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REMARKS ON INHERITANCE SYSTEMS

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Abstract

We try a conceptual analysis of inheritance diagrams, first in abstract terms, and then compare to “normality” and the “small/big sets” of preferential and related reasoning. The main ideas are about nodes as truth values and information sources, truth comparison by paths, accessibility or relevance of information by paths, relative normality, and prototypical reasoning.

AMS Classification: 68T27, 68T30

1 INTRODUCTION

Inheritance networks were introduced about 20 years ago (see e.g. [Tou84], [Tou86], [THT87]), and exist in a multitude of more or less differing formalisms, see e.g. [Sch97-2] for a brief discussion. There still does not seem to exist a satisfying semantics for these networks. The author's own attempt [Sch90] is an a posteriori semantics, which cannot explain or criticise or decide between the different formalisms. On the other hand, inheritance networks seem a quite natural way of treating defeasible information, so an attempt to present a semantics (which then, necessarily, will have to discuss natural concepts used in treating defeasible information) seems worth while. This is what we do here.

We discuss first an abstract semantics, which seems quite natural, in terms of information, information sources, accessibility of information sources, or relevance of the information from those sources, and comparison of information strength. We then discuss the resulting

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concept of normality, i.e. relative normality and not absolute normality, as present e.g. in preferential structures, and consequently in related systems like P or R (see e.g. [Sch04] for a presentation). Finally, we look at the resulting properties of (again relative) concepts of “small” and “big” subsets, and the (weak) laws which seem adequate.

Thus, the discussion is cut into the parts “Basic ingredients”, “Normality” “Small and big sets” (and “Various remarks” in conclusion).

The text is NOT self-contained, and familiarity with the basic concepts of inheritance systems and nonmonotonic logics in general is assumed. For a presentation, the reader might look into [Sch97-2] and [Sch04].

2 THE DISCUSSION

Our basic ingredients

(B1) Direct arrows represent information, valid for their source. Thus, in a set reading, if there is an arrow $A \rightarrow B$ in the diagram, most elements of A will be in B , in short: “most A 's are B 's”.

Consequently, our reading covers also enriched diagrams, where arbitrary information can be “appended” to a node.

(B2) Nodes are information sources. If $A \rightarrow B$ is in the diagram, A is the source of the information “most A 's are B 's”.

(B3) A valid, composed or atomic positive path σ from U to A makes the information of source A accessible to U . One might also say that A 's information becomes relevant to U .

(B4) An alternative way to see a source of information is to see it as a reason to believe the information it gives. U needs a reason to believe s.t., i.e. a valid path from U to the source of the information, and also a reason to disbelieve, i.e. if U' is below U , and U' does NOT believe some information of A , then either it has stronger information to the contrary, or there is not a valid path to A any more (and neither to any other possible source of this information).

(“Reason”, a concept very important in this context, was introduced by A.Bochman into the discussion.)

(B5) A valid, composed or atomic positive path σ from A' to A allows to compare the strength of information source A' with that of A : A' is stronger than A . (In the set reading, this comparison is the result of specificity: more specific information is considered more reliable.) If there is no such valid path, we cannot resolve contradictions between information from A and A' .

(B6) Thus, nodes in an inheritance diagram are also information sources of different

strengths or different truth values. (For simplicity, and to connect to other situations, I will speak now about truth values.) Valid positive paths allow to compare these truth values.

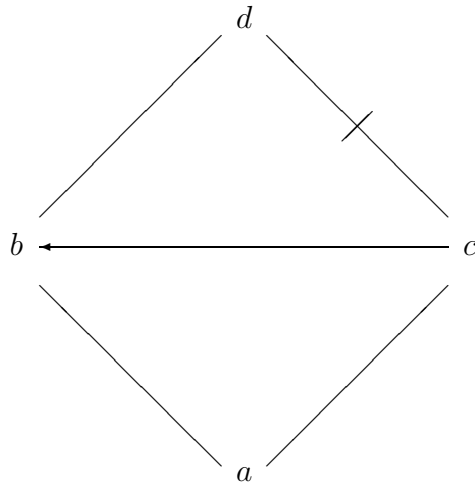
(B7) Thus, in a given node U , information from A is accessible iff there is a valid positive path from U to A , and if information from A' is also accessible, and there is a valid positive path from A' to A , then, in case of conflict, information from A' wins over that from A , as A' has a better truth value.

(In the usual drawings of preclusion diagrams, “access” corresponds to the vertical paths in the lower part of the diagram, and “comparison” to the horizontal ones. The upper part of the diagram represents just information. Tweety has access to penguins and birds, the horizontal link from penguin to bird compares the strengths, and the fly/not fly arrows are the information.)

Example 2.1

The Tweety diagram:

Read: a =Tweety, c =Penguins, b =Birds, d =Flying animals



In all diagrams, arrows point upwards, unless specified otherwise

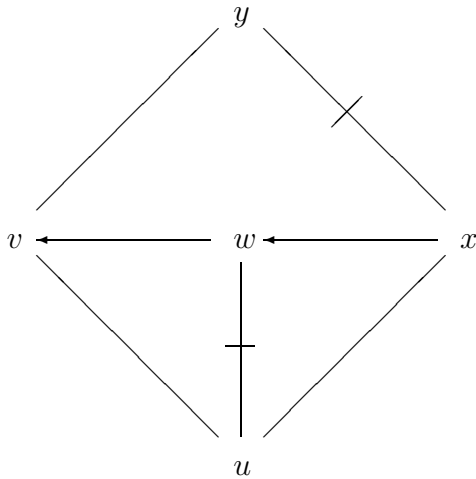
(B8) Thus, a negative direct link can only be information. A positive direct link is information at its source, but it can also be a comparison of truth values, or it can give

access from its source to information at its end. A valid positive, composed path can only be comparison of truth values, or give access to information, it is NOT information itself.

(B9) This interpretation results in split validity preclusion: the comparison between information sources A' and A is absolute, and does NOT depend on the U from which both may be accessible - as can be the case with total validity preclusion - see Diagram 6.9 in [Sch97-2] and its discussion there, which we repeat now. Here, the path $x \rightarrow w \rightarrow v$ is valid, so is $u \rightarrow x$, but not the whole preclusion path $u \rightarrow x \rightarrow w \rightarrow v$.

Example 2.2

Split vs. total validity preclusion:



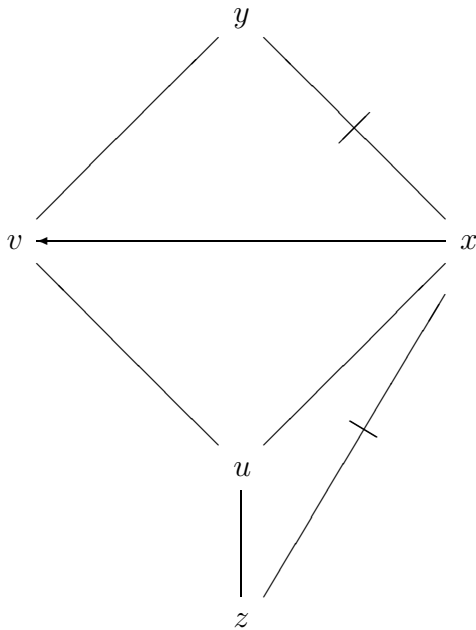
(B10) Inheritance diagrams in this interpretation do not only represent reasoning with many truth values, but also reasoning ABOUT those truth values: their comparison is done by the same underlying mechanism.

(B11) We obtain automatically again that direct information is stronger than any other information: If A has information ϕ , and there is a valid path from A to B , making B 's information accessible to A , then this same path also compares strength, and A 's information is stronger than B 's information. Seen from A , i.e. just considering information accessible to A , A 's own information will always be best.

(B12) Our interpretation underlines the importance of initial segments: Initial segments make information accessible. Thus, initial segments have to be valid. Diagram 6.8, p. 179, in [Sch97-2] (which might be due to folklore of the field) shows requiring downward chaining would be wrong:

Example 2.3

The problem of downward chaining:



Preclusions valid above (here at u) can be invalid at lower points (here at z), as part of the relevant information is not any more accessible (or becomes accessible). We have $u \rightarrow x \not\rightarrow y$ valid, by downward chaining, any valid path $z \rightarrow u \dots y$ has to have a valid final segment $u \dots y$, which can only be $u \rightarrow x \not\rightarrow y$, but intuition says that $z \rightarrow u \rightarrow v \rightarrow y$ should be valid. Downward chaining prevents such changes, and thus seems inadequate, so we decide for upward chaining. (Already preclusion itself underlines upward chaining: In the Tweety diagram, we have to know that the path from bottom up to penguins is valid. So at least some initial subpaths have to be known - we need upward chaining.) (The rejection of downward chaining seems at first sight to be contrary to the intuitions carried by the word “inheritance”.)

(B13) Otherwise, information is considered independent from each other - only (valid) paths create the dependencies.

A discussion of normality

(N1) Normality in the sense of preferential structures is absolute: if x is not in $N(X)$ ($= \mu(X)$), we do not know anything beyond classical logic. This is the dark Swedes' problem: even dark Swedes should probably be tall. Inheritance systems are different: If birds usually lay eggs, then penguins, though abnormal wrt. flying, will still usually lay eggs. Penguins are fly-abnormal birds, but will continue to be egg-normal birds - unless we have again information to the contrary. Normality in inheritance is relative, and as much normality as possible is preserved.

(N2) So the absolute, simple $N(X)$ of preferential structures splits up into many, by default independent, normalities, $N(X, \phi)$ for ϕ -normal etc. Preferential structures seem to be often an oversimplification.

(N3) This corresponds to intuition: There are no absolutely normal birds, each one is particular in some sense, so $\bigcap\{N(X, \phi) : \phi \in \mathcal{L}\}$ may well be empty, even if each single $N(X, \phi)$ is almost all birds. (Compare to the problem to define "human being".)

(N4) We try to preserve as much normality as possible. This can be pictured as follows: each ϕ -normality is a kind of wall, and normal elements are inside. If an element reveals itself as ϕ -abnormal, then we push it out of the ϕ -normality-wall, but keep it inside the other normalities, if possible. In addition, we cancel any properties which are based essentially on ϕ -normality.

(N5) So normality is treated by inertia: we give up only as little as necessary. An exception is: If the reason A to believe ϕ and ψ is not any more accessible, then we give up all information appended at A - but not more.

(N6) Both properties are based on default independence: by default, we presume properties to be independent, but if they are based on the same reason, then they all depend on the reason being accessible (if there is no better information to the contrary accessible).

(N7) This results in a minimal distance semantics: the universally normal situation is the prototype, and we try to stay as close as possible. If the situation is not prototypical, we do not give up all, but only as far as needed, which is the Hamming distance.

The reason is an economical one: we can stay (in thinking about it) close to the prototype, and its known properties, and an ontological one: our language was created to give by default independence of properties. If this independence does not exist, then we have a reason for it, and we know it. (This should still be further elaborated.)

(N8) What are the laws of relative normality? Following above discussion, $N(X, \phi)$ and $N(X, \psi)$ will be largely independent (except for trivial situations, where $\phi \leftrightarrow \psi$, ϕ is a tautology, etc.). (In softening independence, can we say that $N(X, \phi)$ and $N(X, \psi)$ are close, if ϕ and ψ are close?)

(N9) Note that, by relative normality, $N(X, \phi)$ might be defined, and $N(X, \psi)$ not. Thus,

if there is no arrow, or no path, between X and Y , then $N(X, Y)$ and $N(Y, X)$ - where X, Y are also properties - need not be defined. This will get rid of the unwanted connections found with absolute normalities: Penguins don't fly, birds do, penguins are a subclass of birds, so normal birds are not penguins. Yet, there is no arrow $Birds \not\rightarrow Penguins$ in the usual Tweety diagram. But there are NO normal birds in the absolute sense, only ϕ -normal birds. $N(bird, penguin)$ is simply not defined. This is like a new truth value, "undefined".

(N10) Up to now, we have not really used the relative normalities. We expressed doubts that this can be done in a meaningful way "horizontally", combining $N(X, \phi)$ and $N(X, \psi)$ etc., but there seems a natural way to work with them "vertically", combining $N(X, Y)$ and $N(Y, \phi)$ etc. in a kind of normality transfer. It seems that this is done better in the language of "small" and "big" subsets, but we have to stress that these notions are not absolute for given X , but also depend on ϕ, ψ etc. We turn to such sets.

Small sets

(S1) The usual language of inheritance networks motivates an interpretation by sets and soft set inclusion - $A \rightarrow B$ means that "most A 's are B 's". Just as with normality, the "most" will have to be relativized, i.e. there is a B -normal part of A , and a B -abnormal one, and the first is B -bigger than the second - where "bigger" is relative to B , too. A further motivation for this set interpretation is the often evoked specificity argument for preclusion. Thus, we will now translate our remarks about normality into the language of big and small subsets.

(S2) Recall our remarks about relative normality. $N(X, \phi)$ is, a priori, independent of $N(X, \psi)$, and $N(X, \phi)$ might be defined, but not $N(X, \psi)$. Thus, we will have ϕ -big subsets of X , and ψ -big subsets, and the two are independent, may have empty intersection, only one may be defined, etc.

(S3) Consider now the system P (with CUM). Small sets are used in two conceptually very distinct ways: $\alpha \sim \beta$ iff the set of $\alpha \wedge \neg\beta$ -cases is a small subset (in the absolute sense, there is just one system of big subsets of the α -cases) of the set of α -cases. The second use is in information transfer, used in CUM, or CM more precisely: if the set of $\alpha \wedge \neg\gamma$ -cases is a small subset of the set of α -cases, then $\alpha \sim \beta$ carries over to $\alpha \wedge \gamma : \alpha \wedge \gamma \sim \beta$. (See also the discussion in [Sch04], page 86, after Definition 2.3.6.) It is this transfer which we will consider here, and not things like AND, which connect different $N(X, \phi)$ for different ϕ .

(S4) Information transfer: If information ϕ is appended at A , and A is accessible from U (and there is no better information about ϕ available), ϕ will be valid at U . For simplicity, suppose there is a direct positive link from U to A , written sloppily $U \rightarrow A \models \phi$. In the big subset reading, we will interpret this as: $A \wedge \phi$ is a ϕ -big subset of A . We read now $U \rightarrow A$ also as: $U \cap A$ is an A -big subset of U . We want to conclude by transfer that

$U \cap \phi$ is a ϕ -big subset of U . We do this in two steps: First, we conclude that $U \cap A \cap \phi$ is a ϕ -big subset of $U \cap A$, and then, as $U \cap A$ is an A -big subset of U , $U \cap \phi$ itself is a ϕ -big subset of U . We do NOT conclude that $(U - A) \cap \phi$ is a ϕ -big subset of $U - A$, this is very important, as we want to preserve the reason of being ϕ -big subsets - and this goes via A !

Now, for the two steps in a slightly modified notation, corresponding to the diagram $X \rightarrow Y \rightarrow Z$:

(1) If $Y \cap Z$ is a Z -big subset of Y (by $Y \rightarrow Z$), and $X \cap Y$ is a Y -big subset of X (by $X \rightarrow Y$), then $X \cap Y \cap Z$ is a Z -big subset of $X \cap Y$.

(2) If $X \cap Y \cap Z$ is a Z -big subset of $X \cap Y$, and $X \cap Y$ is a Y -big subset of X (by $X \rightarrow Y$) again, then $X \cap Z$ is a Z -big subset of X , so $X \dots \rightarrow Z$.

Note that (1) is very different from CUM or even RM, as we do not say anything about X in comparison to Y : X need not be any big or medium size subset of Y .

Seen as strict rules, this will not work, as it is transitivity, and thus monotony: we have to admit exceptions, as there might just be a negative arrow $X \not\rightarrow Z$ in the diagram.

We try now to give justifications for the two (defeasible) rules. They will be philosophical and can certainly be contested and/or improved.

For (1):

We look at Y . By $X \rightarrow Y$, Y 's information is accessible at X , so, as Z -big is defined for Y , it will be defined for $Y \cap X$. Moreover, there is a priori nothing which prevents X from being independent from Y , i.e. $Y \cap X$ to behave like Y wrt. Z - by default: of course, there could be a negative arrow $X \not\rightarrow Z$, which would prevent this.

Thus, as $Y \cap Z$ is a Z -big subset of Y , $Y \cap X \cap Z$ should be a Z -big subset of $Y \cap X$. By the same argument (independence), we should also conclude that $(Y - X) \cap Z$ is a Z -big subset of $Y - X$. The definition of Z -big for $Y - X$ seems, however, less clear.

To summarize, $Y \cap X$ and $Y - X$ behave by default wrt. Z as Y does, i.e. $Y \cap X \cap Z$ is a Z -big subset of $Y \cap X$ and $(Y - X) \cap Z$ is a Z -big subset of $Y - X$. The reasoning is downward, from supersets to subsets, and symmetrical to $Y \cap X$ and $Y - X$. If the default is violated, we need a reason for it.

This default is an assumption about the adequacy of the language. Things do not change wildly from one concept to another (or, better: from Y to $Y \wedge X$), they might change, but then we are told so - by a corresponding negative link in the case of diagrams. See also Example 2.4 below.

For (2):

By $X \rightarrow Y$, X and Y are related, and we assume that X behaves as $Y \cap X$ does wrt. Z . This is upward reasoning, from subset to superset and it is NOT symmetrical: There is

no reason to suppose that $X - Y$ behaves the same way as X or $Y \cap X$ do wrt. Z , as the only reason for Z we have, Y , does not apply.

Note that, putting relativity aside (which can also be considered as being big/small in various, per default independent dimensions) this is close to the reasoning with absolutely big/small sets: $X \cap Y - (X \cap Y \cap Z)$ is small in $X \cap Y$, so a fortiori small in X , and $X - (X \cap Y)$ is small in X , so $(X - (X \cap Y)) \cup (X \cap Y - (X \cap Y \cap Z))$ is small in X by the filter property, so $X \cap Y \cap Z$ is big in X , so a fortiori $X \cap Z$ is big in X .

Thus, in summary, we conclude by default that,

(3) If $Y \cap Z$ is a Z -big subset of Y , and $X \cap Y$ is a Y -big subset of X , then $X \cap Z$ is a Z -big subset of X .

(This analysis might need elaboration in future work.)

(S5) Treatment of longer paths: Suppose we have a valid composed path from X to Y , $X \dots \rightarrow Y$, and not any longer a direct link $X \rightarrow Y$. By induction (upward chaining!) we argue - use directly (3) - that $X \cap Y$ is a Y -big subset of X , and conclude by (3) again that $X \cap Z$ is a Z -big subset of X .

(S6) Treatment of conflicts:

(1) Conflict with solution by preclusion:

We have valid paths $X \dots \rightarrow Y$, $X \dots \rightarrow Y'$, $Y \dots \rightarrow Z$, and direct links $Y \rightarrow Z$, $Y' \not\rightarrow Z$. $Y \rightarrow Z$ is considered the stronger information, by specificity. So for the conflicting part, Z prevails over $\neg Z$. The conflicting part is $X \cap Y \cap Y'$. We conclude that $X \cap Y \cap Z$ is big in $X \cap Y$, and continue as above, but do NOT conclude that $X \cap (Y' - Y) \cap Z$ is big in $X \cap (Y' - Y)$, on the contrary, $X \cap (Y' - Y) - Z$ is big in $X \cap (Y' - Y)$. This does not give us any information about X and Z itself, but about any parts of $X \cap (Y' - Y)$, i.e. subsets of X , which are in Y' , but not in Y - see again the lower part of Example 2.3 for an illustration: z there is in u and v , but not in x , so information at x is not any longer accessible from z .

(2) Conflict without solution by preclusion:

We have valid paths $X \dots \rightarrow Y$, $X \dots \rightarrow Y'$, and direct links $Y \rightarrow Z$, $Y' \not\rightarrow Z$. $Y \rightarrow Z$ is in unsolvable conflict with $Y' \not\rightarrow Z$, more precisely, we have an unsolvable conflict in $X \cap Y \cap Y'$ - the parts $X \cap Y - Y'$ and $X \cap Y' - Y$ are without conflict. We cannot conclude anything about $X \cap Y$ or $X \cap Y'$. Note that we cannot conclude either that $X \cap Y \cap Y'$ is a big part of $X \cap Y$ or of $X \cap Y'$, as Y and Y' are unrelated to each other, and we do not have absolute size.

(S7) This reading justifies immediately the truth value order via valid paths as a specificity ordering. So it suffices to justify the specificity criterion.

We can do this as follows: If A is more specific than B , and contradicts B concerning ϕ , then there is a reason (A) to do so, but in all subsets of A this reason is by default valid.

This is again due to a good choice of the language. Consider here the following artificial example, which serves to illustrate the importance of a well-chosen language.

Example 2.4

Let the universe be a subset of the integers, $U := [1, \dots, 10] \cup [-11, \dots, -1]$, and define the predicates $P_1 := U - \{-11, -10\}$, $P_2 := P_1 - \{9, 10\}$, etc. Thus, in the universe the majority is negative, in P_1 positive, in P_2 negative again, etc., so P_n is better characterized by P_{n-2} than by P_{n-1} , and specificity gives the wrong answer - when we interpret “normal” by majority.

(S8) Perhaps the deepest part of the analysis can be described as follows: Relevance is coded by positive arrows, and valid positive paths, and thus is similar to Kripke structures for modality. But, relevance (in this reading, which is closely related to causation) is profoundly non-monotonic, and any purely monotonic treatment of relevance would be insufficient. This seems to correspond to intuition. Relevance is then expressed formally by the possibility to combine different small/big sets.

This is, of course, a special form of relevance, there might be other forms of relevance.

Various remarks

(V1) The problem of intersection of extensions vs. direct scepticism goes far beyond inheritance networks. It is a general question how to treat contradictory and absent information, if they are equivalent or not. (The fundamental difference between intersection of extensions and direct scepticism for defeasible inheritance was shown in [Sch93].)

First, note that absolute normality introduces indirectly already a fourth truth value: a can be a normal ϕ -element, a ϕ -element without being a normal ϕ -element, a normal $\neg\phi$ -element, and a $\neg\phi$ -element without being a normal one. The situation is evidently more complicated, when we consider relative normality.

But it is also not at all clear that contradictory information is the same as absence of information. This is (almost) true in a classical situation. If something can only be true or false, then, if we cannot decide, but see that both cases give the same answer to a specific question, we can take the intersection, and have the result.

(V2) Inheritance networks combine many aspects of human reasoning, that’s why they are popular - and difficult.

(V3) Some questions to be answered in the future:

- is this analysis satisfying?
- does it still correspond sufficiently to standard procedures?
- can we generalize our ideas to other situations, revision, update, etc.?

3 ACKNOWLEDGEMENTS

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