Robust stability analysis and control design for switched uncertain polytopic systems

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ROBUST STABILITY ANALYSIS AND CONTROL DESIGN FOR SWITCHED UNCERTAIN POLYTOPIC SYSTEMS

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Abstract: This paper is dedicated to the study of robust stability analysis and control synthesis for discrete time uncertain switching systems under arbitrary switching. Polytopic uncertainties are considered. We show that Lyapunov functions that depend on the uncertain parameter and that take into account the structure of the system may be used in order to reduce the conservatism related to uncertainty problems. New LMI condition are obtained. We introduce the switched parameter dependent Lyapunov functions, i.e. functions that are based on a structure similar to that of the uncertain switched system. Necessary and sufficient LMI conditions for the existence of these functions are presented. A numerical example illustrates the conservatism of existing results. The approach is extended to switched state feedback design or state reconstruction for uncertain switched systems.

Keywords: Switched systems, Robust stability, LMI, Polytopic uncertainty.

1. INTRODUCTION

We call switched systems a class of hybrid systems consisting of a family of continuous/discrete subsystems and a rule that orchestrates the switchings between them (Liberzon and Morse, 1999). Stability and control synthesis are two important topics in the switched system field (DeCarlo *et al.*, 2000). Switched system stability can be checked via switched Lyapunov functions, i.e. a set of functions that are quadratic on the system state and that switch according to active subsystem (Daafouz *et al.*, 2002; Johansson, 2002; Branicky, 1998). For this approach, the stability conditions may be expressed in terms of linear matrix inequalities (LMI) (Boyd *et al.*, 1994), which have a notable practical interest due to the existence of powerful numerical solvers. The stability problem is very complex when parameter uncertainty is considered. In this case, the dynamic of each mode is affected by uncertainty. Until now, few results are concerned with robust stability in the context of switched systems. In the specialized literature (Zhai *et al.*, 2003; Z.Ji *et al.*, 2003*a*; Z.Ji *et al.*, 2003*b*) the results obtained for the non-uncertain case are directly applied : a common Lyapunov function is defined over the uncertainty of a subsystem. The Lyapunov functions switch similarly to the subsystem but they do not vary according to the uncertain parameters. The drawback of such methods is the conservatism inherent to the use of a common function over the whole uncertainty.

In the robust control domain, conditions based on parameter dependent Lyapunov functions (PDLF) are proposed in order to reduce the conservatism related to uncertainty problems (Dasgupta *et al.*, 1994; Feron *et al.*, 1996). Recently, in publication (J.Daafouz and Bernussou, 2001), these Lyapunov functions were used to analyze the stability of discrete systems with polytopic uncertainty. The solution is a class of Lyapunov functions that depends in a polytopic way on the uncertain parameter and that can be derived from LMI conditions.

In this paper we intend to study the robust stability and control synthesis of discrete time uncertain switching systems under arbitrary switching. We will consider that the uncertainty can be modeled in a polytopic way. We intend to reduce the conservatism related to uncertainty problems using Lyapunov functions that depend on the uncertain parameter and that take into account the structure of the system. Two approaches will be presented. First, we will prove that stability analysis for switched uncertain system with polytopic uncertainty can be addressed as the problem of analyzing an unique equivalent matrix polytope for which the parameter dependent Lyapunov function approach presented in publication (J.Daafouz and Bernussou, 2001) can be applied. The results are very good when the conservatism reduction is concerned. However, the method is not obvious to apply for control synthesis. Second, we will show that using parameter dependent Lyapunov functions that have a structure similar to the system leads to LMI conditions for both stability analysis and control synthesis. A new concept will be introduced : the switched parameter dependent Lyapunov functions, i.e. functions that depend in a polytopic way on the uncertainty of each mode and that are switched following the structure of the system. We will obtain a necessary and sufficient LMI condition for the existence of these functions. A numerical example will illustrate the advantage of these approaches compared to other conditions : the proposed conditions lead to Lyapunov functions that prove the asymptotic stability when all the other conditions are too conservative to be satisfied. It will also be shown how this functions are applied to the control synthesis problem : a stabilizing switched state feedback is constructed when the arbitrary switching can be detected in real time.

2. PRELIMINARIES

In this section we recall a stability result based on parameter dependent Lyapunov functions and show how it can help for robust stability analysis in the case of switched uncertain systems with polytopic uncertainty. Consider the uncertain discrete time system :

$$x(k+1) = \sum_{i=1}^{N} \alpha_i(k) A_i x(k), \ \sum_{i=1}^{N} \alpha_i(k) = 1, \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $k \in \mathbb{Z}^+$ is the discrete time and the $\alpha_i \geq 0$ represent the uncertain parameters. In (J.Daafouz and Bernussou, 2001), the stability of such a system is checked using PDLFs of the form :

$$V(k) = x^T(k) \sum_{i=1}^{N} \alpha_i(k) P_i x(k), P_i = P_i^T > 0. \quad (2)$$

A system is said *poly-quadratically stable* if there exists a PDLF (2) whose difference is negative definite (J.Daafouz and Bernussou, 2001).

Theorem 1. (J.Daafouz and Bernussou, 2001) System (1) is poly-quadratically stable iff there exists symmetric positive definite matrices S_i, S_j and matrices G_i of appropriate dimension such that :

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T A_i^T \\ A_i G_i & S_j \end{bmatrix} > 0 \quad (3)$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, N$. In this case, the Lyapunov function is given with $P_i = S_i^{-1}$.

Consider the uncertain switched system :

$$x(k+1) = \hat{A}_{\sigma(k)}(k) x(k) \quad (4)$$

where $\left\{ \hat{A}_i : i \in I \right\}$ with $I = \{1, 2, .., N\}$, is a family of matrices and $\sigma : Z^+ \to I$ is the switching signal. $\hat{A}_\sigma(k)$ is the uncertain matrix :

$$\hat{A}_\sigma(k) = \sum_{j=1}^{n_\sigma} \alpha_{\sigma j}(k) A_{\sigma j}, \ \sum_{j=1}^{n_\sigma} \alpha_{\sigma j}(k) = 1, \ \alpha_{\sigma j}(k) \geq 0$$

where the coefficients α_{ij} describe the polytopic uncertainty of the i^{th} mode of the systems, $A_{\sigma j}$ denote the extreme points of the polytope $\hat{A}_\sigma$ and n_σ is the number of such points. Consider $\mathcal{S} = \{A_{11}, \ldots, A_{1n_1}, \ldots, A_{N1}, \ldots, A_{Nn_N}\}$ the set of all vertices defining the dynamic of system (4) and $\mathcal{E} = \{E \mid E \in \mathcal{S}, \ co\mathcal{S} \neq co(\mathcal{S} - \{E\})\}$, $\mathcal{E} = \{E_1 \ldots E_M\}$, the set of extreme points of $co\mathcal{S}$. Here $co\mathcal{S}$ is the convex hull of $\mathcal{S}$ and M is the number of extreme points of $\mathcal{S}$ (numerical methods for convex hull and extreme points computation are described in (Ottmann *et al.*, 1995; Avis and Fukuda, 1992)).

Theorem 2. System (4) is stable if there exists symmetric positive definite matrices S_i, S_j and matrices G_i of appropriate dimension such that :

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T E_i^T \\ E_i G_i & S_j \end{bmatrix} > 0 \quad (5)$$

for all $i = 1, \ldots, M$ and $j = 1, \ldots, M$. The parameter dependent Lyapunov function is constructed with $P_i = S_i^{-1}$.

Proof. The proof is based on the fact that a convex combination of convex polytopes is also a convex polytope, in other words, on the fact that the system (4) may be expressed on the form (1), for which we have a stability criterion. System (4) is equivalent to

$$x(k+1) = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \xi_i(k)\alpha_{ij}(k)A_{ij}x(k), \quad (6)$$

$\xi_i : \mathbb{Z}^+ \rightarrow \{0,1\},\ \sum_{i=1}^{N} \xi_i(k) = 1,\ \forall k \in \mathbb{Z}^+$. Here, the switching function σ is replaced by the parameters ξ_i; $\xi_i = 1$ when $\sigma = i$ and 0 otherwise. This representation of the system is strictly equivalent with (4). Therefore, no additional conservatism is introduced. Consider the notation $\mathcal{A}$:

$$\mathcal{A} = \sum_{i=1}^{N} \xi_i(k)\hat{A}_i = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \xi_i(k)\alpha_{ij}(k)A_{ij} \quad (7)$$

which is $\mathcal{A}$ is a convex combination of $A_{ij} \in \mathcal{S}$. As E_p are the extreme points of $co\mathcal{S}$, we can write

$$\mathcal{A} = \sum_{p=1}^{M} \Lambda_p(k)E_p,\ \Lambda_p \geq 0,\ \sum_{p=1}^{M} \Lambda_p(k) = 1.$$

Therefore $\mathcal{A}$ is a polytopic uncertainty similar to that in equation (1). From equation (6), it can be noticed that any switched uncertain system with polytopic uncertainty (4) may be expressed as a simple uncertain system of the form (1). By applying Theorem 1, the proof is obvious.

Similar to publication (J.Daafouz and Bernussou, 2001), this approach can be directly applied to the switched state feedback stabilization problem, when the input matrix is known and constant for all system modes. However, in the general case, when both the dynamic and the input matrix are switched and uncertain, the construction of a switched state gain is not obvious.

3. SWITCHED PARAMETER DEPENDENT LYAPUNOV FUNCTIONS

To achieve the control synthesis, the switched parameter dependent Lyapunov function is introduced. It can be used for both proving the asymptotic stability of switched uncertain systems and constructing a switched state feedback when both the dynamic and the input matrix are switched and uncertain.

3.1 Robust stability analysis

Consider the switched uncertain system (4) and its equivalent representation (6). Based on a structure similar to the uncertainty description, we look for switched parameter dependent Lyapunov functions (SPDLF) :

$$V(k) = x^T(k)\hat{P}_\sigma(k)x(k),\ \hat{P}_\sigma(k) = \sum_{j=1}^{n_\sigma} \alpha_{\sigma j}(k)P_{\sigma j}$$
$$= x^T(k)\sum_{i=1}^{N}\sum_{j=1}^{n_i} \xi_i(k)\alpha_{ij}(k)P_{ij}\ x(k) \quad (8)$$

where $P_{ij} = P_{ij}^T > 0$. System (6) is asymptotically stable if the difference of the Lyapunov function along the solutions of (6) $\mathcal{L} = V(k+1) - V(k)$ satisfies :

$$\mathcal{L} = x^T(k)(\mathcal{A}^T\mathcal{P}_+\mathcal{A} - \mathcal{P})x(k) < 0 \quad (9)$$

$$\text{where } \mathcal{A} = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \xi_i(k)\alpha_{ij}(k)A_{ij},$$
$$\mathcal{P} = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \xi_i(k)\alpha_{ij}(k)P_{ij},$$
$$\mathcal{P}_+ = \sum_{i=1}^{N}\sum_{j=1}^{n_i} \xi_i(k+1)\alpha_{ij}(k+1)P_{ij}$$
$$= \sum_{m=1}^{N}\sum_{n=1}^{n_i} \xi_m(k)\alpha_{mn}(k)P_{mn},$$

$\forall k,\ \forall x(k),\ \forall \xi_i$ and $\forall \alpha_{ij}$ defined in (6).

Theorem 3. A switched parameter dependent Lyapunov function whose difference satisfy (9) can be constructed iff there exists symmetric positive definite matrices S_{ij}, S_{mn} and matrices G_{ij} of appropriate dimension such that :

$$\begin{bmatrix} G_{ij} + G_{ij}^T - S_{ij} & G_{ij}^T A_{ij}^T \\ A_{ij}G_{ij} & S_{mn} \end{bmatrix} > 0 \quad (10)$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, n_i$, $m = 1, \ldots, N$ and $n = 1, \ldots, n_m$. The switched parameter dependent Lyapunov function is constructed with $P_{ij} = S_{ij}^{-1}$.

Proof. To prove sufficiency, assume that the condition is feasible. Then

$$G_{ij} + G_{ij}^T - S_{ij} > 0.$$

Therefore G_{ij} is non singular and as S_{ij} is strictly positive definite, we have :

$$(S_{ij} - G_{ij})^T S_{ij}^{-1}(S_{ij} - G_{ij}) \geq 0,$$

which is equivalent to

$$G_{ij}^T S_{ij}^{-1} G_{ij} \geq G_{ij}^T + G_{ij} - S_{ij}.$$

Therefore the relation (10) implies

$$\begin{bmatrix} G_{ij}^T S_{ij}^{-1} G_{ij} & G_{ij}^T A_{ij}^T \\ A_{ij}G_{ij} & S_{mn} \end{bmatrix} > 0. \quad (11)$$

Pre- and post- multiplying the inequality (11) by $diag(G_{ij}^{-T}, S_{mn}^{-1})$ and its transpose gives

$$\begin{bmatrix} S_{ij}^{-1} & A_{ij}^T S_{mn}^{-1} \\ S_{mn}^{-1}A_{ij} & S_{mn}^{-1} \end{bmatrix} > 0 \quad (12)$$

Defining $P_{ij} = S_{ij}^{-1}$ the inequality (12) becomes

$$\begin{bmatrix} P_{ij} & A_{ij}^T P_{mn} \\ P_{mn}A_{ij} & P_{mn} \end{bmatrix} > 0$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, n_i$, $m = 1, \ldots, N$ and $n = 1, \ldots, n_m$. Repeatedly multiplying by the appropriate coefficients and summing one obtains :

$$\begin{bmatrix} \mathcal{P} & \mathcal{A}^T\mathcal{P}_+ \\ \mathcal{P}_+\mathcal{A} & \mathcal{P}_+ \end{bmatrix} > 0$$

From the Schur complement, this is equivalent to

$$\mathcal{A}^T\mathcal{P}_+\mathcal{A} - \mathcal{P} < 0 \quad (13)$$

which implies the existence of the switched parameter dependent Lyapunov function (8).

To prove necessity, assume that $\mathcal{L}$ satisfies (9). Then (13) is true, which implies that

$$P_{ij} - A_{ij}^T P_{mn} A_{ij} > 0$$

for all $i, m = 1..N$, $j = 1..n_i$, $n = 1..n_m$. A development similar to the one presented in (J.Daafouz and Bernussou, 2001) allows to end the proof.

3.2 Control Synthesis

In this subsection, the control synthesis problem via switched state feedback is considered for the following switching uncertain system :

$$x(k+1) = \hat{A}_\sigma(k)x(k) + \hat{B}_\sigma(k)u(k), \quad (14)$$

where

$$\hat{A}_\sigma = \sum_{j=1}^{na_\sigma} \alpha_{\sigma j}(k)A_{\sigma j}, \text{ and } \hat{B}_\sigma = \sum_{l=1}^{nb_\sigma} \beta_{\sigma l}(k)B_{\sigma l},$$

$$\sum_{j=1}^{na_\sigma} \alpha_{\sigma j}(k) = 1, \ \alpha_{\sigma j}(k) \geq 0, \quad (15)$$

$$\sum_{l=1}^{nb_\sigma} \beta_{\sigma l}(k) = 1, \ \beta_{\sigma l}(k) \geq 0, \ \forall\, k \in \mathbb{Z}^+$$

represent the uncertainty on the dynamic and input matrix, respectively. The switching signal is given by σ. Here $\alpha_{\sigma j}$ and $\beta_{\sigma l}$ are the uncertain parameters while na_σ and nb_σ represent the number of extreme points in the uncertainty $\hat{A}_\sigma$ and $\hat{B}_\sigma$, respectively. This system can also be expressed as :

$$x(k+1) = \sum_{i=1}^{N} \xi_i(k)\hat{A}_i(k)x(k) + \sum_{i=1}^{N} \xi_i(k)\hat{B}_i(k)u(k), \quad (16)$$

The closed-loop dynamic with the switched state feedback

$$u(k) = \sum_{i=1}^{N} \xi_i(k)K_i x(k) \quad (17)$$

is described by the equation:

$$x(k+1) = \sum_{i=1}^{N} \xi_i(k)(\hat{A}_i + \hat{B}_i K_i)x(k).$$

Notice that the switching signal σ and the switching parameters $\xi_i(k)$ are assumed to be available in the real time. With the uncertainty description (15), the equation becomes

$$x(k+1) = \sum_{i=1}^{N} \xi_i(k)\Big(\sum_{j=1}^{na_i} \alpha_{ij}(k)A_{ij} + \sum_{l=1}^{nb_i} \beta_{il}(k)B_{il}K_i\Big)x(k)$$

which is the same as :

$$x(k+1) = \sum_{i=1}^{N}\sum_{j=1}^{na_i}\sum_{l=1}^{nb_i} \xi_i(k)\alpha_{ij}(k)\beta_{il}(k)H_{ijl}x(k)$$

where $H_{ijl} = A_{ij} + B_{il}K_i$.

The switched parameter dependent Lyapunov function is given by :

$$V(k) = x^T(k)\mathcal{P}x(k)$$

with

$$\mathcal{P} = \sum_{i=1}^{N}\sum_{j=1}^{na_i}\sum_{l=1}^{nb_i} \xi_i(k)\alpha_{ij}(k)\beta_{il}(k)P_{ijl}. \quad (18)$$

The difference along the system trajectories is :

$$V(k+1) - V(k) = x(k)(\mathcal{H}^T\mathcal{P}_+\mathcal{H} - \mathcal{P})x(k),$$

where

$$\mathcal{H} = \sum_{i=1}^{N}\sum_{j=1}^{na_i}\sum_{l=1}^{nb_i} \xi_i(k)\alpha_{ij}(k)\beta_{il}(k)H_{ijl},$$

and

$$\mathcal{P}_+ = \sum_{i=1}^{N}\sum_{j=1}^{na_i}\sum_{l=1}^{nb_i} \xi_i(k+1)\alpha_{ij}(k+1)\beta_{il}(k+1)P_{ijl}$$
$$= \sum_{m=1}^{N}\sum_{u=1}^{na_m}\sum_{v=1}^{nb_m} \xi_m(k)\alpha_{mu}(k)\beta_{mv}(k)P_{muv}.$$

Theorem 4. System (16) is stabilizable via the control law (17) if there exists symmetric positive definite matrices S_{ijl} and S_{muv}, and matrices G_i and R_i, solutions of the LMI :

$$\begin{bmatrix} G_i + G_i^T - S_{ijl} & G_i^T A_{ij}^T + R_i^T B_{il}^T \\ A_{ij}G_i + B_{il}R_i & S_{muv} \end{bmatrix} > 0, \quad (19)$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, na_i$, $l = 1, \ldots, nb_i$, $m = 1, \ldots, N$, and $u = 1, \ldots, na_m$ and $v = 1, \ldots, nb_m$. The switched state feedback control is given by (17) with

$$K_i = R_i G_i^{-1}.$$

Proof. Assume the condition is feasible. By introducing K_i in (19) one obtains :

$$\begin{bmatrix} G_i + G_i^T - S_{ijl} & G_i^T A_{ij}^T + G_i^T K_i^T B_{il}^T \\ A_{ij}G_i + B_{il}K_iG_i & S_{muv} \end{bmatrix} > 0,$$

which is equivalent to

$$\begin{bmatrix} G_i + G_i^T - S_{ijl} & G_i^T H_{ijl}^T \\ H_{ijl}G_i & S_{muv} \end{bmatrix} > 0. \quad (20)$$

Similar to the proof of Theorem 3, one can show that the inequality (20) is the same as :

$$\begin{bmatrix} S_{ijl}^{-1} & H_{ijl}^T S_{muv}^{-1} \\ S_{muv}^{-1} H_{ijl} & S_{muv}^{-1} \end{bmatrix} > 0 \qquad (21)$$

Defining $P_{ijl} = S_{ijl}^{-1}$, the inequality (21) can be written as

$$\begin{bmatrix} P_{ijl} & H_{ijl}^T P_{muv} \\ P_{muv} H_{ijl} & P_{muv} \end{bmatrix} > 0$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, na_i$, $l = 1, \ldots, nb_i$, $m = 1, \ldots, N$, and $u = 1, \ldots, na_m$ and $v = 1, \ldots, nb_m$. By repeatedly multiplying by the appropriate coefficients and summing one gets:

$$\begin{bmatrix} \mathcal{P} & \mathcal{H}^T \mathcal{P}_+ \\ \mathcal{P}_+ \mathcal{H} & \mathcal{P}_+ \end{bmatrix} > 0$$

From the Schur complement, this is equivalent to

$$\mathcal{H}^T \mathcal{P}_+ \mathcal{H} - \mathcal{P} < 0$$

which implies the existence of a Lyapunov function of the form (18).

Remark. By duality, the results obviously apply to the state reconstruction problem for uncertain switched systems with uncertain output matrix.

4. NUMERICAL EXAMPLES

Example 1. To illustrate the LMI stability conditions derived in the previous sections, we will consider a switched uncertain system with affine uncertainty of the form :

$$x(k+1) = \hat{A}_\sigma x(k) \text{ where}$$

$$\hat{A}_\sigma(k) = A_{0\sigma} + \rho(k) A_\sigma, \ \rho(k) \in [-1, 1].$$

This kind of uncertainty can be expressed as a norm bounded uncertainty

$$\hat{A}_\sigma(k) = A_{0\sigma} + D_\sigma F(k) E_\sigma, \ F(k) \in [-1, 1],$$

with $\rho = F$ and $A_\sigma = D_\sigma E_\sigma$, and as a polytopic uncertainty

$$\hat{A}_\sigma(k) = \alpha_{\sigma 1}(k) A_{\sigma 1} + \alpha_{\sigma 2}(k) A_{\sigma 2},$$

with

$$A_{\sigma 1} = A_{0\sigma} + D_\sigma E_\sigma \text{ and } A_{\sigma 2} = A_{0\sigma} - D_\sigma E_\sigma,$$

$$\alpha_{\sigma 1}(k), \alpha_{\sigma 2}(k) > 0, \ \alpha_{\sigma 1}(k) + \alpha_{\sigma 2}(k) = 1, \ \forall k \in \mathbb{Z}^+.$$

Such an uncertain switched system can be used for comparing the previous LMI conditions, based on quadratic stability (Z.Ji *et al.*, 2003*a*) and on a norm bounded uncertainty approach (Z.Ji *et al.*, 2003*b*), to the conditions of our paper. Let

$$A_{01} = \begin{bmatrix} 0.2 & 0.2 & 0.3 & 0.1 & -0.5 \\ 0.8 & 0 & -0.1 & -0.3 & 0.3 \\ 0 & -0.3 & -0.4 & 0 & 0 \\ 0 & 0.3 & 0.1 & 0.3 & 0.5 \\ -0.2 & 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$A_{02} = \begin{bmatrix} -0.7 & -0.7 & 0 & 0 & 0.2 \\ 0.5 & 0.3 & 0.3 & -0.3 & 0 \\ 0.3 & 0.4 & 0.3 & 0.6 & 0.3 \\ 0.3 & -0.8 & 0 & 0 & 0 \\ 0.1 & -0.7 & 0.1 & -0.3 & 0.3 \end{bmatrix},$$

$$D_1^T = [0.2 \ 0.5 \ -0.1 \ 0.3 \ 0.2],$$

$$D_2^T = [-0.5 \ 0.38 \ 0.5 \ 0.2 \ 0.5],$$

$$E_1 = [-0.3 \ -0.3 \ -0.5 \ 0.2 \ 0.3]$$

$$E_2 = [-0.2 \ 0.1 \ -0.1 \ -0.05 \ 0.7].$$

In this case, all the existing LMI conditions (Z.Ji *et al.*, 2003*a*; Z.Ji *et al.*, 2003*b*) prove to be too conservative and do not have any solution.

Yet, the LMI conditions (5) and (10) here presented have solutions. The existence of these solutions was tested by a numerical LMI solver (SEDUMI). Hence, a parameter dependent and a switched parameter dependent Lyapunov function can be constructed using conditions (5) and (10).

Example 2. We will consider the uncertain switched system :

$$x(k+1) = \hat{A}_\sigma(k) x(k) + \hat{B}_\sigma(k) u(k), \qquad (22)$$

where

$$\hat{A}_\sigma(k) = A_{0\sigma} + D_\sigma F E_\sigma^A,$$

$$\hat{B}_\sigma(k) = B_{0\sigma} + D_\sigma F E_\sigma^B, F \in [-1, 1],$$

represent norm bounded uncertainties similar to the previous example. One should remark that the two uncertainties, $\hat{A}_\sigma$ and $\hat{B}_\sigma$, are varying with the same uncertain parameter F. This choice of F is useful when comparing with the norm bounded uncertainty approach in publication (Z.Ji *et al.*, 2003*b*). These uncertainties can be expressed in the polytopic form:

$$\hat{A}_\sigma(k) = \lambda_{\sigma 1}(k) A_{\sigma 1} + \lambda_{\sigma 2}(k) A_{\sigma 2},$$

$$\hat{B}_\sigma(k) = \lambda_{\sigma 1}(k) B_{\sigma 1} + \lambda_{\sigma 2}(k) B_{\sigma 2},$$

with

$$A_{\sigma 1} = A_{0\sigma} + D_\sigma E_\sigma^A, \ A_{\sigma 2} = A_{0\sigma} - D_\sigma E_\sigma^A,$$

$$B_{\sigma 1} = B_{0\sigma} + D_\sigma E_\sigma^B, \ B_{\sigma 2} = B_{0\sigma} - D_\sigma E_\sigma^B,$$

$$\lambda_{\sigma 1}(k), \lambda_{\sigma 2}(k) > 0, \ \lambda_{\sigma 1}(k) + \lambda_{\sigma 2}(k) = 1, \ \forall k \in \mathbb{Z}^+.$$

This equivalent polytopic uncertainty is a particular case of equation (15) with $j = l$, $na_\sigma = nb_\sigma = 2$, and $\alpha_{\sigma j} = \beta_{\sigma l} = \lambda_{\sigma j}$. Therefore, the LMI condition (19) becomes :

$$\begin{bmatrix} G_i + G_i^T - S_{ij} & G_i^T A_{ij}^T + R_i^T B_{ij}^T \\ A_{ij} G_i + B_{ij} R_i & S_{uv} \end{bmatrix} > 0, \qquad (23)$$

for $i, u = 1..N$, $j, v = 1..2$, where S_{ij}, S_{uv} are positive definite symmetric matrices.

We will consider that not all the $\hat{A}_i(k)$ matrices are Hurwitz and we will apply theorem (4) in order to construct a switched state feedback of the form (17). For $N = 2$,

$$A_{01} = \begin{bmatrix} -0.1 & 0.7 & -0.2 \\ -0.4 & 0.7 & 1 \\ 0.3 & 0.3 & 0 \end{bmatrix}, \; A_{02} = \begin{bmatrix} 1 & 0.7 & 0.7 \\ 0.4 & 0.6 & 0.2 \\ 1 & 0.7 & 0 \end{bmatrix},$$

$$B_{01} = [0.1\ 0.8\ 0.8]^T, \; B_{02} = [0.2\ 0.9\ 0.2]^T$$
$$D_1 = [-0.2328\ 0.4340\ -0.4590]^T,$$
$$D_2 = [-0.2645\ 0.2681\ 0.9316]^T,$$
$$E_1^A = [0.7461\ -0.4767\ 0.1131],$$
$$E_2^A = [-0.4787\ 0.4671\ 0.4731],$$
$$E_1^B = 0.8194 \text{ and } E_2^B = -0.7610$$

The LMI condition (23) is found to be feasible and the following stabilizing switched gains

$$K_1 = [0.1956\ -0.8403\ -0.7902]$$
$$K_2 = [-1.1285\ -1.1554\ -0.353]$$

are derived. One should remark that the LMIs proposed in (Z.Ji *et al.*, 2003*a*; Z.Ji *et al.*, 2003*b*) have no solution and do not allow to build a stabilizing switched state feedback.

5. CONCLUSION

This paper was dedicated to the robust stability analysis and control synthesis for switched uncertain systems with polytopic uncertainties. Using Lyapunov functions that are based on a structure similar to the uncertain switched system, less conservative LMI conditions have been proposed. The approach was extended to the case of control synthesis via a switched state feedback. By duality, these results obviously apply to the state reconstruction problem for uncertain switched systems with uncertain output matrix. In the future, one could extend this approach to the output feedback stabilization problem and reduce the number of conditions through convex analysis methods.

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