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# Thermomechanical Description of Moving Discontinuities, Application to Fracture and Wear

C. Stolz

*The propagation of moving surface inside a body is analysed within the framework of thermomechanical couplings, when the moving surface is associated with an irreversible change of mechanical properties. The moving surface is a surface of heat sources and of entropy production, intensities of which are related to particular energy release rates defined in terms of Hamiltonian gradients. As examples, we analyse the evolution of partial damage in a composite sphere and a model for study the contact wear phenomena between two bodies.*

## 1 Introduction

In the recent past, the propagation of damage has been studied in connection with fracture mechanics, and different approaches based on macroscopic or microscopic descriptions of mechanical degradation properties have been proposed.

During a loading history damage in continuum mechanics can be induced by the initiation and the growth of micro-cracks and micro-cavities. These descriptions, which are based on the evolution of the microscopic properties, propose to take the growth of pores or micro-cracks into account, through the idea that when some threshold value is reached, the material can not support further tensile loading.

Variational formulations were performed to describe the evolution of the surface between the sound and damaged material (Bui et al., 1981; Pradeilles-Duval and Stolz, 1995). In the framework of thermomechanical coupling as in fracture mechanics the analysis defines two different energy release rates associated with heat production and entropy production. (Stolz, 1995 ; Stolz and Pradeilles-Duval, 1996).

This paper is concerned mostly with the description of damage involved on the evolution of a moving interface along which mechanical transformation occurs. Some connections can be made with the notion of configurational forces, (Gurtin, 1995 ; Maugin, 1995 ; Truskinovski, 1987 ; Grinfeld, 1980,1991).

## 2 General Features

The domain  $\Omega$  is composed of two distinct volumes  $\Omega_1, \Omega_2$  of two materials with different mechanical characteristics. The bond between the two phases is perfect and the interface is denoted by  $\Gamma$ , ( $\Gamma = \partial\Omega_1 \cap \partial\Omega_2$ ). The external surface  $\partial\Omega$  is decomposed in two parts  $\partial\Omega_u$  and  $\partial\Omega_T$  on which the displacement  $u^d$  and the loading  $T^d$  are prescribed respectively.

The material 1 changes into material 2 along the interface  $\Gamma$  by an irreversible process. Hence  $\Gamma$  moves with the normal velocity  $c = \phi N$  in the reference state,  $N$  is the outward  $\Omega_2$  normal, then  $\phi$  is positive. When the surface  $\Gamma$  is moving, all the mechanical quantities  $f$  can have a jump denoted by  $[f] = f_1 - f_2$ , and any volume average has a rate defined by

$$\frac{d}{dt} \int_{\Omega(\Gamma)} f d\omega = \int_{\Omega(\Gamma)} \dot{f} d\omega - \int_{\Gamma} [f] c.N da$$

The state of the system is characterized by the displacement field  $u$ , from which a strain field  $\varepsilon$  is derived. The other parameters are the temperature  $\theta$  and the spatial distribution of the two phases given by the position of the boundary  $\Gamma$ . We analyse quasistatic evolution of  $\Gamma$  under a given loading prescribed on the boundary  $\partial\Omega$ .

The behaviour of the phase  $i$  is given by the free energy density  $w_i$ , function of the strain  $\varepsilon$  and of the

temperature  $\theta$ . The mass density of the two phases is the same  $\rho$ . The state equations of each phase are

$$\sigma_i = \rho \frac{\partial w_i}{\partial \varepsilon}, \quad s_i = -\frac{\partial w_i}{\partial \theta}$$

where  $\sigma_i$  is the reversible stress and  $s_i$  the entropy. If the materials have no viscosity then  $\sigma_i$  is the stress satisfying the momentum equation. If each  $w_i$  is a quadratic form, the two phases are linear elastic materials. The potential energy of the structure  $\Omega$  ( $\Omega_1 \cup \Omega_2$ ) has the following form

$$\mathcal{P}(u, \Gamma, T^d) = \sum_{i=1,2} \int_{\Omega_i} \rho w_i(\varepsilon(u), \theta) d\omega - \int_{\partial\Omega_T} T^d \cdot u da$$

The potential energy represents the global free energy in a thermodynamical description ; we can notice that the position of the interface  $\Gamma$  becomes an internal parameter for the global system. The characterization of an equilibrium state is given by the stationarity of the potential energy

$$\frac{\partial \mathcal{P}}{\partial u} \cdot \delta u = \sum_{i=1,2} \int_{\Omega_i} \rho \frac{\partial w_i}{\partial \varepsilon} : \varepsilon(\delta u) d\omega - \int_{\partial\Omega_T} T^d \cdot \delta u da = 0$$

for all  $\delta u$  kinematically admissible field satisfying  $\delta u = 0$  over  $\partial\Omega_u$ . This formulation is equivalent to the set of local equations :

- local constitutive relations

$$\sigma_i = \rho \frac{\partial w_i}{\partial \varepsilon}$$

- momentum equations

$$\text{div } \sigma = 0, [\sigma] \cdot N = 0 \text{ over } \Gamma, \sigma \cdot n = T^d \text{ over } \partial\Omega_T$$

- compatibility relations

$$2\varepsilon = \nabla u + \nabla^t u, [u] = 0 \text{ over } \Gamma, u = u^d \text{ over } \partial\Omega_u$$

At a given state of equilibrium for a given value of the prescribed loading ( $u^d, T^d$ ), the position of the interface  $\Gamma$  is known. At this time we apply a variation of the loading, the mechanical quantities evolve and propagation of the interface can occur according to a given evolution law. For a prescribed history of the loading, we must determine the rate of all mechanical fields and the normal propagation  $\phi$  to characterize the position of the interface  $\Gamma$  at each time.

Let us introduce the convected derivative  $D_\phi$  of a function  $f(x_\Gamma, t)$  defined along  $\Gamma$  as

$$D_\phi f = \lim_{\Delta t \rightarrow 0} \frac{f(x + \phi N \Delta t, t + \Delta t) - f(x, t)}{\Delta t}$$

As the contact is perfect between the phases the displacement and the stress vector are continuous along  $\Gamma$ . Their rates have discontinuities according to the general compatibility equations of Hadamard, rewritten with the convected derivative :

$$\begin{aligned} D_\phi [u]_\Gamma &= [v]_\Gamma - \phi [\nabla u]_\Gamma \cdot N = 0 \\ [D_\phi(\sigma \cdot N)]_\Gamma &= [\dot{\sigma}]_\Gamma \cdot N + \text{div}_\Gamma([\phi \sigma]) = 0 \end{aligned}$$

where  $\text{div}_\Gamma F = \text{div} F - N \cdot \nabla F \cdot N$ . The discontinuities of  $\sigma$  and  $\nabla u$  have a property of orthogonality as pointed by Hill (1986):  $[\sigma] : [\nabla u] = 0$ .

### 3 Dissipation Analysis

The mass conservation leads to the continuity of the mass flux  $m = \rho\dot{\phi}$ . The first law and the second law of thermodynamics give rise to local equations inside the volume and along the moving surface  $\Gamma$  :

$$\begin{aligned}\rho\dot{e} &= \sigma : \dot{\varepsilon} - \text{div } q, \text{ over } \Omega \\ 0 &= m[e] + N \cdot \sigma \cdot [v] - N \cdot [q], \text{ on } \Gamma\end{aligned}$$

where  $e$  is the internal energy density ( $e = w + \theta s$ ), and  $q$  is the heat flux associated to the heat conduction. Thanks to the Hadamard compatibility equations, the heat power supply is given in terms of a release rate of the internal energy as an objective quantity defined along  $\Gamma$

$$N \cdot [q] = G_{th}\dot{\phi}, \text{ with } G_{th} = \rho[e] - N \cdot \sigma \cdot [\nabla u] \cdot N = \rho[e] - \sigma : [\varepsilon]$$

The value  $G_{th}$  is obtained by considering the orthogonality condition of Hill. When  $\dot{\phi} = 0$ , in the reference state, the interface  $\Gamma$  does not move, and the normal flux is continuous. When the transformation occurs the moving interface is a surface of heat sources, the intensities of which are given by  $G_{th}\dot{\phi}$ . The total internal energy of the structure is

$$E(u, \Gamma, \theta, T^d) = \int_{\Omega(\Gamma)} e \, d\omega - \int_{\partial\Omega_T} T^d \cdot u \, da = \mathcal{P} + \int_{\Omega(\Gamma)} s\theta \, d\omega$$

For a quasistatic evolution, the first law of thermodynamics is written as follows :

$$\frac{dE}{dt} - \frac{\partial E}{\partial T^d} \cdot \dot{T}^d = - \int_{\partial\Omega} q \cdot n \, da$$

and taking into account of the momentum conservation, we have

$$\frac{\partial E}{\partial \Gamma} \cdot \dot{\Gamma} = - \int_{\Gamma} [q] \cdot N \, da = - \int_{\Gamma} G_{th}\dot{\phi} \, da$$

Then the derivative of the total energy relatively to the position of the interface is the source of heat due to the irreversible process.

The entropy production is given by

$$\int_{\Omega} \left( \rho\dot{s} + \frac{\text{div } q}{\theta} - q \cdot \frac{\nabla\theta}{\theta^2} \right) d\omega + \int_{\Gamma} \left( -m[s] + N \cdot \left[ \frac{q}{\theta} \right] \right) da \geq 0$$

Under the assumption of separability of the two dissipations, the term inside the volume is reduced to the conduction and the term along the surface is then

$$D_{\Gamma} = \frac{\rho[w] - N \cdot \sigma \cdot [\nabla u] \cdot N}{\theta} \dot{\phi} = \frac{G_s}{\theta} \dot{\phi}$$

where  $G_s$  is the release rate of free energy. This quantity has an analogous form to the driving traction force acting on a surface of strain discontinuity proposed by Abeyaratne and Knowles (1990). The criteria which guide the evolution of the interface may be written as function of this quantity.

In a thermomechanical coupling, two different release rates must be distinguished. One, defined in terms of variation of the total internal energy with respect to the position of the interface, determines the heat source associated with the moving surface ; the second one describes the production of entropy. In the case of an isothermal evolution, the total dissipation is given in terms of the derivative of the potential energy relatively to the position of the interface

$$\frac{d\mathcal{P}}{d\Gamma} \cdot \dot{\Gamma} = - \int_{\Gamma} G_s \dot{\phi} \, da$$

where  $G_s = \rho[w] - \sigma : [\nabla\varepsilon]$ . In this case, there is only one energy release rate to characterize the propagation. It gives the sources of heat production and the dissipation.

These relations can be generalized in the dynamical case, by replacing the internal energy of the system by its Hamiltonian, (Stolz, 1995, Stolz and Pradeilles-Duval 1996,1997).

#### 4 Quasistatic Evolution

In isothermal evolution, complementary relations must be considered to describe irreversibility. An energy criterion is chosen as a generalized form of the well known theory of Griffith. Then, we assume

$$\phi \geq 0, \text{ if } G_s = G_c \text{ on } \Gamma, \text{ and } \phi = 0, \text{ otherwise}$$

This is a local energy criterion. At each equilibrium state, the interface  $\Gamma$  can be decomposed into two subsets where the propagation is either possible or not. Let us denote by  $\Gamma^+$  the subset of  $\Gamma$  where the critical value  $G_c$  is reached. The evolution of the interface is governed by the consistency condition, during the evolution of  $\Gamma$  ; if at the geometrical point  $x_\Gamma(t)$  the criterion is reached

$$G_s(x_\Gamma(t), t) = G_c$$

then the derivative of  $G_s$  following the moving surface vanishes  $D_\phi G_s = 0$ . This leads to the consistency condition written for all points inside  $\Gamma^+$

$$(\phi - \phi^*) D_\phi G_s \geq 0, \forall \phi^* \geq 0, \text{ over } \Gamma^+$$

otherwise  $\phi = 0$ . With Hadamard relations the derivative defined above takes the final form

$$\begin{aligned} D_\phi G_s &= [t.\sigma].\nabla v_{1,t} - N.\dot{\sigma}_2.[\nabla u].N - \phi K \\ K &= t.\text{div}_\Gamma \sigma_2.[\nabla u].N - t.\sigma_2.\nabla([\nabla u].N) + [\sigma : \nabla \varepsilon.N] - N.\sigma.(\nabla \nabla u.N).N \end{aligned}$$

In that case, the evolution is determined by the functional

$$F(v, \phi, \dot{T}^d) = \int_\Omega \frac{1}{2} \varepsilon(v) : \frac{\partial^2 w}{\partial \varepsilon \partial \varepsilon} : \varepsilon(v) \, dw - \int_{\partial\Omega_T} \dot{T}^d . v \, d\mathbf{a} + \int_\Gamma (\phi [t.\sigma].\nabla v_{1,t} + \frac{\phi^2}{2} K) \, d\mathbf{a}$$

Then the evolution is given by :

$$\frac{\partial F}{\partial v} . (v - v^*) + \frac{\partial F}{\partial \phi} (\phi - \phi^*) \geq 0$$

for all  $v^*$  kinematically admissible fields, and  $\phi^*(s) \geq 0$  along  $\Gamma^+$ . The discussion of the stability and bifurcation along an evolution process can be investigate as presented in the paper Pradeilles-Duval and Stolz (1995). We consider the rate of the displacement  $v$  which is solution of the local equations :

$$\begin{aligned} \text{div } \dot{\sigma} &= 0, \dot{\sigma} = \frac{\partial^2 w}{\partial \varepsilon \partial \varepsilon} : \varepsilon(v), \text{ inside } \Omega \\ v &= v^d \text{ on } \partial\Omega_u, \dot{\sigma} . n = \dot{T}^d \text{ on } \partial\Omega_T \end{aligned}$$

and satisfying non-classical boundary conditions on  $\Gamma$ , for any given velocity  $\phi$  :

$$D_\phi(\sigma.N) = 0, D_\phi[u] = 0$$

We define the value  $W$  of  $F$  for such a field  $v(\phi, v^d, \dot{T}^d)$ , then  $W(\phi, v^d, \dot{T}^d) = F(v(\phi, v^d, \dot{T}^d), \phi, \dot{T}^d)$ . The stability of the actual state is determined by the condition of the existence of a solution

$$\delta\phi \frac{\partial^2 W}{\partial \phi \partial \phi} \delta\phi \geq 0, \delta\phi \geq 0 \text{ on } \Gamma^+, \delta\phi \neq 0$$

and the condition of uniqueness or non-bifurcation is given by

$$\delta\phi \frac{\partial^2 W}{\partial \phi \partial \phi} \delta\phi \geq 0, \delta\phi \neq 0 \text{ on } \Gamma^+$$

## 5 Composite Spheres Assemblage

In this section, the composite spheres assemblage of Hashin is analyzed, (Christensen and Lo, 1979). The system is composed by the compact assemblage of spheres with external radii in order to fill the whole domain. The microscopic structure is constituted by composite spheres with a core made of material 2 and the shell of material 1, both materials are linear elastic and homogeneous. As in the general case, materials 1 transforms into material 2 ; the transformation is irreversible and the criterion is a generalized Griffith's criterion based on the energy release rate of the transformation. Applying the same method than in (Hervé and Zaoui, 1991), the assemblage is considered as well-disordered. Using the particular three phase model, the homogeneous equivalent medium denoted by material 0 is unknown. In phase  $i$ , the local characteristics are the bulk modulus  $k_i$  and shear modulus  $\mu_i$ . In what follows,  $k_1$  is assumed to be larger than  $k_2$ .

### 5.1 Macroscopic Behavior with One Family of Spheres

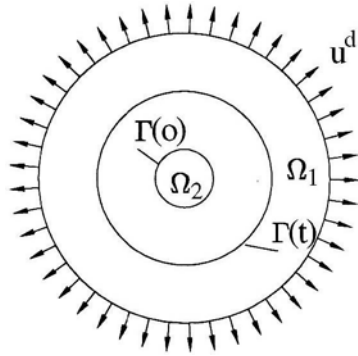
There exists only one family of composite spheres in the structure ;  $c$  is the concentration of material 2. Using analytical results obtained in (Hervé and Zaoui, 1991), one gets the bulk modulus of the material 0, denoted by  $k_o$

$$k_o = k_1 + c \frac{(k_2 - k_1)}{1 + \frac{3(k_2 - k_1)(1 - c)}{(4\mu_1 + 3k_1)}}$$

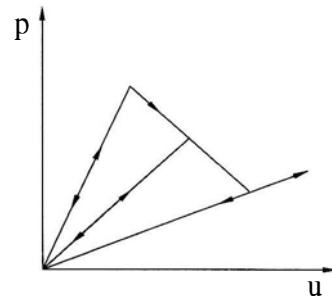
On the interface, the energy release rate is

$$G = \frac{\theta_o^2 (4\mu_1 + 3k_1)(4\mu_1 + 3k_2)(k_1 - k_2)}{2(4\mu_1 + 3k_2 + 3c(k_1 - k_2))^2}$$

where  $\theta_o$  represents the uniform strain ( $\varepsilon = \theta_o \mathbf{I}$ ) given at the infinity. When a generalized Griffith criterion is taken into account for the damage transformation, as  $G$  reaches the critical value  $G_c$ , the ratio  $c$  increases such that  $G$  equals to  $G_c$ . The behavior takes the form plotted in the following figures.



The composite sphere



The response of the composite sphere

### 5.2 Macroscopic Behavior with Two Families of Spheres

In what follows, we consider the macroscopic evolution of a composite spheres assemblage when two different families ( $I$  and  $II$ ) coexist in the structure. They are supposed to be perfectly disordered.  $c_I$  (respectively  $c_{II}$ ) denotes the volume fraction of material 2 in the first family (in the second one). If we denoted by  $G_I$  (resp.  $G_{II}$ ) the release rate of energy for the family  $I$  (resp.  $II$ ), we can shown the inequality

$$(G_I - G_{II})(c_I - c_{II})(\mu_1 - \mu_2) > 0$$

So, the global behavior of the system is the following one : at the beginning the macroscopic behaviour is linear elastic, until the energy criterion is reached and at this moment

- If  $\mu_1 > \mu_2$ , the difference between the two concentrations ( $c_I - c_{II}$ ) increases until the larger reaches 1.
- If  $\mu_1 < \mu_2$ , the difference between the two concentrations decreases until they are identical. That means that the two different families become only one.
- If  $\mu_1 = \mu_2$ , both concentrations could increase.

When whole volume is transformed, the behavior is the mechanical behavior of material 2. Even if the system is composed by only one family, the local response to the loading increment is non-unique. In fact, there can exist many kinds of bifurcations: one part of the structure can be damaged, that is some kind of damage localization ; or very well-ordered configurations can appear for which specific space distributions of the constituents phases are obtained

One gets order among disorder. If  $\mu_1 > \mu_2$  then a new perfectly disordered family can appear along the first one. In that case there is more disorder in the structure.

Here, it is to be underlined that the total dissipation is only due to the change of mechanical characteristic along a moving surface. The macroscopic behavior is dissipative while the components are always in a reversible process. The transformation between the two materials corresponds to a volume change at the macroscopic scale.

## 6 Dissipation Analysis in Dynamical Case

Now, we take the inertia effects into account. Thus the two thermodynamic principles must be rewritten. The mass conservation leads to the continuity of the mass flux  $m = \rho\phi$ . The first law and the second law of thermodynamics give rise to local equations inside the volume and along the moving surface  $\Gamma$  :

$$\begin{aligned} \rho\dot{e} &= \sigma : \dot{\epsilon} - \text{div } q, \text{ in } \Omega \\ 0 &= m\left[\dot{e} + \frac{v^2}{2}\right] + N \cdot [\sigma \cdot v] - N \cdot [q], \text{ on } \Gamma \end{aligned}$$

Then taking the conservation of the momentum and the continuity of the displacement into account :

$$[u] = 0, [\sigma] \cdot N = m[v]$$

we obtain the heat power supply defined by the internal release rate  $G_{th}$  ( $\bar{\sigma} = \frac{1}{2}(\sigma_1 + \sigma_2)$ ):

$$N \cdot [q] = G_{th}\phi, G_{th} = \rho[e] - N \cdot \bar{\sigma} \cdot [\nabla u] \cdot N$$

The total Hamiltonian of the structure is the sum of the kinetic energy and the total internal energy, the potential energy is defined as above :

$$H = \int_{\Omega} \frac{1}{2} \rho v^2 d\omega + \mathcal{P} + \int_{\Omega} \rho s \theta d\omega$$

The momentum conservation is then defined by the set of equations

$$\frac{\partial H}{\partial p} \bullet \delta p = \int_{\Omega} v \cdot \delta p d\omega, \frac{\partial H}{\partial u} \circ \delta u = -\frac{d}{dt} \int_{\Omega} p \cdot \delta u d\omega$$

where  $p$  is the momentum. These equations lead to the classical equations of motion. The first law of thermodynamics can be written as follows :

$$\frac{dH}{dt} - \frac{\partial H}{\partial T^d} \cdot \dot{T}^d = \int_{\partial\Omega} -q \cdot n da$$

and taking into account of the momentum conservation, we have

$$\frac{\partial H}{\partial \Gamma} \cdot \dot{\Gamma} = - \int_{\Gamma} [q] \cdot N \, da = - \int_{\Gamma} G_{th} \phi \, da$$

The second law has the same form as previously. The interface is perfect at each time. Under the assumption of separability of the two dissipations, the term inside the volume is reduced to the conduction and the term along the surface is then :  $D_{\Gamma} = \phi G_s / \theta$  where  $G_s$  has the form of a release rate of energy.

$$G_s = \rho[w] - N \cdot \bar{\sigma} \cdot [\nabla u] \cdot N$$

In a thermomechanical coupling, two different release rates must be distinguished, one defined in terms of variation of the Hamiltonian gives rise for the heat source associated with the moving surface, the second one describes the production of entropy.

In the case of an isothermal evolution, we can define another Hamiltonian

$$H = \int_{\Omega} \frac{1}{2} \rho v^2 d\omega + \mathcal{P}$$

and the total dissipation is then given by :

$$\frac{dH}{dt} - \frac{\partial H}{\partial T^d} \cdot \dot{T}^d = \frac{\partial H}{\partial \Gamma} \cdot \dot{\Gamma} = - \int_{\Gamma} G_{dyn} \phi da$$

where  $G_{dyn} = \rho[w] - N \cdot \bar{\sigma} \cdot [\nabla u] \cdot N$ .

## 7 Contact - Wear Phenomena

Wear phenomena are due to contact and relative motion between two solids and characterized by a loss of material from at least one of them. In the contact area the products of wear or lost particles and the damaged subsurfaces of both solids define an interface. Particles detach, asperities are cracked, failure occurs in this interface making its description difficult. Many works on wear are based on experimental observations which depend closely on operating conditions (for example the Archard law, Archard, 1953) and can not provide enough information for studying the wear of a structure (Meng and Ludema, 1995). We propose to characterize the continuous evolution of the boundary separating both solids from their common interface. These surfaces between sound and damaged materials are moving surfaces according with the loss of material due to wear phenomena. In the present work, we propose to derive a criterion for wear of both contacting bodies describing in a more fundamental manner the local quantities involved in the wear process.

We analyse a system of two contacting bodies. The two bodies are decomposed into a sound part  $\Omega_1$  (resp.  $\Omega_2$ ) and a damaged part  $\Omega'_1$  (resp.  $\Omega'_2$ ) separated by a perfect interface  $\Gamma_1$  (resp.  $\Gamma_2$ ). Along the contact area a third body appears  $\Omega_3$  composed by the area of contact, the detached particles, the two damaged materials  $\Omega'_i$ . The sound materials are characterized by known constitutive laws defined by a given free energy and a potential of dissipation. Damaged materials  $\Omega'_i$  correspond to the process zone where the wear takes place. They are transition zones between sound material to granular system. When the wear occurs, each  $\Gamma_i$  moves with a normal velocity  $\phi_i N_i$  where the normal  $N_i$  is oriented inward  $\Omega_i$ . All the conservation laws are written as previously

- the conservation of mass over  $\Gamma_i$  is reduced to

$$m_i = \rho_i \phi_i$$

- the conservation of the energy is

$$m_i [w + sT]_{\Gamma_i} - N_i \cdot \bar{\sigma} \cdot [v]_{\Gamma_i} + [q] \cdot N_i = 0$$



- the second law gives us the production of entropy as

$$D_{\Gamma_i} = \frac{m_i[w] - N \cdot \bar{\sigma} \cdot [v]}{T} \geq 0$$

Each interface  $\Gamma_i$  is a perfect bond, the displacement is continuous. If the mass flux  $m_i$  is null, the jump of the velocity vanishes. Then the dissipation  $D_{\Gamma_i}$  is characteristic of the loss of material associated with the wear phenomenon.

The third body  $\Omega_3$  has a small thickness and its average surface  $\Gamma$  defines the contact area between the bodies. An element of  $\Omega_3$  is defined by a small cylinder with area  $dS$  of  $\Gamma$  and a length  $h$  for the thickness. By unit area of contact  $dS$ , the total dissipation of the interface is then

$$D = \int_h D_3 dz + \sum_i D_{\Gamma_i} \geq 0$$

where  $D_3$  is the volumetric contribution to the dissipation of the interface  $\Gamma$  :

$$D_3 = -q_3 \cdot \frac{\nabla T}{T} + \frac{1}{T} (\sigma : \text{grad} v - \rho(\dot{w} + s\dot{T}))$$

This total dissipation contains two contributions, one due to friction associated to the relative motion of the two solids and the other related to the loss of mass  $m_i$ . The internal structure of  $\Omega_3$  could be analyzed by considering some mechanical characteristics. The solution of the problem of films of thickness  $h$  submitted to a uniform loading and shear gives information on the level of the two contributions.

On each  $\Gamma_i$ , the displacement is continuous. The mass flux is  $m_i = \rho\phi_i$ , where  $\phi_i$  is small relatively to the gliding motion between the two bodies. Then the stress vectors are continuous between the two bodies. The Hadamard compatibility equations give

$$[v]_{\Gamma} + \phi[\nabla u]_{\Gamma} \cdot N = 0$$

the dissipation can now be rewritten as

$$D = \int_h D_3 dz + \sum_i \frac{\phi_i}{T_i} (G_i - G_i^3) \geq 0$$

where the quantities

$$G_i = \rho w_i - N \cdot \sigma \cdot \nabla u \cdot N, \quad G_i^3 = \rho w_3 - N \cdot \sigma \cdot \nabla u_3 \cdot N$$

In the expression of the dissipation the nature of the two terms are different. The first is a dissipation inside the interface along the surface of contact  $\Gamma$ , which is essentially the term of friction, the condition of friction depends on the behaviour of the interface material. The others are those due to wear.

Assume now that the temperature is uniform and the evolution is isothermal. Moreover assuming that the behaviour of the interface is perfect plastic. When the shear reaches the yield stress  $\tau_c$ , the rate  $v$  is approximated by  $v = \frac{V}{h} z e_x$  so the dissipation is reduced to

$$D = \frac{1}{T} (\tau_c V + \sum_i \frac{\phi_i}{T_i} (G_i - G_i^3)) \geq 0$$

the first term takes the form of a friction law. A wear criterion is defined by a law between the propagation  $\phi_i$  and the thermodynamical force  $\gamma_i = G_i - G_i^3$ . If a potential of dissipation is given as a convex function of  $\gamma_i$ , applying the normality law ensures the positivity of the production of entropy. This gives rise to a local definition of wear. The identification of a law for wear, or such a potential remains still now the main difficulty.

## 8 Conclusion

We have considered the transformation of a material to another one with different mechanical characteristics along a moving interface as an irreversible process. This point of view can describe many situations corresponding to the transition between sound materials and damage materials.

The application of this idea for the description of wear-contact phenomena gives a definition of the mechanical characteristics associated to wear description. The description is local and makes in the dissipation the part due to friction and the part due to wear.

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