since the lead time *L* can be random, the protection interval can also be random.

In the *r* and *S* inventory levels, the component which covers the variability of the demand during the protection interval is called "Safety Stock". Since the demand is supposed stationary, the safety stock is computed and optimized, once and for all, in the beginning of the horizon in order to guarantee a target service level.

If the demand is normally distributed with parameters $(\overline{D},\sigma_D)$, and the protection interval is also normally distributed with parameters $(\overline{PI},\sigma_{PI})$, the safety stock (*Ss*) necessary to guarantee a target service level (*CSL*) is given by :

$$Ss = F_{D_{PI}}^{-1}(CSL) - \overline{D}.\overline{PI} = F^{-1}(CSL)\sigma_{D_{PI}} = F^{-1}(CSL)\sqrt{\overline{PI}\,\sigma_D{}^2 + (\overline{D})^2\,\sigma_{PI}{}^2}$$

If lead-times are constant, the safety stock is given by :

$$Ss = F^{-1}(CSL)\sqrt{PI}.\sigma_D$$

Where $F(.)$ is the standard normal cumulative probability distribution and $F_{D_{PI}}(.)$ is the cumulative probability distribution of the demand over the protection interval.

3.2 Forecast Based Inventory Management Policies

3.2.1 Demand Structure

We recall that in the standard inventory management approach, most of the studies uses a probability distribution to model the demand. Note that this is not judicious in the case of non-stationary demand since it is not appropriate to model a non-stationary demand over a large horizon with a fixed probability distribution. Thus, the necessity to consider an appropriate structure of the demand.

In this section, we present the structure of the demand we consider. In fact, we suppose that the demand is given in the form of uncertain forecasts. It means that, on each period, we have the forecast value and the probability distribution of the forecast uncertainty, as shown in Figure 4.

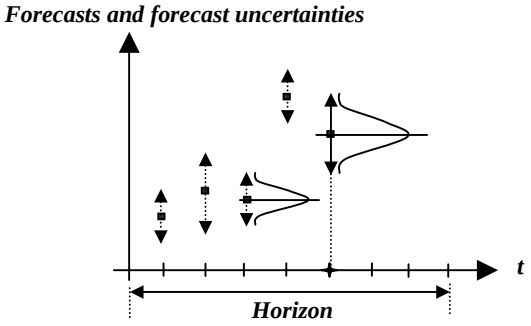

Figure 4. Demand structure in the forecast based approach

Even if the forecast model is pertinent, the forecast obtained from the forecast model is an average value, and the actual demand does not match in general with the average forecast. So, to have a good estimate of expected demand, we must determine a measure of the forecast uncertainty. The forecast uncertainty is also used to analyze whether past forecasts were accurate or not and to understand which mistakes were done when establishing the forecasts.

In practice, the forecast uncertainty statistics can be determined from the historic of the forecasts as the difference between the forecasts and actual demand. These statistics can be determined not only for each period (the individual forecast uncertainty) but also over some intervals composed by several single periods (the cumulative forecast uncertainty over an interval).

Note that it is necessary to have the cumulative forecast uncertainty over a given interval, because in the inventory management, it is useful to estimate the future forecasted demand during the protection interval. From these statistics, the probability distributions of individual and cumulative forecast uncertainties can be obtained and can be updated in real time. In this paper, we suppose that these probability distributions are obtained once and for all in the beginning of the horizon, meaning that they are not updated. This assumption remains valid if the horizon is not very large, since the probability distributions of forecast uncertainties will not change too much.

From the cumulative probability distribution of the cumulative forecast uncertainty over the protection interval (CFU_{PI}), we can determine a "maximal cumulative forecast uncertainty" corresponding to a certain target service level x, which we note $CFU_{PI}(x)$ (i.e. $CFU_{PI}(x) = F^{-1}_{CFU_{PI}}(x)$).

It should be noted that the forecast uncertainty may be absolute or relative. Forecast uncertainty is absolute if it is independent of forecasts values, and it is relative if it is proportional to the forecasts values. We show in this paper that the type of the forecast uncertainty has a significant impact on the parameters of the policies.

3.2.2 The (r_k,Q) and (T,S_k) Policies

Below, we present the two forecast based inventory management policies, namely the (r_k,Q) which is a dynamic reorder point policy and the (T,S_k) policy which is a dynamic order-up-to policy.

Here is the notation that is used throughout the paper :

F_k : Forecast at period k
PI : Protection Interval
L : Replenishment lead-time
T_f : Elementary forecast period (Here we assume that $T_f = 1$)
T : Review period
I_k : Inventory position at the end of period k
S_k : Replenishment level at period k
r_k : Reorder point at period k
CSL : Cycle Service Level

We suppose that L, T , PI are multiples of T_f.

3.2.3 The (r_k,Q) Policy

The (r_k,Q) policy is a "continuous" review policy. In this policy, the inventory is controlled at the beginning of each forecast period T_f. If the inventory position at the end of period k-1 (I_{k-1}) is less than the reorder point r_k, a quantity Q is ordered (Figure 5). We put the continuous word between quotation marks because the forecasts are expressed in terms of periods, so the review is not really continuous. But, it is obvious that if the T_f goes to 0, the model goes to a continuous review.

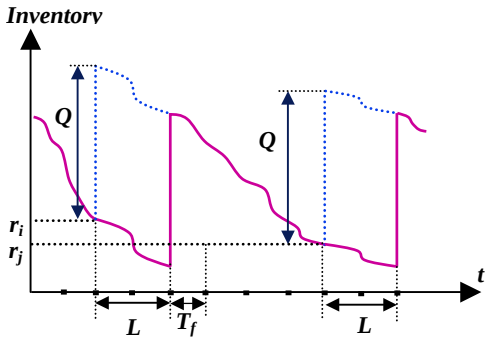

Figure 5. The (r_k,Q) policy

In the (r_k,Q) policy, the protection interval PI corresponds to the replenishment lead-time L and a single forecast period T_f, i.e. $PI = L+T_f = L+1$. In fact, the single period added to the formula of PI is due to the discrete time aspect in the (r_k,Q) policy.

The reorder point r_k corresponds to the cumulative forecasts and the maximal cumulative forecast uncertainty during $L+ T_f$, and it is given by :

$$r_k = \sum_{i=1}^{L+T_f} F_{k+i-1} + CFU_{L+T_f}(CSL)$$

On the contrary to the standard (r,Q) policy where the reorder point is constant, the reorder point in the (r_k,Q) policy is dynamic. In fact, the forecasts are dependant on time, and the cumulative forecast uncertainty over the protection interval can also be dependant on time (relative forecast uncertainty). Then, inevitably, the reorder point r_k is dynamic.

In this paper, we suppose the ordered quantity Q fixed and given. But, in practice, it can be computed using for example Wilson's formula, or the Silver-Meal heuristic.

3.2.4 The (T,S_k) Policy

In this policy, at the beginning of each period T, if the inventory position (I_{k-1}) is less than the replenishment level S_k, a quantity Q_k is ordered so that the stock is replenished up to the level S_k (Figure 6).

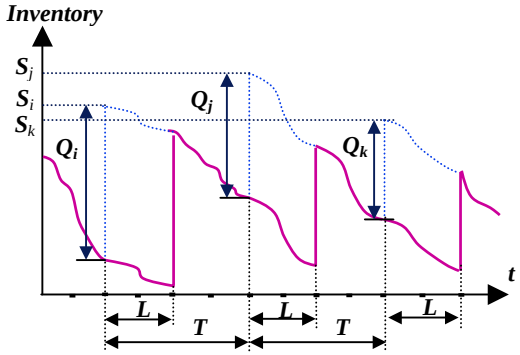

Figure 6. The (T,S_k) Policy

In the (T,S_k) policy, the protection interval PI corresponds to the replenishment lead-time L and the review period T ($PI = L+T$).

The replenishment level S_k is equal to the cumulative forecasts and the maximal cumulative forecast uncertainty during $L+T$, and it is given by :

$$S_k = \sum_{i=1}^{T+L} F_{k+i-1} + CFU_{T+L}(CSL)$$

On the contrary to the standard *(T,S)* policy where the replenishment level S is constant, in the *(T,Sk)* policy the replenishment level S_k is dynamic. The reasoning is the same as for the reorder point in the (Q,r_k) policy, since the formula of S_k is identical to that of r_k, and only the protection interval changes.

The ordered quantity Q_k is also computed dynamically. In fact, at each review period T, we order exactly the quantity that meets the maximal cumulative demand forecast over the protection interval. Mathematically,

$$Q_k = \max\{S_k - I_{k-1}\ ,\ 0\}$$

In this paper, we also suppose that the period T is given. But, in practice, it can be computed using for example Wilson's formula, or the Silver-Meal heuristic.

3.2.5 Safety Quantity

In forecast based inventory management, the forecasts are dependant on time and so is the forecast uncertainty. Consequently, the safety stock should be variable as well in order to match the forecasted demand. Thus, in forecast based inventory management, we use the term "Safety Quantity" (SQ) instead of "Safety Stock". The safety quantities play the same role as the safety stock but they are variable and are given in a dynamic way to cover forecast uncertainties with a specified service level. The safety quantity corresponds, in the (r_k,Q) and (T,S_k) formulas given above, to the term $CFU_{PI}(CSL)$.

To propose a practical computational approach of the safety quantities, assume that individual forecast uncertainty is normally distributed with parameters $(0,\sigma_k)$. In addition, assume that forecast uncertainties over the various periods are independent. Thus, the

cumulative forecast uncertainty over the protection interval is normally distributed with parameters $(0, \sigma_{CFU_{PI}})$, where :

$$\sigma_{CFU_{PI}} = \sqrt{\sum_{i=1}^{PI} \sigma_{k+i-1}^2}$$

The computation of safety quantities is done directly from the obtained probability distribution of the cumulative forecast uncertainty given a target service level *CSL*. The safety quantity, at a period *k*, is given by :

$$SQ_k = F_{CFU_{PI}}^{-1}(CSL) = F^{-1}(CSL)\sigma_{CFU_{PI}} = F^{-1}(CSL)\sqrt{\sum_{i=1}^{PI} \sigma_{k+i-1}^2}$$

Where $F(.)$ is the standard normal cumulative probability distribution and $F_{CFU_{PI}}(.)$ is the cumulative probability distribution of the cumulative forecast uncertainty over the protection interval.

4. Comparative Study Results

We summarize our comparative study in the table below, assuming we have a constant lead-time *L*. We provide all the parameters of the policies we described above. The periodic review and continuous review lines in the table correspond respectively to the (*T*,*S*) and (*r*,*Q*) policies in the Standard inventory management, and to the (T,S_k) and (r_k,Q) policies in the forecast based inventory management.

		Standard Inventory Management	**Forecast Based Inventory Management**
Protection Interval	Periodic review	$PI = L+T$	$PI = L+T$
	Continuous review	$PI = L$	$PI = L+1$
Reorder point	Continuous review	The reorder point is constant $r = \overline{D}.PI + Ss$	The reorder point is dynamic $r_k = \sum_{i=1}^{PI} F_{k+i-1} + SQ_k$
Replenishment level	Periodic review	The replenishment level is constant $S = \overline{D}.PI + Ss$	The replenishment level is dynamic $S_k = \sum_{i=1}^{PI} F_{k+i-1} + SQ_k$
Ordered Quantity	Continuous review	*Q* is computed using the Wilson's formula	*Q* is given *(Q* may also be computed by the Wilson's formula, or the Silver-Meal heuristic)
	Periodic review	$Q_k = \max\{S - I_{k-1}, 0\}$	$Q_k = \max\{S_k - I_{k-1}, 0\}$
Safety Parameter		Safety Stock $Ss = F^{-1}(CSL).\sqrt{PI}.\sigma_D$	Safety Quantity $SQ_k = F^{-1}(CSL)\sqrt{\sum_{i=1}^{PI} \sigma_{k+i-1}^2}$

Table 1. Policies' parameters comparison

It should be noted that if the forecasts are constant, the forecast based inventory management policies (r_k,Q) and (T,S_k) are respectively equivalent to the standard inventory management policies (r,Q) and (T,S). In fact, in this case the forecasts are equal to the average of the demand, and there is no difference between relative and absolute forecast uncertainty. Therefore, all the respective parameters of the policies are equal.

Proposition

If the forecast uncertainty is absolute and the lead-times are constant, then the safety quantities are also constant. Thus, the safety quantities are equivalent to a Safety Stock.

Proof

The safety quantity, at a period k, is given by : $SQ_k = F^{-1}(CSL)\sqrt{\sum_{i=1}^{PI} \sigma^2_{k+i-1}}$

If the forecast uncertainty is absolute, i.e. $\sigma_k = \sigma \ \forall k$, the safety quantities are given by :

$$SQ_k = F^{-1}(CSL).\sqrt{PI}.\sigma$$

So, the safety quantities are constant :

$$SQ_k = F^{-1}(CSL).\sqrt{PI}.\sigma \ \forall k$$

Thus, we obtain an equivalent safety stock : $SQ = F^{-1}(CSL).\sqrt{PI}.\sigma$. □

The formula of the safety quantity given above is the same than that of the safety stock with a constant lead-time, except that in SQ we use the standard deviation of the forecast uncertainty, whereas in Ss we use the standard deviation of the demand. In the case of a non-stationary demand, the standard deviation of the demand is generally greater than the standard deviation of the forecast uncertainty. Therefore, the equivalent safety stock in the case of the forecast based inventory management is lower than the safety stock in the case of the standard inventory management, which involves a considerable reduction in the inventory holding costs. Obviously, in the case of stationary demand with low variance, and with a forecast uncertainty variance close to the demand variance, the two safety stocks are very close.

5. Conclusions

In the first part of this paper, we provided an outline and a classification of several inventory management policies. We distinguished two approaches : the standard inventory management approach where there is no advance demand information, and the inventory management approach based on advance demand information given as firm orders or forecasts.

In the second part of the paper, we summarized some results of the well known (r,Q) and (T,S) policies, and then we investigated two new forecast based inventory management policies, which we called (r_k,Q) and (T,S_k) policies. We compared the (r_k,Q) and (T,S_k) parameters to the (r,Q) and (T,S) ones. We showed the value of using forecasts when the demand is non-stationary. We concluded that if the demand is stationary and the forecasts are constant, the two forecast based inventory management policies are equivalent to the two standard inventory management ones.

We also provided practical approach to compute the safety quantity in the forecast based inventory management policies. The safety quantity may be considered as a dynamic safety stock, that adjusts better to the variability of the demand. We showed that when the forecast uncertainty is absolute, the safety quantities are constant and they are equivalent to a safety stock.

It will be interesting to compare numerically the parameters of the policies. Moreover, in standard inventory management there are several other approaches to compute the policies' parameters, especially the safety stock parameter. It will be interesting also to extended this study by applying these approaches to the forecast based inventory management policies.

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