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Models and methods for frequency assignment with cumulative interference constraints

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Abstract: *In this paper, a realistic modeling of interferences for frequency assignment in hertzian telecommunication networks is presented. In contrast with traditional interference models based only on binary interference constraints involving two frequencies, this new approach considers the case of cumulative disruptions that are modeled through a unique non-binary constraint. To deal with these complex constraints, we propose extensions of classical integer linear programming formulations. On a set of realistic instances, we propose hybrid constraint programming and large neighborhood search solution methods to solve minimum interference and minimum span frequency assignment problems. We compare their performances with those of existing heuristics. Finally, we show how the end-user benefits from using the cumulative model instead of the traditional one.*

Keywords: *Frequency Assignment, Cumulative Interference Constraints, Linear Programming, Constraint Programming, Large Neighborhood Search.*

1 Frequency assignment with cumulative interference constraints

This paper presents a frequency assignment problem (FAP) in an hertzian telecommunication network. The network consists of geographic sites on which antennas are implanted. Each antenna is connected to senders and/or receivers. A given site may include several antennas. Two distinct geographic sites can be connected by one or several unidirectional links. Each link is directed from the sender of an antenna located on the first site to the receiver of an antenna located on the second one.

Let T denote the set of links. Frequency assignment aims at giving to each link a frequency value which guarantees a good communication quality.

Each link $i \in T$ is associated to a frequency domain F_i which defines the set of discrete frequencies that can be allocated to i. These domains result from legal issues, hardware limitations and geographic equipments localization.

Communication quality is evaluated through electromagnetic compatibility computations. Such compatibility consists, for a given receiver, in taking into account the different emissions of neighbor senders that may disrupt it. For instance, the carrier to interference ratio (C/I criterion) expresses an acceptable threshold between the useful power received by the disrupted receiver and the received power coming from all neighbor senders.

Usually, the "right to disturb" (Aardal *et al.*, 2003), defined through the C/I criterion, is equally distributed among all the disrupters. Such a distribution allows to transform a situation with N senders disrupting a receiver into N binary situations with a single disrupter sender and a single disrupted receiver.

Let CEM_1 denote the set of links pairs concerned by such binary interference constraints. Then, the classical interference constraints can be expressed as follows:

$$|f_i - f_j| \geq \delta_{ij} \qquad \forall (i,j) \in \text{CEM}_1 \tag{1}$$

where δ_{ij} is a minimal gap between frequency f_i allocated to link i and frequency f_j allocated to link j.

However, the uniform distribution assumption is made according to practical solution issues without any realistic justification. In this paper, this simplification is dropped by considering a new model issued by the French Armament Electronics Center (CELAR[1]).

Indeed, distance constraints (1) can be replaced by weaker but more complex constraints which simultaneously take the N disrupter links into account. Such realistic models have been studied only recently in the literature. For example, (Dunkin *et al.*, 1998, Mannino and Sassano, 2003) consider a simplified model compared to the one presented here. The differences are explained below.

In this paper, we replace the distance matrix by a function T_{ij} of $\mathbb{N} \rightarrow \mathbb{R}$ such that $T_{ij}(x)$ represents the disruption of link j on link i when $|f_i - f_j| = x$. Such a model allows to consider disruptions with $x > 1$, which is not the case of the model presented in (Mannino and Sassano, 2003). Function T_{ij}, called the perturbation function, is positive, decreasing and tends to 0 as x increases, as shown in Figure 1.

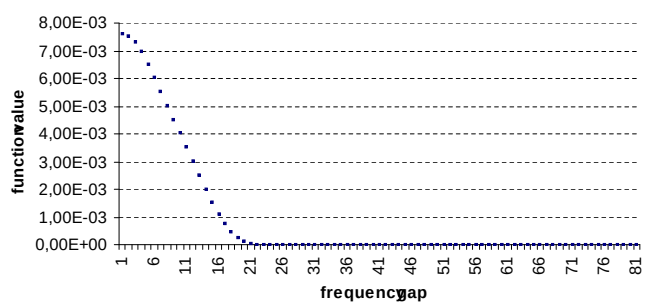

Figure 1: Example of a perturbation function T_{ij}

Another difference with the model considered in (Mannino and Sassano, 2003) concerns the influence of a disrupter link $j \neq i$, weighted by a multiplier λ_{ij} that takes into account the geographical distance and the respective orientations of both disrupting and disrupted links. Finally, only a subset P_i of links (the ones that are able to disrupt link i) is involved in a cumulative constraint. Let Λ_i denote an acceptable threshold for the receiver of link i, computed according to the C/I criterion, and let CEM_2 denote the set of disrupted links

[1]Centre d'ÉLectronique de l'ARmement

involved in a cumulative constraint. The new interference constraints are now expressed as follows:

$$\sum_{j \in P_i} \lambda_{ij} T_{ij}(|f_i - f_j|) \leq \Lambda_i \qquad \forall i \in \text{CEM}_2 \tag{2}$$

More precisely, two cases have to be considered:

- The case where the disrupting senders are located on the same site as the disrupted receiver, ("near field" disruptions). In this case, the constraints are kept in the binary form (1).
- The case where the disrupting senders are not located on the same site as the disrupted receiver ("far field" disruptions). In this case, new formulation (2) is involved.

In addition, other imperative binary constraints are considered in the studied problem: these are fixed distance (3) and forbidden distance (4) constraints. Such constraints, named duplex constraints, appear between two links connecting the same sites. CI_1 and CI_2 denote the sets of links pairs involved in each type of duplex constraints, respectively.

$$|f_i - f_j| = \epsilon_{ij} \quad \forall (i,j) \in \text{CI}_1 \tag{3}$$
$$|f_i - f_j| \neq \epsilon_{ij} \quad \forall (i,j) \in \text{CI}_2 \tag{4}$$

The set of constraints defines an interference graph $G = (T, E)$ where each node of T represents a link and each edge $(i, j) \in E$ represents a (CI or CEM) constraint between links i and j. Working on this graph allows frequency assignment problems involving only binary interference constraints to be represented as graph coloring problems.

For a realistic problem, satisfying all interference constraints (1) and (2) reveals itself impractical. Thus, a solution approach consists in relaxing these constraints and minimizing the weighted sum of their respective violations. Such a problem is known as the minimum interference frequency assignment problem (MI-FAP). When a solution can be found without violating any interference constraint, a secondary objective is to minimize the span, *i.e.* the difference between the maximum and minimum assigned frequencies. This problem is denoted as the minimum span frequency assignment problem (MS-FAP).

Because of the originality and practical importance of the model involving cumulative interferences, the CELAR organized, in 2002, a contest where three independent teams had to solve the problem using three different methods. The first objective was to find the best solution method. Eventually, it aimed at checking whether the methods were able to take advantage of the flexibility introduced by the new model or, on the contrary, if the model was too complex to be useful in practice. Related work on classical frequency assignment problems and other methods presented in the contest are described in Section 2. Integer programming and constraint programming formulations of the frequency assignment problem with cumulative constraints are presented in Sections 3 and 4, respectively. Section 5 presents the problem instances proposed by the CELAR as well as the preprocessing technique, exact method and large neighborhood search heuristic we propose. In Section 6, the results of the large neighborhood search heuristic are compared with the methods presented in the contest. The interest of considering the new cumulative model is underlined in Section 7. Concluding remarks are drawn in Section 8.

2 State-of-the-art approaches for frequency assignment and methods presented in the CELAR contest

Most of the approaches encountered in the literature deal with the MI-FAP or the MS-FAP with binary interference constraints. A complete description of these approaches can be found in (Aardal *et al.*, 2003).

Due to the strong connections between graph coloring and frequency assignment, many encountered methods involve techniques that have been shown very effective on the former problem class. These methods rank from the simplest constructive algorithms to complex metaheuristics. Among the constructive methods, the generalization of the DSATUR procedure (Brélaz, 1979) constitutes the basis of Costa's work (Costa, 1993). Slight modifications can be performed to tackle the specificity of the MI-FAP (Borndörfer *et al.*, 1998). The concept of generalized saturation degree (Valenzuela *et al.*, 1998) falls into this category. Other constructive methods are based on the analogy between graph coloring and frequency assignment, like the generalized sequential packing procedure (Sung and Wong, 1997). Local search approaches (Borgne, 1994) and metaheuristics, like genetic algorithms (Valenzuela *et al.*, 1998), are also based on graph coloring methodologies.

Integer linear programming formulations have been proposed for frequency assignment, but only for the binary interference case (Aardal *et al.*, 2003). A constraint programming approach has been described, restricting also to the binary case and for span minimization only (Walser, 1996).

Local search constitutes a common approach for solving (binary) frequency assignment problems, as already observed above for graph coloring based methods (Tsang and Voudouris, 1998). Standard metaheuristics have also been developed, like simulated annealing (Knälmann, 1994) and tabu search (Capone and Trubian, 1999, Hao, 1996). Under the category of evolutionary approaches , genetic algorithms (Crompton *et al.*, 1994, Kolen, 1999) and ant colony optimization methods (Maniezzo and Carbonaro, 2000, Montemanni *et al.*, 2002) can be found.

For problems involving cumulative interferences, a methodology called Solve and Extend, has been applied in (Smith, 1998) and (Mannino and Sassano, 2003). As already underlined in Section 1, the authors consider a particular case of the model presented in this paper. The procedure is executed in two distinct phases. During the first phase, a significant sub-problem is chosen and solved (Solve phase); then, it is extended in order to obtain a solution to the global problem (Extend phase). The two-phase process is iterated until a stopping criterion is met.

To solve the problem considered in this paper, three teams were in competition in the context of the CELAR contest. A simulated annealing procedure was designed by (Sarzeaud and Berny, 2003), involving Gibbs sampling for the choice of neighbor. The neighborhood was defined by all the possible changes of a single frequency value, each move being associated to a probability. The Gibbs sampling introduces a variant compared to the traditional simulated annealing scheme. At each iteration, it performs an exhaustive search of the neighborhood to possibly find an improving solution. The probabilistic selection of the neighbor is performed only if the best neighbor does not improve the best known solution. In addition, learning techniques are used: based on statistics on the previously explored solutions, the probabilities of all moves likely to lead to good solutions are increased.

(Vlasak and Vasquez, 2003) tested the consistent neighborhood tabu search method (CN-tabu) which obtained excellent results for other frequency assignment problems, see *e.g.* (Vasquez *et al.*, 2005). As CN-tabu solves only search problems, a linear search from an upper bound is performed for the two considered criteria. We refer to (Vasquez *et al.*, 2005) for a precise description of the CN-tabu method. It differs from the classical tabu search method on two main characteristics. First, the search is performed on partial solutions, where only

a subset of frequencies are instantiated, satisfying all constraints. A move consists in selecting an unassigned frequency and fixing its value. This instantiation may lead to an inconsistency and generate a conflict set of frequencies that have to be unfixed by a repairing process. The neighborhood of a partial solution is obtained by considering all partial solutions that can be reached by such a move. The second main component of the CN-tabu method is the use of specific local consistency checking techniques to compute efficiently which variables have to be uninstanciated at each move.

In the remaining of the paper we describe the models and methods our team developed for this contest and we compare the results of our best method with the ones developed by (Sarzeaud and Berny, 2003) and (Vlasak and Vasquez, 2003).

3 Integer linear programming formulations

We propose two mixed integer linear programming (ILP) formulations of the cumulative MI-FAP and MS-FAP, respectively. Inspired by the classical ILP formulations of the binary MS-FAP and MI-FAP (Aardal *et al.*, 2003), we use variables indexed by the frequency value. For each link $i \in T$ and for each possible value $v \in F_i$ of f_i, we introduce a binary variable x_{iv} equal to 1 if and only if $f_i = v$.

Finding the optimal solution for MI-FAP consists in solving the following integer linear program:

$$\min \quad \alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij} + \beta \sum_{i\in \text{CEM}_2} d_i \tag{5}$$

$$\text{s-t} \quad \sum_{v\in F_i} x_{iv} = 1 \qquad \forall i \in T \tag{6}$$

$$x_{iv} \leq x_{j(v+\epsilon_{ij})} + x_{j(v-\epsilon_{ij})} \qquad \forall (i,j) \in \text{CI}_1, \forall v \in F_i \tag{7}$$

$$x_{j(v+\epsilon_{ij})} + x_{j(v-\epsilon_{ij})} + x_{iv} \leq 1 \qquad \forall (i,j) \in \text{CI}_2, \forall v \in F_i \tag{8}$$

$$x_{iv} + \sum_{u\in V_{ijv}} x_{ju} \leq 1 + c_{ij} \qquad \forall (i,j) \in \text{CEM}_1, \forall v \in F_i, V_{ijv} \neq \emptyset \tag{9}$$

$$\sum_{j\in P_i} \lambda_{ij} \sum_{w\in F_j} T_{ijvw} x_{jw} \leq \Lambda_i + M(1 - x_{iv} + d_i) \qquad \forall i \in \text{CEM}_2, \forall v \in F_i \tag{10}$$

$$d_i \in \{0,1\} \qquad \forall i \in \text{CEM}_2 \tag{11}$$

$$c_{ij} \in \{0,1\} \qquad \forall (i,j) \in \text{CEM}_1 \tag{12}$$

$$x_{iv} \in \{0,1\} \qquad \forall i \in T, \forall v \in F_i \tag{13}$$

where

- Constraints (6) state that one and only one frequency has to be assigned to each link.
- Constraints (7) and (8) represent fixed distance constraints (3) and forbidden distance constraints (4), respectively.
- Constraints (9) correspond to classic binary interference constraints (1). V_{ijv} denote the set of frequencies $u \in F_j$ such that $|v - u| < \delta_{ij}$, i.e. assignments of f_j violating the constraint when $f_i = v$. Hence for a distance constraint (1), there are as many linear constraints as possible values v for f_i such that V_{ijv} is non-empty. Binary variable c_{ij} indicates if the constraint is violated. Indeed, the distance δ_{ij} between f_i and f_j is respected if and only if $c_{ij} = 0$.

- Constraints (10) correspond to cumulative interference constraints (2). Let $T_{ijvw} = T_{ij}(|v - w|)$ denote the interference value of link j induced on link i if $f_i = v$ and $f_j = w$. Then, the left member of the constraint represents the sum of interferences on link i when $f_i = v$. If $x_{iv} = 0$, the constraint is always satisfied, provided that constant M is large enough. Binary variable d_i allows to verify if the constraint is violated. If $x_{iv} = 1$ and if and only if $d_i = 0$, the constraint is satisfied and the interference sum is not greater than the threshold Λ_i.
- Objective function (5) aims at minimizing the weighted sum of violated interference constraints, where α is the weight of binary constraints CEM$_1$ while β is the weight of cumulative constraints CEM$_2$.

Setting $c_{ij} = 0, \forall (i, j) \in$CEM$_1$ and $d_i = 0, \forall i \in$CEM$_2$, the MS-FAP can be expressed as follows:

$$\min \quad f_{\max} - f_{\min} \tag{14}$$

$$\text{s-t} \quad (6), (7), (8), (9), (10), (13)$$

$$f_{\max} \geq \sum_{v \in F_i} v x_{iv} \qquad \forall i \in T \tag{15}$$

$$f_{\min} \leq \sum_{v \in F_i} v x_{iv} \qquad \forall i \in T \tag{16}$$

$$f_{\max}, f_{\min} \geq 0 \tag{17}$$

Constraints (15) and (16) are used to compute the maximal and minimal frequencies, respectively. Objective function (14) corresponds to span minimization.

For both formulations, the *big* M constraints (10) needed to represent the cumulative interferences are known to give poor relaxations. For each cumulative constraint $i \in$ CEM$_2$ (2), there is a set of equivalent constraints of *cover inequalities* yielding better relaxations:

$$x_{iv} + \sum_{(j,w) \in P} x_{jw} \leq |P| + d_i, \forall i \in T, \forall v \in F_i, \forall P \in \mathcal{P}_{iv} \tag{18}$$

where $P \in \mathcal{P}_{iv}$ is a set of pairs (link, value) violating the cumulative constraint for $f_i = v$. Because of their exponential number, these constraints can be added only when needed, through a branch-and-cut technique.

As we know, the resulting integer linear programming formulation is intractable for practical problems, see e.g. (Mehrotra and Trick, 1996). This is due to the possibly huge amount of integer variables and to the symmetry of the formulation. Hence, the proposed model, even if enhanced by cutting plane techniques, can only be used to solve small problems. Consequently, it is worth investigating other exact and heuristic solution frameworks.

4 Constraint programming formulations

The constraint programming (CP) formulations of the MI-FAP and the MS-FAP are very close to the natural ones. They are based on the following decision variables:

- f_i for all $i \in T$ with domain F_i. These variables represent the frequencies assigned to the links.

- d_{ij} for any links pair (i, j) involved in any CI or CEM constraint. d_{ij} represents the gap value between the frequencies assigned to links i and j.
- t_{ij} for any link pair (i, j) involved in a cumulative constraint CEM_2. t_{ij} represents the discrete perturbation function value.
- c_{ij} for all $(i, j) \in \text{CEM}_1$ with domain $\{0, 1\}$ represent (as in the ILP model) the violation indicators of the corresponding binary interference constraints.
- d_i for all $i \in \text{CEM}_2$ with domain $\{0, 1\}$ represent (as in the ILP model) the violation indicators of the cumulative interference constraints.

The MI-FAP model can be expressed as follows:

$$\min \quad \alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij} + \beta \sum_{i\in \text{CEM}_2} d_i \tag{19}$$

$$d_{ij} = \epsilon_{ij} \qquad \forall (i, j) \in \text{CI}_1 \tag{20}$$

$$d_{ij} \neq \epsilon_{ij} \qquad \forall (i, j) \in \text{CI}_2 \tag{21}$$

$$d_{ij} < \delta_{ij} \Rightarrow c_{ij} = 1 \qquad \forall (i, j) \in \text{CEM}_1 \tag{22}$$

$$\sum_{j\in P_i} \lambda_{ij} t_{ij} > \Lambda_i \Rightarrow d_i = 1 \qquad \forall i \in \text{CEM}_2 \tag{23}$$

$$d_{ij} = |f_i - f_j| \tag{24}$$

$$t_{ij} = T_{ij}[d_{ij}] \tag{25}$$

Constraints (22) and (23) represent the soft binary and cumulative interference constraints as “implication” constraints. Constraints (25) force variable t_{ij} to be the element of an array (the perturbation function) indexed by a finite-domain variable (here being the distance d_{ij}). Such constraints are commonly known as “Element” constraints and were introduced in (Hentenryck and Carillon, 1988).

The MS-FAP model can be expressed as follows, replacing soft constraints by hard constraints:

$$\min \quad f_{max} - f_{min} \tag{26}$$

$$s - t \quad (20), (21), (24), (25)$$

$$d_{ij} \geq \delta_{ij} \qquad \forall (i, j) \in \text{CEM}_1 \tag{27}$$

$$\sum_{j\in P_i} \lambda_{ij} t_{ij} \leq \Lambda_i \qquad \forall i \in \text{CEM}_2 \tag{28}$$

$$f_{max} \geq f_i \qquad \forall i \in T \tag{29}$$

$$f_{min} \leq f_i \qquad \forall i \in T \tag{30}$$

$$f_{max}, f_{min} \geq 0 \tag{31}$$

The use of such a CP model to solve realistic instances will be investigated in Section 5.3.

5 Solution methods

The methods we propose to solve the cumulative frequency assignment problem follow a framework set by the CELAR contest. After an initial preprocessing phase, the search is performed in two phases. First, the methods aim at solving the MI-FAP. Second, and only when the previous phase ends with a number of violated constraints equal to 0, the MS-FAP is solved.

The problem instances proposed during the CELAR contest are presented in Section 5.1. The common preprocessing phase is described in Section 5.2. Section 5.3 presents a hybrid constraint programming and combinatorial optimization exact method. Section 5.4 is devoted to a large neighborhood search heuristic.

5.1 The CELAR instances

The CELAR generated 30 instances of the frequency assignment problem with cumulative interference constraints, named FAPPG. These instances were generated from military applications issued data [2]. The CELAR used the instances generated during the CALMA project (Combinatorial ALgorithms for Military Applications) considering only classical interference constraints and extended them to cumulative interferences. For details about the CALMA project, refer to the "FAP web" site (`http://fap.zib.de`) and (Aardal *et al.*, 2002). For a presentation of practical extensions of the CALMA instances (including the FAPPG instances), refer to the "Frequency Assignment Problems" site `http://www.fap.ema.fr`.

As shown in Table 1, the instances are named x_n, where x represents the instance number and n the number of links. The number of links varies from 16 to 2166. The number of type (3) and (4) imperative constraints ($|C_I|$ columns) varies from 10 to 1229. The number of binary interference constraints (1) ($|C_E|$ columns) varies from 16 to 4155. Last, the number of cumulative interference constraints (2) ($|C_C|$ columns) varies from 16 to 2015.

Additionally, we give the number K of connected components of the interference graph G for each instance, as each connected component corresponds to an independent sub-problem, w.r.t. the MI-FAP.

instance	K	$\|C_I\|$	$\|C_E\|$	$\|C_C\|$	instance	K	$\|C_I\|$	$\|C_E\|$	$\|C_C\|$	instance	K	$\|C_I\|$	$\|C_E\|$	$\|C_C\|$
01_0016	1	10	16	16	11_0164	3	84	439	101	21_1088	1	546	2789	1081
02_0018	1	11	24	18	12_0902	23	453	2354	572	22_0768	2	386	1604	757
03_0066	2	35	100	50	13_0306	2	155	1142	244	23_0034	1	19	53	24
04_0064	2	34	88	48	14_0194	1	99	587	163	24_0048	1	38	80	39
05_0064	1	34	80	64	15_2454	4	1229	5135	2015	25_0106	1	68	181	63
06_0182	4	93	245	172	16_0038	1	21	128	37	26_0140	2	85	219	83
07_0182	4	93	245	172	17_0040	1	22	92	36	27_0154	1	92	255	134
08_0608	20	306	812	484	18_0052	1	28	116	42	28_0398	9	199	821	340
09_1460	65	732	1862	1123	19_0770	1	387	2276	770	29_0526	17	263	980	471
10_1698	73	851	705	1292	20_1930	136	967	3896	1075	30_2166	46	1083	4155	1985

Table 1: Characteristics of the 30 CELAR instances

[2] The instances can be downloaded from `http://www.fap.ema.fr/save.php/fr/Local/fap/dir/instances/fapg/archives/bench_fappg.zip`, last visit July 2007

5.2 Preprocessing

For each link $i \in T$, the frequency domain can be reduced by applying the following simple filtering rule:

$$v \in F_i, \quad \text{if } \nexists w \in F_j : |f_i - f_j| = \epsilon_{ij} \Rightarrow F_i = F_i - v$$

Applying this filtering rule trough a basic constraint propagation algorithm for the imperative fixed distance constraints (3) has a positive impact. On 14 of the 30 instances, the percentage of values removed from the links domains ranges from 2.07% to 33.33%. The impact is especially significant on 6 instances where more than 20% of the values are removed.

We also propose a special preprocessing taking jointly into account the cumulative constraints and the fixed distance constraints (3). On one hand, the cumulative constraints depend on the distances between f_i and $f_j \in P_i$ for any $i \in \text{CEM}_2$. On the other, the fixed distance constraints provide the exact value ϵ_{ij} of this distance for any $(i, j) \in \text{CI}_1$. Hence we can use these values directly in the cumulative constraints where pairs of frequencies linked with duplex constraints are involved. This preprocessing is sufficient to prove that instances 25 and 27 of Table 1 admit a lower bound of 2 violated cumulative constraints.

5.3 Hybrid constraint programming and combinatorial optimization exact method

The CP model presented in Section 4 can be solved through any general-purpose constraint programming solver by specifying the branching rule while using the standard constraint propagation algorithms of the solver. A simple branching rule consists in selecting the variable f_i with the smallest domain and exploring the values of domain F_i in an increasing order. For optimization under the CP framework, feasibility problems are iteratively solved by setting constraints on the objective function through linear or binary search, yielding a basic exact method.

To enhance this CP-based method, we have coupled the solving of the CP model with the solving of a relaxation based on the concept of cliques in a constraint graph. Considering the cumulative interference constraints (2), we can derive binary interference constraints (1) as follows:

$$|f_i - f_j| \geq \min_{e \in IN} \{e : \lambda_{ij} T_{ij}(e) \leq \Lambda_i\} \quad \forall j \in P_i, i \in \text{CEM}_2 \tag{32}$$

Using these constraints and the ones of CEM_1, we build a constraint graph where the nodes represent the links and the edges are the original binary interference constraints (1) and fixed distance constraints (3) plus the inferred binary interference constraints (32). Each edge is weighted by the distance, *i.e.* the right-hand side of the corresponding constraint. Consider a k-clique in the so-defined constraint graph. Solving any relaxation of the traveling salesman problem in the clique gives a lower bound for the span criterion. Hence we can show that the MS-FAP admits no solution when the lower bound is greater than the difference between the largest and the smallest frequency values of all frequency domains. The perfect matching relaxation of the traveling salesman problem is used to prune the search at each node of the CP search phase.

Though this hybrid method is not extremely effective, the relaxation based on the perfect matching was able to find lower bounds for the number of violated constraints on some instances. Solutions with no violated interference constraints and minimal span were found for instances 1, 23 and 24 of Table 1.

Table 2 summarizes the resulting non-zero lower bounds for the number of violated constraints. The optimal solutions found for the span criterion by the hybrid method are displayed in Table 3.

Instance	16	17	19	20	25	27
LB MI	1	2	4	1	2	2

Table 2: Non zero lower bounds for the number of violated constraints obtained by the combinatorial bound

Instance	1	23	24
opt span	548	380	410

Table 3: Optimal spans found by the hybrid method

5.4 A large neighborhood search heuristic

For most instances, the high number of variables and the important domains size do not allow the use of an exact ILP or CP solution method. We propose a heuristic scheme based on the Large Neighborhood Search (LNS) methodology. A preliminary version of this work was presented in (Palpant *et al.*, 2002). This LNS method has shown its effectiveness and applicability to large problems. In particular, it has been able to find all the solutions proved optimal by the hybrid CP method.

The method consists in generating and solving iteratively sub-problems of the global problem. Regarding frequency assignment problems, the idea is not entirely new since a hybrid method called Solve and Extend has previously been designed for a particular case of the problem considered in this paper Smith *et al.*, 1998, Mannino and Sassano, 2003), see also Sections 1 and 2. In (Smith *et al.*, 1998), a clique of level p in the constraint graph is used to generate a first sub-problem which is solved by a specific heuristic. The extension of the partial solution obtained at the end of this first phase to a complete solution is then performed heuristically, too. The process is iterated by adding each time vertices of maximal saturation to the initial clique. In the approach of (Mannino and Sassano, 2003), an implicit enumeration scheme is used to solve and extend the sub-problems. The latter are obtained one from each other by adding links to an initial sub-problem considering connectivity criteria in the constraint graph.

The method we present in this paper differs from the two cited above since we define a large neighborhood search framework. The sub-problems are obtained roughly by solving the global problem after fixing a subset of the decision variables to their current values. It follows that the "Extend" phase becomes trivial. It results a local search procedure that explores iteratively large neighborhoods of the current solution.

For each phase of the optimization process, the algorithm works on the K problems $P_1, \ldots, P_K$, of respective size $n_1, \ldots, n_K$, defined by the connected components of the interference graph G. Regarding interference minimization (MI-FAP), solving independently each problem P_k is equivalent to solving the whole problem, while this is not the case for span minimization (MS-FAP) since the objective function may relate to links belonging to distinct connected components. Nevertheless, because of the specific strategies involved in the approach, the second phase of the optimization process is also based on this decomposition. The general execution scheme, given in Figure 2, consists in generating and solving, at each iteration s, K distinct sub-problems. For each problem P_k, sub-problem SP_k^s, of size $p_k \leq n_k$, is built using current solution $\overline{F}^{s-1}$ and considering the current objective (interference or span). It is then solved by an appropriate method in order to obtain partial solution $\overline{F}_k^s$. Then, the solutions of the K sub-problems are used to compute neighbor solution

$\overline{F}^s$, from which the process is iterated. The algorithm, which returns final solution $\overline{F}^*$, stops when a maximal execution time MAX_CPU is reached.

1. Perform the preprocessing phase.
2. Generate initial solution $\overline{F}^0 = (\overline{F}^0_1, \ldots, \overline{F}^0_K)^t$ with the help of a greedy algorithm.
3. $\overline{F}^* = \overline{F}^0, s = 1$
4. Repeat
5. For each problem P_k
6. Generate SP^s_k from $\overline{F}^{s-1}$ considering current objective (MI-FAP or MS-FAP)
7. Solve SP^s_k to compute partial solution $\overline{F}^s_k$.
8. $\overline{F}^s = (\overline{F}^s_1, \ldots, \overline{F}^s_K)^t$
9. Update $\overline{F}^*$ if needed.
10. $s = s + 1$
11. Until the CPU time reaches MAX_CPU

Figure 2: General algorithm of the LNS method

We describe in the following subsections each key point of the algorithm.

5.4.1 Generation of the initial solution

For each problem P_k, a greedy algorithm is used to compute initial solution $\overline{F}^0_k$. This heuristic comprises n_k steps. At each step, the most constrained link (i.e. the one that appears in the largest number of constraints) is selected and assigned to the lowest possible frequency that minimizes the objective function value. At each step of the procedure, the problem is kept consistent, *i.e.* the value assigned to the selected link must satisfy the imperative constraints.

5.4.2 Sub-problem generation

Each iteration s of the LNS procedure (Figure 2) defines a sub-problem SP^s_k per problem P_k, by defining a subset T^s_k of "freed" links. The remaining links are assigned to their current frequency value in current solution $\overline{F}^{s-1}$. The sub-problem lies in finding a value for the frequency of links T^s_k satisfying the problem constraints and improving the considered objective.

The selection of the freed variables starts from a randomly chosen link, say $T^s_k = \{i\}$. In the case of interference minimization (MI-FAP), this link is assumed to belong to a constraint that is violated by the current solution. Then, T^s_k is extended by selecting every link j involved in the same (CI or CEM) constraint as i, *i.e.* such that vertices representing i and j are adjacent in constraint graph G. At the end of this step, if the sub-problem size p_k is not reached, the process is iterated starting from a link already included in T^s_k. Figure 3 shows an execution of the process. Starting from randomly chosen link 9, links 8, 10, 2, 3 and 12 are first included in T^s_k, in the given order. Considering that $p_k = 8$, the process is iterated from the secondly included link (link 8) and then from link 10, included in third position in T^s_k, which gives as a final freed links subset $T^s_k = \{9, 8, 10, 2, 3, 12, 7, 11\}$.

Once subset T^s_k is defined, the (MI-FAP) sub-problem lies in finding a feasible assignment to the freed links

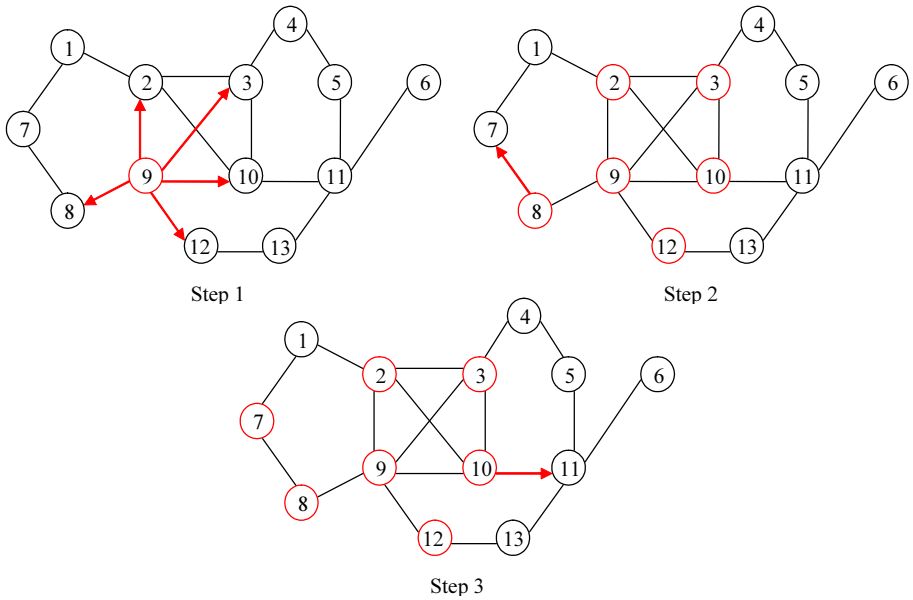

Figure 3: Sub-problem selection process

subject to the following constraints under the CP formulation:

$$\alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij} + \beta \sum_{i\in \text{CEM}_2} d_i < \alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij}^{s-1} + \beta \sum_{i\in \text{CEM}_2} d_i^{s-1} \tag{33}$$

$$(20), (21), (22), (23), (24), (25)$$

$$f_i = f_i^{s-1} \qquad \forall i \notin T_k^s \tag{34}$$

Obviously, fixed links constraints (34) impose reductions on freed links domains via problem constraints (20-25) and objective improvement constraints (33).

In the case of the span minimization phase (MAS-FAP), several strategies are involved in order to tackle problems $P_1, \ldots, P_K$ independently. Each of them consists in performing additional (heuristic) domain reductions on freed links domains. Let $f_+ = \max_{i\in T} f_i^{s-1}$ and $f_- = \min_{i\in T} f_i^{s-1}$ denote the maximum and minimum assigned frequency values in the current solution $\overline{F}^{s-1}$, respectively. The different strategies lie in modifying heuristically the domains of the freed links variables, depending on values f_+, f_- and subset T_k^s. They are described as follows:

- AllMax strategy. This strategy is employed when all links assigned to f_+ belong to T_k^s, and when at least one link assigned to f_- does not. The lowest assigned frequency being fixed, reassigning the frequencies of freed links in the interval $I = [f_-, f_+[$ is sufficient to obtain an improved span value. All values that do not belong to I are consequently removed from the domains of the freed links variables.
- AllMin strategy. This is the symmetric strategy of AllMax. Domains are reduced to the interval $I =]f_-, f_+]$.
- NoMinMax strategy. This strategy is involved when at least a link assigned to f_+ and a link assigned to f_- do not belong to T_k^s. The span value can't be improved by solving the sub-problem. However,

rearranging the frequencies in the interval $I = [f_-, f_+]$ can lead to an improvement of the solution in subsequent steps and contribute to diversify the search. Domain values are kept in this interval.

- AllMinMax strategy. This strategy is used when all links assigned to f_+ and f_- belong to T_k^s. Given $d_- = \min_{i \notin T_k^s} f_i^{s-1} - f_-$ and $d_+ = f_+ - \max_{i \notin T_k^s} f_i^{s-1}$, any solution improving the span value can not contain a link assigned outside the interval $I = [f_- - d_+, f_+ + d_-]$, which sets the domains of the freed links variables.

All these strategies have to be coupled with appropriate objectives. Hence, for the first three strategies, a solution satisfying all interference and interval constraints is searched without optimizing any objective function, while the last strategy involves the search of a minimal span solution for SP_k^s

It is worth noting that the heuristic domain reductions of first two strategies may suppress good or even optimal solutions from the neighborhood. However, reducing the neighborhood size contributes to make its exploration easier and consequently allows to solve sub-problems with larger subsets T_k^s. Moreover, the diversity of the strategies involved during the search, combined with a non-deterministic selection scheme of the freed links may lead to reconsider previously ignored solutions.

5.4.3 Solving the sub-problems

Sub-problem SP_k^s is solved by taking into account two criteria: the considered objective and the global solution strategies. Regarding the optimization parameter, it is worth noting that, due to the several domain reduction strategies and objectives involved during the search, solving a sub-problem does not necessarily lead to the obtaining of an optimal solution, neither of an improved one.

The global solution strategy defines the computing effort spent to find the neighbor solution: a heuristic provides medium-quality solutions but in a very short time, while an exact method needs more computing time to obtain high-quality solutions. The idea is to apply a heuristic scheme when the current solution is likely to be easily improved (*i.e.* during the first iterations) and then apply an exact procedure in order to intensify the search.

The heuristic procedure is the greedy procedure presented in Section 5.4.1. In the case the method does not provide an improved neighbor solution, the current solution remains unchanged. However, we observed that the greedy heuristic behaves effectively to improve medium-quality solutions. Hence, the method is employed at the beginning of the MI-FAP phase, in order to obtain quickly a solution of satisfying quality.

Once this solution is reached, another solution scheme is involved to tackle the sub-problems, consisting of a truncated exact CP-based procedure. The search is stopped as soon as an improved or optimal solution is found, or when a time limit H is reached. This method allows to intensify the search of the neighbor solution as it spends more time to explore the neighborhood. The time limit parameter H is used to tune effectively the procedure: it is high for interference minimization or when AllMinMax strategy is active; on the contrary, it is set to a low value for all other cases of span minimization, in particular when NoMinMax strategy is employed.

6 Comparison of the methods proposed in the CELAR contest

Experiments have been conducted on a PC equipped with a 350 MHz CPU and 256 Mo RAM. As CP solver, we used Ilog Solver 5.0. To establish a comparison, we have compared the LNS approach to the methods proposed

for the CELAR contest: the Simulated Annealing method of (Sarzeaud, 2003) and the Tabu Search method of (Vlasak and Vasquez, 2003), see Sections 1 and 2. To make a fair comparison, all tests have been performed by the CELAR following the same operating mode (1 hour of execution time, same hardware). Table 4 gives the results on the CELAR instances for all approaches in terms of interference and span value. Columns LNS, SA and TS give the results of the Large Neighborhood Search, Simulated Annealing and Tabu Search methods, respectively. For each approach, we have reported the results obtained for the two considered criteria. If the interference value is greater than 0, it is reported in column MI-FAP and the associated span value is indicated in parenthesis; on the contrary, if all electromagnetic constraints have been satisfied, the span value is reported in column MS-FAP. Best solutions are displayed in bold.

instance	LNS		SA (Sarzeaud *et al*)		TS (Vlasak *et al*)	
	MI-FAP	MS-FAP	MI-FAP	MS-FAP	MI-FAP	MS-FAP
01	-	**548**	-	549	-	**548**
02	-	**629**	-	**629**	-	**629**
03	2 (599)	-	**2** (**580**)	-	2 (623)	-
04	-	520	-	520	-	**519**
05	-	**599**	-	623	-	676
06	-	**718**	-	**718**	-	758
07	6 (666)	-	**4** (**698**)	-	8 (687)	-
08	-	**620**	-	646	-	642
09	-	**544**	-	656	-	860
10	-	**412**	-	692	-	849
11	-	604	-	656	-	**601**
12	-	**572**	1 (639)	-	2 (634)	-
13	9 (399)	-	**6** (**399**)	-	8 (380)	-
14	-	398	-	360	-	**354**
15	**31** (**399**)	-	73 (399)	-	44 (399)	-
16	**46** (**146**)	-	**46** (**146**)	-	**46** (**146**)	-
17	46 (99)	-	**45** (**98**)	-	**45** (**98**)	-
18	-	**404**	1 (476)	-	-	408
19	4385 (492)	-	4375 (496)	-	**3998** (**496**)	-
20	**150** (**492**)	-	193 (496)	-	152 (496)	-
21	-	**982**	2 (964)	-	4 (994)	-
22	-	**788**	-	818	1	894
23	-	**380**	-	**380**	-	**380**
24	-	**410**	-	430	-	**410**
25	2* (540)	-	**2** (**490**)	-	2* (540)	-
26	-	**480**	1 (492)	-	-	**480**
27	**2** (**490**)	-	4 (490)	-	**2** (**490**)	-
28	-	**610**	-	646	-	638
29	-	**542**	-	852	-	866
30	**23** (**912**)	-	27 (912)	-	32 (912)	-

Table 4: Computational results on the FAPPG CELAR instances

The LNS method is in general superior to the SA and TS methods, with the notable exception of instance 19. This particular instance is the only one involving different weights in the expression of the sum of violated constraints, which could explain the bad performance of our LNS approach on it. Indeed, the sub-problems selection process appears inadequate to this case, since it does not take weights into account for the selection

of the freed links. Another remark that can be raised from these results concerns the good behavior of the LNS method on highly decomposable instances when minimizing the span criterion (interference free instances 8, 9, 10, 12 and 29). In this case, LNS significantly outperforms the results obtained by the two others approaches. This tends to demonstrate the effectiveness of the neighborhood reduction rules involved during the MS-FAP phase. An interesting outcome could then consist in extending this methodology to non-decomposable instances, which could be divided into several components regarding connectivity criteria in the constraint graph. This would allow the LNS method to work on smaller problems during the MS-FAP phase.

7 Comparison between traditional and cumulative formulations

An important objective of the study was to establish a comparison between the model including the cumulative interference constraints, and the traditional model that replaces these constraints by more constrained binary interference constraints. Recall that the classical representation of the interference constraints can be obtained by replacing all CEM_2 constraints (2) by CEM_1 constraints (1), through a uniform distribution of the "right to disrupt":

$$\lambda_{ij}|f_i - f_j| \geq \frac{\Lambda_i}{|P_i|} \quad \forall j \in P_i \tag{35}$$

In terms of span minimization, it is in theory always possible to find a solution for the problem with the cumulative constraints at least as good as the best solution of the classical model. However, the objective of the current study is to determine whether the methods are able to find these solutions in a reasonable amount of time. In other words, is the increase in complexity of the constraints balanced by the quality of the obtained solutions?

From the CELAR instances, we generated the set of corresponding instances for the classical binary model. The results displayed in Table 5 are obtained with the LNS method. If a solution satisfies all CEM constraints, the span value is indicated in column MS-FAP. Otherwise, the interference value is displayed on column MI-FAP. Values in parenthesis indicate the results obtained on the model with cumulative constraints. The experiments have been conduced as described in Section 6.

The results show that the model with cumulative constraints obtains a larger number of interference free solutions than the model with only binary interference constraints. Furthermore, for the interference free solutions, the cumulative model yields significantly lower span values. This clearly shows the benefit of introducing the cumulative interference constraints for solving practical frequency assignment problems.

8 Conclusion

We have performed an experimental comparison of two models and several methods to solve frequency assignment problems with cumulative interferences.

The good behavior of the large neighborhood search approach tends to prove that efficient methods can be designed for this practical problem. Our study establishes that suitable heuristics can take advantage of a direct representation of the cumulative constraints, despite their complexity. Such a result is of practical importance to solve real frequency assignment problems since one would benefit from switching, at least partially, to the new model.

Instance	MI-FAP	MS-FAP	Instance	MI-FAP	MS-FAP
01	-	571 (548)	16	57 (46)	-
02	1 (0)	-	17	55 (46)	-
03	3 (2)	-	18	-	488 (404)
04	-	718 (520)	19	5629 (4385)	-
05	1 (0)	-	20	178 (150)	-
06	-	(761) 718	21	24 (0)	-
07	17 (6)	-	22	10 (0)	-
08	-	620 (620)	23	-	380 (380)
09	-	650 (544)	24	-	440 (410)
10	-	500 (412)	25	2 (2)	-
11	-	617 (604)	26	-	480 (480)
12	-	600 (572)	27	2 (2)	-
13	13 (9)	-	28	-	634 (610)
14	1 (0)	-	29	-	548 (542)
15	31 (31)	-	30	27 (23)	-

Table 5: Benefits of using the cumulative interference formulation

As suggested by our encouraging results, efficient constraint propagation for the cumulative interference constraints may besides improve the efficiency of the methods. A future research direction may consist in designing such propagation techniques.

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