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**STATISTICAL AND NUMERICAL ANALYSIS  
OF THE NONLINEAR BEHAVIOR OF A POROUS MATERIAL  
WITH A HETEROGENEOUS DISTRIBUTION OF PORES**

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**ABSTRACT**

The yield stress of a ductile 2D porous material with a heterogeneous distribution of pores is investigated by means of a numerical tool based on the Fast Fourier Transform. Random distributions of circular voids, with a uniform or variable pore size and with overlapping or clearly separated pores, are compared, for a same overall porosity, to microstructures obtained as the intersection of the previous ones with a boolean scheme with a larger characteristic size. For pure shear and even more under purely hydrostatic load, the numerical simulations show that the pore distribution has a strong effect on the yield stress, with the so-called “Hashin Composite Cylinders Assemblage” being the hardest microstructure and the connected clusters the softest, while disconnected clusters lead to intermediate responses. An image analysis tool for the statistical characterization of the pore distribution is then proposed. It is based on the distribution function of the distance function, the width of which increases with the presence of clusters. An additional analysis allows one to distinguish microstructures with connected clusters from those with disconnected ones.

**INTRODUCTION**

The effective behavior of a porous material strongly depends on the spatial distribution of the pores as pointed out by Michel and al [5] using the numerical method based on the Fast Fourier Transform (F.F.T.) proposed by Moulinec and Suquet [6] [7] for the computation of the effective properties of heterogeneous materials with complex microstructure. This paper is concerned, first, with the study of the effect of the clustering of cylindrical voids on the yield stress using the F.F.T. approach and, second, with the application of a statistical analysis tool, based on the distance function to distinguish microstructures with a uniform distribution of pores from those with clusters. Attention is restricted to the class of periodic microstructures, as required by the F.F.T approach, but with large unit cells, so that the modeled microstructures can be assumed to behave nearly as totally random microstructures. The principle of the simulation of two-dimensional microstructures, with and without clusters, is first described. Then the effect of the heterogeneous distribution of voids on the yield stress is analyzed by means of numerical simulations. Finally, the statistical analysis tool based on the distance function is presented and applied to different microstructures.

## SIMULATION OF TWO-DIMENSIONAL PERIODIC MICROSTRUCTURES

The situation that we wish to model is that of a random distribution of cylindrical voids supposed to represent the microstructure of a 2D porous media. The boolean model is a good candidate to generate such a microstructure (see for example [3]). A binary image of the microstructure is obtained by the implementation, on a random point process, of primary grains of circular form. Depending on the assumption made on the implementation of these primary grains, different types of models are generated as represented in Figure 1.

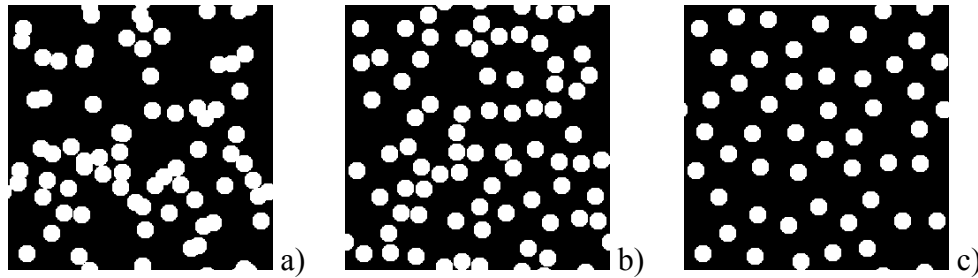


Figure 1. Examples of two-dimensional microstructures with identical pores size  
a) Classical boolean model b) Hard spheres model c) Cherry pit model.

In the classical boolean model, no restriction is made on the position of the disks, which can overlap (Figure 1a). In the two other models, a condition on the minimal euclidian distance between the edges of neighbor disks has to be fulfilled. In the hard spheres model, these distance is zero: the disks can't overlap but they can come into contact (Figure 1b); in the cherry pit model, the minimal distance is strictly positive so that the disks can neither overlap, nor come into contact (Figure 1c). To satisfy the periodicity of the images required by the FFT computations, one needs to take care of two additional conditions. First, a disk overlapping the edge of the image is split into several parts. Second, in the case of the hard spheres and the cherry pit models, disks in the neighboring images need to be taken into account.

## SIMULATION OF TWO DIMENSIONAL MICROSTRUCTURES WITH CLUSTERS

Starting from the above presented models of random microstructures, there are several possibilities to generate more complex structures in order to mimic, for instance, the fluctuations of the local volume fraction of defects, observed in real materials. In our study, such microstructures are obtained by the intersection of two independent random sets. This method consists in simulating a second random microstructure with a larger characteristic size and superpose it on the first one, as presented in figure 2. Any scheme can be used for the simulation of both random microstructures (boolean, hard spheres or cherry pit). However, both microstructures must satisfy the periodicity conditions. Microstructures with disconnected or connected clusters are then obtained by preserving only the disks resulting from the first set (scale of defects) whose centers are either inside or outside the disks of the second set (scale of clusters) as represented respectively in figures 2a and 2b.

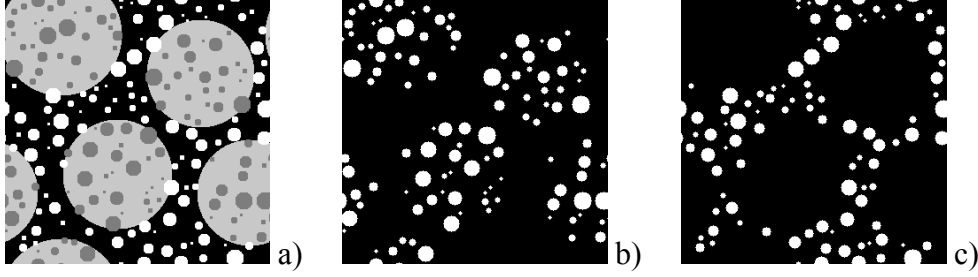


Figure 2. a) Superposition of two random microstructures with different characteristic sizes  
 b) Intersection of boolean schemes: disconnected clusters  
 c) Complementary of the intersection: connected clusters.

### EFFECTIVE FLOW SURFACE OF POROUS MATERIALS

Gurson [1] proposed to describe the behavior of a two-dimensional porous material, characterized by a macroscopic porosity  $f$  and a yield stress of the matrix  $\sigma_0$ , by the following criterion, where  $\Sigma_{\alpha\alpha} = \Sigma_{11} + \Sigma_{22}$  and  $\Sigma_{eq}$  is the von Mises stress:

$$\frac{\Sigma_{eq}^2}{\sigma_0^2} + 2f \cosh\left(\frac{\sqrt{3}}{2} \frac{\Sigma_{\alpha\alpha}}{\sigma_0}\right) - 1 - f^2 = 0. \quad (1)$$

This relation is exact for the hollow cylinder and, in fact, for any microstructure described by a Hashin's Composite Cylinders Assemblage [2], when submitted to a macroscopic stress state in the form (vector  $e_3$  being along the axis of the cylinders)

$$\Sigma = \Sigma_{11} (e_1 \otimes e_1 + e_2 \otimes e_2) + \Sigma_{33} e_3 \otimes e_3. \quad (2)$$

The effect of the heterogeneous distribution of cylindrical voids on the nonlinear behavior of a material with a perfectly plastic matrix is studied by means of the numerical method based on the Fast Fourier Transform (F.F.T.) proposed by Moulinec and Suquet [6, 7]. An augmented lagrangian method, recently proposed by Michel and al [5] to solve the nonlinear problem when the contrast between the phase is infinite, is used for our simulations. Six different microstructures, and four realizations of each of them, with a total porosity of  $f = 4.6\%$  are generated (Figure 3) according to the methods described in the previous sections.

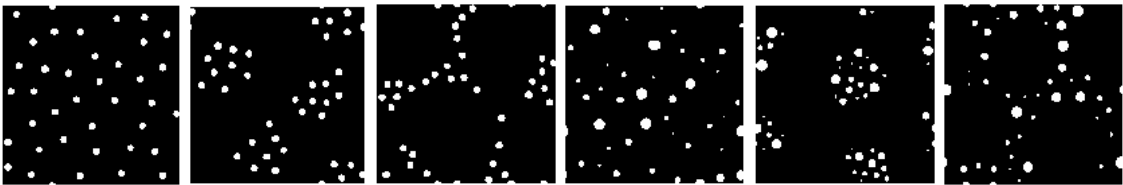


Figure 3. Periodic microstructures with identical and variable pore sizes, and with the same void volume fraction  $f = 4.6\%$ . Voids shown in white.

Each image is discretized into 1215×1215 pixels. Calculations are performed in the framework of generalized plane strains [4]. The overall stresses, considered in this study, are of the following form

$$\Sigma = T(e_1 \otimes e_1 + e_2 \otimes e_2 + e_3 \otimes e_3) + \gamma(e_1 \otimes e_2 + e_2 \otimes e_1). \quad (3)$$

For a purely hydrostatic load ( $T \neq 0$  and  $\gamma = 0$ ) and for pure shear in the transverse plane ( $T = 0$  and  $\gamma \neq 0$ ), microstructures with uniform distribution of pores with identical and variable pore sizes have a yield stress softer than Gurson's bound (eq. 1). Clustered microstructures have an even softer yield stress than those with uniform distribution of voids, and, in addition, there is a dependence on the connectedness of the clusters: microstructures with connected clusters exhibit a lower yield stress than those with disconnected clusters, as shown on Figures 4 and 5. Note that on these plots the error bars indicate the observed fluctuations on the four realizations of the microstructures. In general, the sensitivity of the overall yield stress to the microstructure is higher for hydrostatic load ( $\approx 50\%$  variation on the macroscopic yield stress) than for pure shear ( $\approx 10\%$  fluctuations).

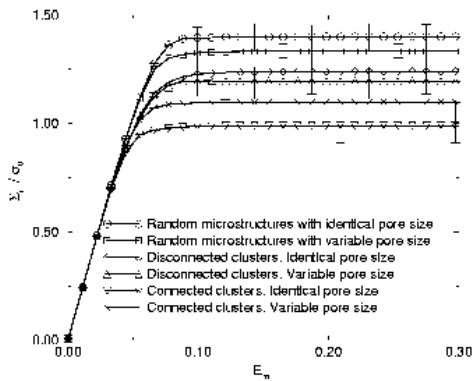


Figure 4. Hydrostatic load

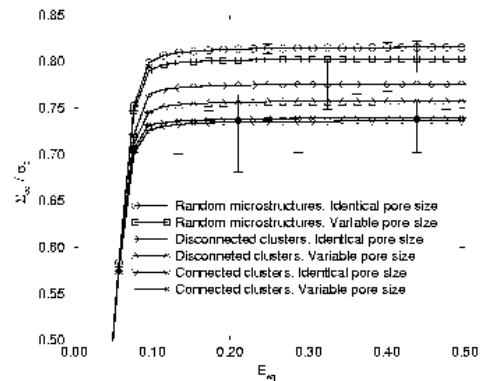


Figure 5. Pure shear

The variations of the overall yield stress are related to the fact that strains tend to localize into bands (in white in figures 6 and 7) whose orientation is governed by the overall load: horizontal or vertical for a pure shear. For a purely hydrostatic load, their orientation is only determined by the local configuration of the pores. Note that for microstructures described by Hashin's Composite Cylinders Assemblage all the matrix is plastified.

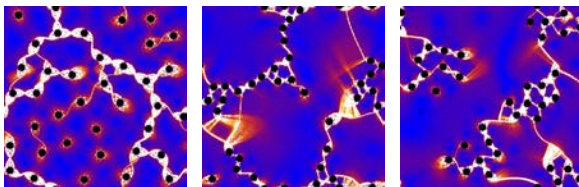


Figure 6. Contours of the local equivalent strain for a hydrostatic load

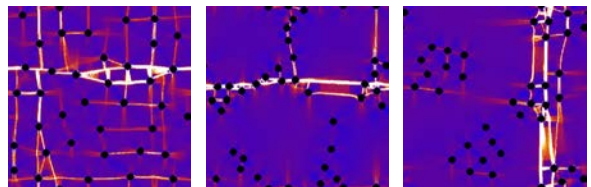


Figure 7. Contours of the local equivalent strain for a pure shear.

These simulations show how important the effect induced by the clustering of defect can be on the effective plastic behavior. It might thus be useful to define a tool allowing to detect clusters and their connectedness on real microstructures.

## STATISTICAL INFORMATIONS BASED ON THE DISTANCE FUNCTION

The statistical tool used to detect the presence of several microstructural scales (related to pores and clusters) is the distance function which gives for each point in the matrix (black phase in figures 3) the euclidian distance to the nearest pore. An extension of the distance function which takes into account the dispersion of the pore size in the material consists in calculating for each point  $i$  in the matrix the euclidian distance to the edge of the defect  $j$ , noted  $d_i^j$ , normalized by its radius  $r_j$  and taking its minimal value:  $d_i = \text{Min}_j \{d_i^j / r_p^j\}$ . The distribution of the distance function and of its normalized extension calculated on each microstructures presented in figure 3 is plotted in figures 8 and 9 respectively. Note that in figure 8 distances are scaled in such a way that the width of the unit cell is equal to 1.

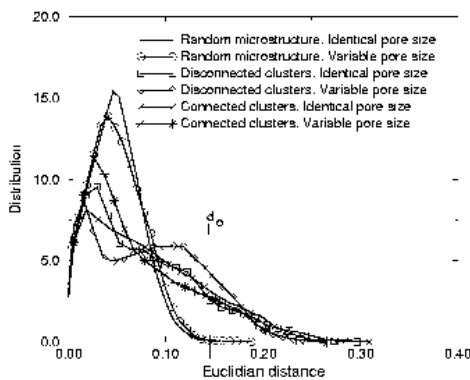


Figure 8. Distribution of the distance function

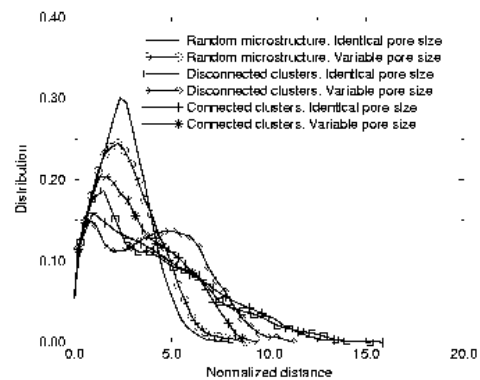


Figure 9. Distribution of the normalized distance function

The width of the distribution of the distance function is clearly larger for microstructures with clusters than for microstructures with a uniform distribution of pores, so that this tool can be used to characterize the tendency of real microstructures to form clusters. But this first approach does not allow one to distinguish connected and disconnected microstructures.

This is possible by thresholding the image of the distance function and analyzing the connectedness of the obtained binary image, which gives the set of matrix points (in gray in figures 11) whose distance to the nearest neighbor pore is lower than a given threshold distance  $d_0$  (Figure 8). The variation of the number of connex components of this gray set as a function of the threshold is given in Figure 10. When  $d_0$  is very small, the number of gray zones coincides with the number of pores in the microstructure. When  $d_0$  increases, this number decreases because of the overlapping of the gray zones. This decrease occurs much earlier and faster for clustered microstructures than for random ones, since the pores are closer to each other in the clusters. When  $d_0$  increases more, a horizontal straight line appears for the microstructures with disconnected clusters but not for the other ones, the number of gray zones at this plateau corresponding to the number of disconnected clusters. Microstructures with disconnected clusters can be differentiated from those with connected clusters this way.

This procedures can be applied to real microstructures, and even to 3D microstructures, even though the determination of the 3D spatial distribution of the pores would require costly microtomographic investigations of the porous material.

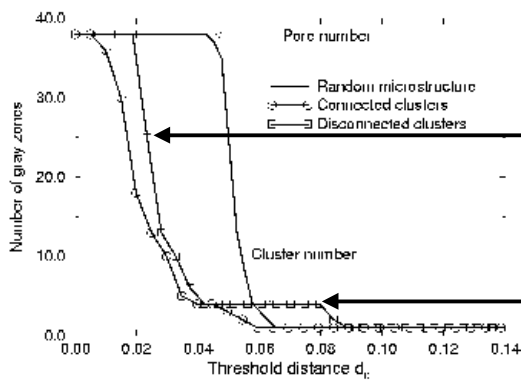


Figure 10. Number of gray zones. Case of a microstructure with identical pore size.

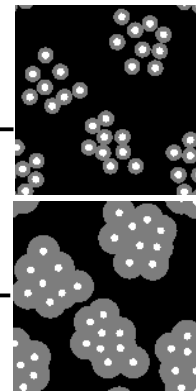


Figure 11. Thresholded distance function; two levels:  $d_0=0,023$  and  $d_0=0,066$

## CONCLUSION

Numerical simulations based on the Fast Fourier Transform show that the pore distribution has a strong effect on the yield stress, with the so-called “composite cylinders assemblage” being the hardest microstructure and the connected clusters the softest, while disconnected clusters lead to intermediate responses. A statistical analysis based on the distance function allows one to distinguish microstructures without clusters, with connected clusters or with disconnected clusters. Numerical investigations using the F.F.T. approach are currently under progress in order to quantify the effect of a heterogeneous distribution of pores on the local fields and on the macroscopic nonlinear properties in the three-dimensional case. Statistical analysis of the spatial pore distribution can also be extended to 3D microstructures.

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