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Abstract: *We consider a realistic modeling of interferences for frequency allocation in hertzian telecommunication networks. In contrast with traditional interference models based only on binary interference constraints, this new approach considers the case of a receiver disrupted simultaneously by several senders yielding cumulative disruptions that are modeled through a unique non-binary constraint. To deal with these complex constraints, we propose extensions of classical integer linear programming formulations. On a set of realistic instances, we propose hybrid constraint programming and large neighborhood search solution methods. We also compare the performances of our best method with those of existing heuristics and we show how the end-user benefits from using the cumulative model instead of the traditional one.*

Keywords: *frequency allocation, cumulative interference constraints, linear programming, constraint programming, large neighborhood search*

1 Frequency allocation with cumulative interference constraints

In this paper, we consider a frequency assignment problem (FAP) in an hertzian telecommunication network. The network is made of geographic sites on which antennas are located, each antenna being connected with senders and/or receivers. A given site may include several antennas. Two distinct geographic sites can be connected by one or several unidirectional links between two antennas, each link being defined from the sender of the first antenna to the receiver of the second one.

Let T denote the set of links for a given problem. Frequency allocation aims at giving to each link a frequency value which guarantees a satisfying communication quality.

Each link i of T is associated with a frequency domain F_i which defines the set of discrete frequencies that can be allocated to i. The domain results from legal issues, hardware limitations and geographic localization of the equipments.

Communication quality is based on electromagnetic compatibility computations. These computations consist, for a given receiver, to take into account the different emissions of neighbor senders that may disrupt it. For instance, the "C/I" criterion expresses an acceptable threshold between the useful power received by the disrupted receiver and the received power coming from all neighbor senders.

Usually (Aardal *et al.*, 2003) the "right to disturb", defined through the "C/I" criterion, is equally distributed among all the disrupters. Such a distribution allows to transform a situation with N senders disrupting a receiver

into N elementary situations with a single disrupter sender and a single disrupted receiver.

This binary structure gives the following usual interference constraints, where δ_{ij} is a minimal gap between frequency f_i allocated to link i and frequency f_j allocated to link j. Let CEM_1 denote the set of pairs of links concerned by the binary interference constraints.

$$|f_i - f_j| \geq \delta_{ij} \qquad \forall (i,j) \in \text{CEM}_1 \tag{1}$$

Nevertheless, this simplifying equal distribution is made considering practical solution issues without any realistic justification. In this paper, we drop this simplification by considering a new model issued by the French armament electronics center (CELAR).

Indeed, distance constraints (1) can be replaced by weaker but more complex constraints which simultaneously take the N disrupter links into account. Such realistic models have been studied only recently in the literature (Dunkin *et al.*, 1998), (Mannino and Sassano, 2003), though these authors consider a simplified model compared to the one presented in this paper, as explained below.

The distance matrix is here replaced by a function T_{ij} of $\mathbb{N} \rightarrow \mathbb{R}$ such that $T_{ij}(x)$ represents the disruption of link j on link i when $|f_i - f_j| = x$, and allows perturbation for $x > 1$, which is not the case of the model presented in (Mannino and Sassano, 2003). Function T_{ij}, called the perturbation function, is positive, decreasing and tends to 0 as measure as x increases, as shown in Figure 1.

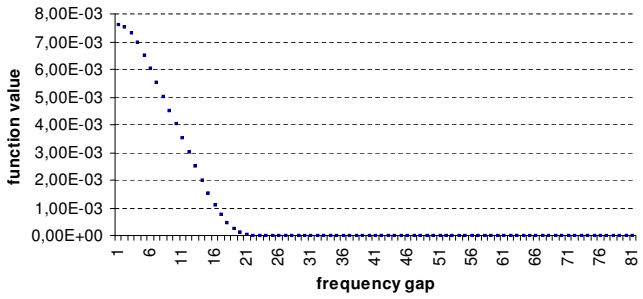

Figure 1: Example of perturbation function T_{ij}

Another difference concerns the influence of a disrupter link $j \neq i$ which is weighted by a multiplier λ_{ij}, taking into account the geographical distance and the respective orientations of both disrupter and disrupted links. Finally, only a subset P_i of links (the ones that are able to disrupt link i) is involved in a cumulative constraint. Let Λ_i denote an acceptable threshold for the receiver of link i, computed according to the "C/I" criterion, and CEM_2 the set of disrupted links involved in a cumulative constraint, the new interference constraints are now expressed as follows:

$$\sum_{j \in P_i} \lambda_{ij} T_{ij}(|f_i - f_j|) \leq \Lambda_i \qquad \forall i \in \text{CEM}_2 \tag{2}$$

More precisely two distinct cases are considered:

- when the disrupter senders are located on the same site as the disrupted receiver, ("near field" disruptions) the constraints are kept in the binary form (1);
- when the disrupter senders are not on the same site as the disrupted receiver ("far field disruptions"), the new formulation (2) is involved.

In addition, other imperative binary constraints are considered in the studied problems: these are fixed distance (3) and forbidden distance (4) constraints. Such constraints appear when two links connect the same sites (duplex constraints). CI_1 and CI_2 denote the set of links pairs involved in each type of imperative constraints, respectively.

$$|f_i - f_j| = \epsilon_{ij} \quad \forall (i,j) \in \text{CI}_1 \tag{3}$$
$$|f_i - f_j| \neq \epsilon_{ij} \quad \forall (i,j) \in \text{CI}_2 \tag{4}$$

The set of constraints of the problem defines a so-called interference graph $G = (T, E)$ such that two vertices representing links that belong to the same constraint are connected in the graph. Working on this graph allows specific assignment problems to be reduced to graph coloring problems.

For problems of realistic size, trying to satisfy all interference constraints (1) and (2) can reveal itself impractical. In consequence, a classical solution approach consists in relaxing these constraints and minimizing the weighted sum of their respective violations. In the case where a solution is found with no violation of any interference constraints, a secondary objective is to minimize the span, i.e. the difference between the maximum and minimum assigned frequencies.

Because of the originality and practical importance of this model, the CELAR organized in 2002 a contest where three independent teams had to solve the problem using three different methods. The objective was, first, to find the best solution method and, second, to check whether the methods are able to take advantage of the flexibility introduced by the new model to provide better solutions or, on the contrary, if the model is too complex to be useful in practice. Related work for some classical frequency assignment problems and the other methods in competition for the CELAR contest are presented in Section 2. Integer programming and constraint programming formulations of the problem are presented in Sections 3 and 4, respectively. Section 5 is devoted to the presentation of the methods we propose for solving the CELAR instances, that are presented in Section 5.1: Section 5.2 introduces a common preprocessing technique used by all approaches, while an exact hybrid CP-based method is presented in Section 5.3 and a heuristic based on large neighborhood search is detailed in Section 5.4. The results of the heuristic are then compared with the methods in competition for the CELAR contest in Section 6. The interest of considering the cumulative model is underlined in Section 7. Concluding remarks are finally drawn in Section 8.

2 State-of-the-art approaches for frequency assignment and methods in competition for the CELAR contest

Since the problem studied in the present work introduces a new formulation, most of the approaches that can be encountered in the literature usually do not deal with it but mostly with classical MI-FAP (Minimum Interference) or MS-FAP (Minimum Span). For a complete description of these approaches, we refer to (Aardal *et al.*, 2003).

Because of the strong connection between graph coloring and frequency assignment, many methods involve techniques that have been shown very effective on the first class of problems. These methods rank from the simplest constructive algorithms to standard metaheuristics. Among the first class of methods, the generalization of the DSATUR procedure (Brélaz, 1979) constitutes the basis of Costa's work (Costa, 1993). Slight modifications can be performed to tackle the specificities of MI-FAP: (Borndörfer, 1998), Generalized Saturation Degree (Valenzuela, 1998). Other constructive methods are based on this analogy like the Generalized Sequential Packing procedure (Sung and Wong, 1997). More sophisticated approaches, like genetic algorithms (Valenzuela, 1998) or local search ones (Borgne, 1994) also invoke graph coloring methodologies.

Integer linear programs have been proposed for the binary interference case (Aardal *et al.*, 2003). Constraint programming has been tested by (Walser, 1996) for span minimization only.

Apart from that kind of methods, standard metaheuristics can be encountered, like evolutionary approaches: genetic algorithms (Crompton, 1994), ANTS (Manezzio and Carbonaro, 2000), (Montemanni, 2002). An original genetic algorithm is described in (Kolen, 1999). The latter works on a redefinition of the crossing and mutation operators allowing to perform optimization operations on the individuals of the population.

Local search also constitutes a common approach for solving frequency assignment: guided local search (Tsang and Voudouris, 1998), simulated annealing (Knälmann, 1994) and tabu search (Capone and Trubian, 1999), (Hao, 1999).

For problems involving cumulative interferences, with a simplified model compared to the one presented in this paper, an original methodology, called Solve and Extend, has been applied in (Smith, 1998) and (Mannino and Sassano, 2003). The procedure executes in two distinct phases: during the first one, a significative sub-problem is chosen and solved (Solve phase); it is then extended in order to obtain a solution to the global problem (Extend phase). The two-phase process is iterated until a stopping criterion is met.

For the CELAR contest and the problem considered in this paper, three teams were in competition. A simulated annealing procedure was designed by (Sarzeaud and Berny, 2003), involving Gibbs sampling for the choice of neighbor. The Gibbs sampling introduces a variant compared to the traditional simulated annealing scheme since at each iteration, it first performs an exhaustive search of the neighborhood to possibly find a solution improving the best known one. The random selection of the neighbor is performed only in the case the best neighbor does not improve the best known solution. In addition, learning techniques are used. The neighborhood is defined by all the possible changes of a single frequency. A distribution probability is associated to each possible frequency value and the learning process increases the probability if the frequency value is likely to be included in a good solution based on statistics on the preceding explored solutions.

(Vlasak and Vasquez, 2003) tested the consistent neighborhood tabu search method (CN-tabu) which obtained excellent results for other frequency assignment problems, see e.g. (Vasquez *et al.*, 2005). As CN-tabu solves only decisional problems, a linear search from an upper bound is performed for the two considered criteria. We refer to (Vasquez *et al.*, 2005) for a precise description of the CN-tabu method. It differs from the classical tabu search method on two main characteristics. First, the search is performed on partial solutions, where only a subset of frequencies are instantiated, satisfying all constraints. A move consists in selecting an unassigned frequency and fixing its value. Regarding to consistency considerations, this instantiation may generate a conflict set of frequencies that have to be unfixed by a repairing process. The neighborhood of a partial solution is obtained by considering all partial solutions that can be reached by such a move. The second main component of the CN-tabu method is the use of specific local consistency checking techniques to compute efficiently which variables have to be unfixed for each move.

In the remaining of the paper we describe the models and methods our team developed for this contest and we

compare the results of our best method with the ones developed by (Sarzeaud and Berny, 2003) and (Vlasak and Vasquez, 2003).

3 Integer linear programming formulations

We propose a mixed integer linear programming formulation of the problem inspired from the classical formulation (Aardal *et al.*, 2003), based on variables indexed by the frequency value. For each link $i \in T$ and for each possible value $v \in F_i$ for f_i, we introduce a binary variable x_{iv} equal to 1 if and only if $f_i = v$. Finding the optimal solution for MI-FAP consists in solving the following integer linear program:

$$\min \quad \alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij} + \beta \sum_{i\in \text{CEM}_2} d_i \tag{5}$$

$$\text{s-t} \quad \sum_{v\in F_i} x_{iv} = 1 \qquad \forall i \in T \tag{6}$$

$$x_{iv} \leq x_{j(v+\epsilon_{ij})} + x_{j(v-\epsilon_{ij})} \qquad \forall (i,j) \in \text{CI}_1, \forall v \in F_i \tag{7}$$

$$x_{j(v+\epsilon_{ij})} + x_{j(v-\epsilon_{ij})} + x_{iv} \leq 1 \qquad \forall (i,j) \in \text{CI}_2, \forall v \in F_i \tag{8}$$

$$x_{iv} + \sum_{u\in V_{ijv}} x_{ju} \leq 1 + c_{ij} \qquad \forall (i,j) \in \text{CEM}_1, \forall v \in F_i, V_{ijv} \neq \emptyset \tag{9}$$

$$\sum_{j\in P_i} \lambda_{ij} \sum_{w\in F_j} T_{ijvw} x_{jw} \leq \Lambda_i + M(1 - x_{iv} + d_i) \qquad \forall i \in \text{CEM}_2, \forall v \in F_i \tag{10}$$

$$d_i \in \{0,1\} \qquad \forall i \in \text{CEM}_2 \tag{11}$$

$$c_{ij} \in \{0,1\} \qquad \forall (i,j) \in \text{CEM}_1 \tag{12}$$

$$x_{iv} \in \{0,1\} \qquad \forall i \in T, \forall v \in F_i \tag{13}$$

where

- constraints (6) state that one and only one frequency has to be assigned to each link.
- constraints (7) and (8) represent fixed distance constraints (3) and forbidden distance constraints (4), respectively.
- constraints (9) correspond to classic binary interference constraints (1). V_{ijv} denote the set of frequencies $u \in F_j$ such that $|f_v - f_u| < \delta_{ij}$, i.e. assignments of f_j violating the constraint when $f_i = v$. Hence for a distance constraint (1) there are as many linear constraints as possible values v for i such that V_{ijv} is non empty. Binary variable c_{ij} indicates if the constraint is violated. Indeed, the distance δ_{ij} between f_i and f_j is respected if and only if $c_{ij} = 0$.
- constraints (10) correspond to cumulative interference constraints (2). Let $T_{ijvw} = T_{ij}(|v - w|)$ denote the interference value of link j induced on link i if $f_i = v$ and $f_j = w$. Then, the left member of the constraint represent the sum of interferences on link i when $f_i = v$. If $x_{iv} = 0$ the constraint is always satisfied whenever constant M is large enough. Binary variable d_i allows to verify if the constraint is violated. If $x_{iv} = 1$, the constraint is satisfied and the interference sum is not grater than threshold Λ_i if and only if $d_i = 0$.
- (5) is the objective function minimizing the weighted sum of violated interference constraints, where α is the weight of binary constraints CEM_1 and β is the weight of cumulative constraints CEM_2.

Setting $c_{ij} = 0, \forall (i,j) \in$CEM$_1$ and $d_i = 0, \forall i \in$CEM$_2$, MS-FAP can be expressed as follows:

$$\min \quad f_{\max} - f_{\min} \tag{14}$$

$$\text{s-t} \quad (6), (7), (8), (9), (10), (13)$$

$$f_{\max} \geq \sum_{v \in F_i} v x_{iv} \qquad \forall i \in T \tag{15}$$

$$f_{\min} \leq \sum_{v \in F_i} v x_{iv} \qquad \forall i \in T \tag{16}$$

$$f_{\max}, f_{\min} \geq 0 \tag{17}$$

Constraints (15) and (16) are used to compute the maximal and minimal frequencies, respectively. Objective function (14) correspond to the minimization of the span.

Returning to the cumulative model, the *big* M constraints needed to represent the cumulative constraints are known to give poor relaxations. For each cumulative constraint $i \in$ CEM$_2$ (2), there is a set of equivalent constraints of the type *cover inequalities* yielding better relaxations:

$$x_{iv} + \sum_{(j,w) \in P} x_{jw} \leq |P| + d_i, \forall i \in T, \forall v \in F_i, \forall P \in \mathcal{P}_{iv} \tag{18}$$

where $P \in \mathcal{P}_{iv}$ is a set of couples (link,value) violating the cumulative constraint for $f_i = v$. Because of their exponential number, these constraints could be added only when needed through branch-and-cut techniques.

Unfortunately, the resulting integer linear programming formulation is still well-known to become intractable for practical problems, see e.g. (Mehrotra and Trick, 1996). This is due to the possibly huge number of integer variables and to the symmetry of the formulation. Hence, the proposed model, even if enhanced by cutting plane techniques, could only be used to solve small problems. It is consequently worst investigating other exact and heuristic resolution frameworks.

4 Constraint programming formulations

The constraint programming formulation of the problem is very close to the natural one already described. It is based on the following decision variables:

- f_i for all $i \in T$ with domain F_i. These variables represent directly the frequencies assigned to the links.
- d_{ij} for any couple of links (i,j) involved in a constraint. d_{ij} represents the gap value between the frequencies assigned to links i and j.
- t_{ij} for any couple of links (i,j) involved in a cumulative constraint CEM$_2$. t_{ij} represents the value of the discrete perturbation function.
- c_{ij} for all $(i,j) \in$ CEM$_1$ with domain $\{0,1\}$ represent (as in the ILP model) the violation indicators of the corresponding binary interference constraints.
- d_i for all $i \in$ CEM$_2$ with domain $\{0,1\}$ represent (as in the ILP model) the violation indicators of the cumulative interference constraints.

The model for the minimization of the violated constraints can be expressed as follows:

$$\min \quad \alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij} + \beta \sum_{i\in \text{CEM}_2} d_i \tag{19}$$

$$d_{ij} = \epsilon_{ij} \qquad \forall (i,j) \in \text{CI}_1 \tag{20}$$

$$d_{ij} \neq \epsilon_{ij} \qquad \forall (i,j) \in \text{CI}_2 \tag{21}$$

$$d_{ij} < \delta_{ij} \Rightarrow c_{ij} = 1 \qquad \forall (i,j) \in \text{CEM}_1 \tag{22}$$

$$\sum_{j\in P_i} \lambda_{ij} t_{ij} > \Lambda_i \Rightarrow d_i = 1 \qquad \forall i \in \text{CEM}_2 \tag{23}$$

$$d_{ij} = |f_i - f_j| \tag{24}$$

$$t_{ij} = T_{ij}[d_{ij}] \tag{25}$$

Constraints (22) and (23) represent the soft binary and cumulative interference constraints as implication constraints. Constraints (25) force variable t_{ij} to be the element of an array (the perturbation function) indexed by a finite-domain variable, here distance d_{ij}. These constraints are commonly known as ELEMENT constraints and were introduced in (Hentenryck and Carillon, 1988).

The model for the minimization of the span value can be expressed as follows, replacing soft by hard constraints:

$$\min \quad f_{max} - f_{min} \tag{26}$$

$$s-t \quad (20), (21), (24), (25)$$

$$d_{ij} \geq \delta_{ij} \qquad \forall (i,j) \in \text{CEM}_1 \tag{27}$$

$$\sum_{j\in P_i} \lambda_{ij} t_{ij} \leq \Lambda_i \qquad \forall i \in \text{CEM}_2 \tag{28}$$

$$f_{max} \geq f_i \qquad \forall i \in T \tag{29}$$

$$f_{min} \leq f_i \qquad \forall i \in T \tag{30}$$

$$f_{max}, f_{min} \geq 0 \tag{31}$$

The used of such a CP model to solve realistic instances will be investigated in Section 5.3.

5 Solution methods

For solving the instances presented on Section 5.1, the solution methods described in Sections 5.3 and 5.4 follow the same framework: after an initial preprocessing phase, as presented in section 5.2, the search performs in two phases. First, the methods aim at minimizing the weighted number of violated constraints; then, and only when the previous phase ends up with a number of violated constraints equal to 0, they attempt to minimize the span.

5.1 The CELAR FAPPG instances

The CELAR generated 30 instances of the frequency allocation problem with cumulative interference constraints, named FAPPG instances, from data issued from military applications [1]. To generate these instances the CELAR enriched the instances generated during the CALMA project (Combinatorial ALgorithms for Military Applications) considering only classical interference constraints. For details about the CALMA project, we refer to the FAP web site (`http://fap.zib.de`) and the paper by (Aardal *et al*, 2002). For a presentation of the considered extension and of other practical extensions considered by the CELAR, we refer to the web site `http://www.fap.ema.fr`, hosted by the Ecole des Mines d'Alès (France).

As shown in Table 1, the instances (named x_n, where x represents the instance number and n the number of links) have from 16 to 2166 links, from 10 to 1229 imperative constraints of type (3) and (4) ($|C_I|$ columns), from 16 to 4155 binary interference constraints (1) ($|C_E|$ columns) and from 16 to 2015 cumulative interference constraints (2) ($|C_C|$ columns). Additionally, the set of links of each instance can be decomposed accordingly to the K connected components of the corresponding interference graph G.

instance	K	$\|C_I\|$	$\|C_E\|$	$\|C_C\|$	instance	K	$\|C_I\|$	$\|C_E\|$	$\|C_C\|$	instance	K	$\|C_I\|$	$\|C_E\|$	$\|C_C\|$
01_0016	1	10	16	16	11_0164	3	84	439	101	21_1088	1	546	2789	1081
02_0018	1	11	24	18	12_0902	23	453	2354	572	22_0768	2	386	1604	757
03_0066	2	35	100	50	13_0306	2	155	1142	244	23_0034	1	19	53	24
04_0064	2	34	88	48	14_0194	1	99	587	163	24_0048	1	38	80	39
05_0064	1	34	80	64	15_2454	4	1229	5135	2015	25_0106	1	68	181	63
06_0182	4	93	245	172	16_0038	1	21	128	37	26_0140	2	85	219	83
07_0182	4	93	245	172	17_0040	1	22	92	36	27_0154	1	92	255	134
08_0608	20	306	812	484	18_0052	1	28	116	42	28_0398	9	199	821	340
09_1460	65	732	1862	1123	19_0770	1	387	2276	770	29_0526	17	263	980	471
10_1698	73	851	705	1292	20_1930	136	967	3896	1075	30_2166	46	1083	4155	1985

Table 1: Characteristics of the 30 CELAR instances

5.2 Preprocessing

For each link $i \in T$, the frequency domain can be reduced by propagating the imperative fixed distance constraints (3) as follows:

$$v \in F_i, \quad \text{if } \nexists w \in F_j : |f_i - f_j| = \epsilon_{ij} \Rightarrow F_i = F_i - v$$

This simple reduction rule has a positive impact on 14 out of 30 instances (from 2.07% to 33.33% of values removed from the links domains), but significantly only on 6 instances (more than 20% of removed values).

We also propose a special preprocessing for the cumulative constraints, taking into account jointly these constraints and the fixed distance constraints (3). On one hand, the cumulative constraints depend on the distances between f_i and $f_j \in P_i$ for any $i \in \text{CEM}_2$. On the other hand, the fixed distance constraints provide us with the exact value ϵ_{ij} of this distance for any $(i,j) \in \text{CI}_1$. Hence we can use these values directly in the cumulative constraints where couples of frequencies linked with duplex constraints are involved. This preprocessing is sufficient to prove that instances 25 and 27 possess a lower bound of 2 violated cumulative constraints.

[1] The instances can be downloaded from `http://www.fap.ema.fr/save.php/fr/Local/fap/dir/instances/fapg/archives/bench_fappg.zip`

5.3 Hybrid constraint programming and combinatorial optimization exact method

The CP model presented in Section 4 is solved by a constraint programming solver (ILOG solver 5.0) by specifying the branching rule and using the standard constraint propagation algorithms of the solver. The simple branching rule we use consists in selecting the variable f_i with the smallest domain and exploring the values of domain F_i in an increasing order. For optimization, series of feasibility problems are solved by generating constraints on the objective function.

To enhance the CP-based method, we have coupled the solving of the CP model with the solving of a relaxation of the problem based on the concept of cliques in a constraint graph. Considering the cumulative interference constraints (2), we can deduce classical binary interference constraints (1) as follows:

$$|f_i - f_j| \geq \min_{e \in IN} \{e : \lambda_{ij} T_{ij}(e) \leq \Lambda_i\} \quad \forall j \in P_i, i \in \text{CEM}_2 \tag{32}$$

Using these constraints, we build a constraint graph where the nodes are the links and the edges are the original binary interference constraints (1) and fixed distance constraints (3) plus the deduced binary interference constraints (32). Each edge is weighted by the distance, i.e. the right term of the corresponding constraint. Let us consider a k-clique in the constraint graph. Solving a relaxation of the traveling salesman problem, that is the perfect matching, in this clique gives a lower bound for the span criterion. Hence we can deduce that there is no solution to the problem when the lower bound is greater than the difference between the largest and the smallest frequency values of all frequency domains. This relaxation is used to prune the search at each node of the CP search phase.

Though this method doesn't reveal itself extremely effective, the relaxation based on the perfect matching was able to find lower bounds for the number of violated constraints on some instances. For the span criterion, the problem is solved to optimality for instances 1, 23 and 24.

We summarize the obtained non zero lower bounds for the number of violated constraints in table 2. The optimal solutions found for the span criterion by the hybrid method are displayed in Table 3.

Instance	16	17	19	20	25	27
LB MI	1	2	4	1	2	2

Table 2: Non zero lower bounds for the number of violated constraints obtained by the combinatorial bound

Instance	1	23	24
opt span	548	380	410

Table 3: Optimal span found by the hybrid method

5.4 Solving the problem with large neighborhood search

For most instances, the high number of variables and the important domains size do not allow the use of an exact ILP or CP solution method. We have then developed a heuristic scheme based on the Large Neighborhood Search (LNS) methodology. A preliminary version of this work was presented in (Palpant *et al.*, 2002). This

LNS method has shown its effectiveness and applicability to big problems. In particular, it has been able to find all the solutions proved optimal by the hybrid CP-ILP method.

The method consists in generating and solving iteratively sub-problems of the global problem. Regarding FAP, the idea is not entirely new since a hybrid ad-hoc method called Solve and Extend has previously been assessed on specific problems. Thus, in (Smith et al., 1998), a clique of level p in the constraint graph is used to generate a first sub-problem which is solved by a specific heuristic. The extension of the partial solution obtained at the end of this first phase to a complete solution is then performed heuristically too. The process is iterated by adding each time vertices of maximal saturation to the initial clique. As for the approach of (Mannino and Sassano, 2003), an implicit enumeration scheme is used to solve and extend the sub-problems. The latter are obtained one from each other by adding links to an initial sub-problem regarding connectivity criterions in the constraint graph.

The method we present in this paper differs from the two ones cited above. Here, sub-problems are not extended in order to obtain a complete solution, but the current complete solution is considered to generate and solve sub-problems that are then used to improve this current solution. It consists then in a local search procedure that explores iteratively neighborhoods of the current solution.

For each phase of the optimization process, the algorithm works on the K problems $P_1, \ldots, P_K$, of respective size $n_1, \ldots, n_K$, defined by the connected components of the corresponding interference graph G. Regarding interference minimization, solving independently each problem P_k is equivalent to solving the whole problem, while this is not the case for span minimization since the objective function may relate to links belonging to distinct connected components. Nevertheless, because of the specific strategies involved in the approach, the second phase of the optimization process also works on this decomposition. The general execution scheme, given in Figure 2, consists in generating and solving, at each iteration s, K distinct sub-problems. For each problem P_k, sub-problem SP_k^s, of size $p_k \leq n_k$, is built using current solution $\overline{F}^{s-1}$ and considering the current objective (interference or span). It is then solved by an appropriate method in order to obtain partial solution $\overline{F}_k^s$. The result of the optimization of the K sub-problems is then used to compute neighbor solution $\overline{F}^s$, from which the process is iterated. The algorithm, which return final solution $\overline{F}^*$, stops when maximal execution time MAX_CPU is reached.

1. preprocessing
2. generate initial solution $\overline{F}^0 = (\overline{F}_1^0, \ldots, \overline{F}_K^0)^t$ with the help of a greedy algorithm
3. $\overline{F}^* = \overline{F}^0, s = 1$
4. repeat
5. for each problem P_k
6. generate SP_k^s from $\overline{F}^{s-1}$ considering current objective
7. solve SP_k^s to compute partial solution $\overline{F}_k^s$
8. $\overline{F}^s = (\overline{F}_1^s, \ldots, \overline{F}_K^s)^t$
9. eventually update $\overline{F}^*$
10. $s = s + 1$
11. until CPU=MAX_CPU

Figure 2: General algorithm

We describe in the following subsections each crucial point of the algorithm.

5.4.1 Generation of the initial solution

For each problem P_k, we compute initial solution $\overline{F}_k^0$ with the help of a greedy algorithm. This heuristic performs in n_k steps, each one selecting the most constrained link (i.e. the one that appears in the biggest number of constraints) and assigning it to the lowest possible frequency that minimizes the increase of the interference value. At each step of the procedure, the problem is kept consistent, i.e. the value assigned to the selected link satisfies the imperative constraints.

5.4.2 Sub-problem generation

At current iteration s, sub-problem SP_k^s is defined for each problem P_k by freeing a subset T_k^s of links. The other links keeping the value they are assigned to in current solution $\overline{F}^{s-1}$, the sub-problem lies then in finding a solution satisfying several constraints. Thus, fixed links impose reductions on freed links domains via imperative and/or co-site constraints. In addition, other reductions may be performed according to some specific strategies involved during the second phase of the optimization process (span minimization).

The selection of the freed variables starts from a randomly chosen link i. In the case of interference minimization, this link is assumed to belong to a constraint violated by the current assignment. We then extend subset T_k^s by selecting every link j appearing simultaneously to i in any constraint, i.e. such that vertices representing i and j are adjacent in constraint graph G. At the end of this step, if p_k is not reached, the process is iterated starting from a link already included in T_k^s. Figure 3 shows an execution of the process. Starting from randomly chosen link 9, links 8, 10, 2, 3 and 12 are firstly included in T_k^s, in the given order. Considering that $p_k = 8$, the process is iterated from the secondly included link, that is link 8, and then from link 10, included in third position in T_k^s, which gives as final result $T_k^s = \{9, 8, 10, 2, 3, 12, 7, 11\}$.

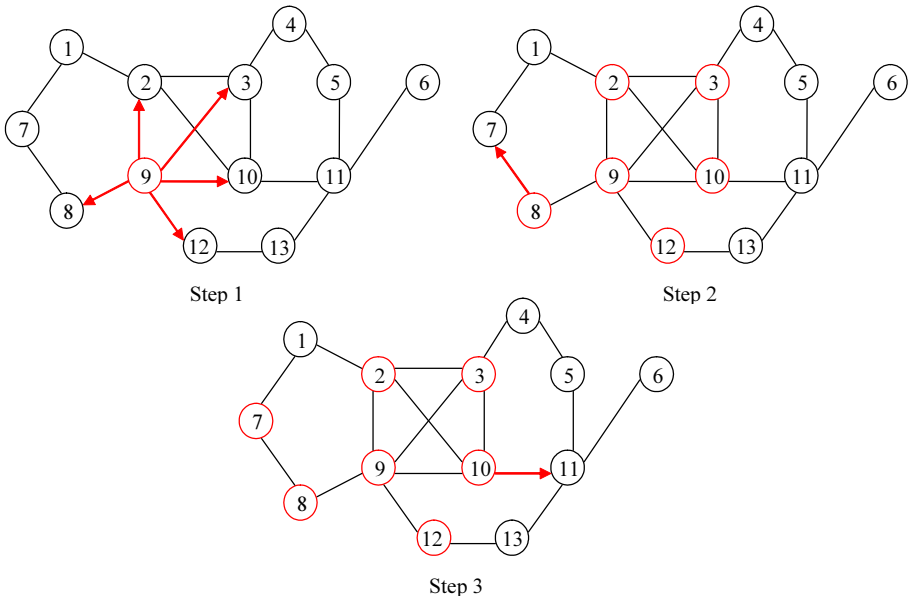

Figure 3: Selection process

Once subset T_k^s defined, the sub-problem lies in finding a feasible assignment to the freed links such that the

following constraints are satisfied (interference minimization phase):

$$\alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij} + \beta \sum_{i\in \text{CEM}_2} d_i < \alpha \sum_{(i,j)\in \text{CEM}_1} c_{ij}^{s-1} + \beta \sum_{i\in \text{CEM}_2} d_i^{s-1} \tag{33}$$

$$(20), (21), (22), (23), (24), (25)$$

$$f_i = f_i^{s-1} \qquad \forall i \notin T_k^s \tag{34}$$

As for the span minimization phase, several strategies may be involved, in order to tackle problems $P_1, \ldots, P_K$ independently. They consist in performing additional domains reductions on freed links domains. Let $f_+ = \max_{i\in T} f_i^{s-1}$ and $f_- = \min_{i\in T} f_i^{s-1}$ denote the values of the maximum and minimum assigned frequencies in current solution $\overline{F}^{s-1}$ respectively, we can now consider the following cases:

- AllMax strategy: it is employed when all links assigned to f_+ belong to T_k^s and at least one link assigned to f_- doesn't. The lowest assigned frequency being fixed, reassigning the frequencies of freed links in the interval $I = [f_-, f_+[$ is sufficient to obtain an improved solution. Consequently, all values that don't belong to I are removed from the domains of the freed links.
- AllMin strategy: it is the symmetric strategy of the previous one. The domains are reduced to the interval $I =]f_-, f_+]$.
- NoMinMax: it is involved when a link that doesn't belong to T_k^s is assigned to f_+ and another one to f_-. Here, the span value can't be improved; however, rearranging the frequencies in the interval $I = [f_-, f_+]$ can lead to a further improve of the solution and also contributes to diversify the search. Domains values are then kept in this interval.
- AllMinMax: it is used when all links assigned to f_+ and f_- belong to T_k^s. Given $d_- = \min_{i\notin T_k^s} f_i^{s-1} - f_-$ and $d_+ = f_+ - \max_{i\notin T_k^s} f_i^{s-1}$, any solution improving the span value can't contain a link assigned outside the interval $I = [f_- - d_+, f_+ + d_-]$.

All these strategies are coupled with appropriate objectives. Hence, for the three first cases, a feasible solution (i.e. a solution satisfying all interference and interval constraints) is searched, while the last strategy involves the search of an optimal solution for SP_k^s.

It is worth noting that the heuristic domains reductions of two first strategies may suppress good or even optimal solutions from the neighborhood. However, by reducing the neighborhood size, they contribute to make its exploration easier and consequently allow to solve sub-problems of bigger size. Moreover, the diversity of the strategies involved during the search, combined with a non-deterministic selection scheme of the freed links may hopefully lead to reconsider previously ignored solutions.

5.4.3 Solving the sub-problems

Sub-problem SP_k^s is solved taking into account two criterions: on one hand, the considered objective; on the other hand, the global solution strategies. Regarding the optimization parameter, it is worth noting that, due to the several strategies and objectives involved during the search, solving a sub-problem doesn't necessarily lead to the obtention of an optimal solution, neither of an improved one, for the considered sub-problem.

The global solution strategy defines the computing effort that will be spent to find the neighbor solution: a heuristic will provide medium-quality solutions but in a very short time, while an exact method will necessitate

more computing time to obtain high-quality solutions. The idea is to apply a heuristic scheme when the current solution is likely to be easily improved (i.e. during the first iterations) and then apply an exact procedure in order to intensify the search.

The heuristic procedure is the one presented above. As previously said, this method doesn't assume to provide an improved neighbor solution, in which case the current solution remains unchanged. However, it globally behaves effectively to improve medium-quality solutions. That's why it is employed at the beginning of the first phase of the optimization process, in order to obtain a solution of satisfying quality.

Once this solution reached, another solution scheme is involved to tackle the sub-problems. The latter consists of a truncated exact procedure. The search is stopped as soon as an improved or optimal solution is found, or when a time limit H is reached. This method allows to intensify the search of the neighbor solution as it spends more time to explore the neighborhood. The time limit parameter is used to tune effectively the procedure: it is high for interference minimization or when AllMinMax strategy is active; on the contrary, it is set to a low value for all other cases of span minimization, in particular when NoMinMax strategy is employed.

6 Comparison of the methods in competition

Experiments have been conduced on a PC equipped with a 350 MHz CPU and 256 Mo RAM. To establish a comparison, we have confronted our LNS approach to the Simulated Annealing method of (Sarzeaud, 2003) and the Tabu Search method of (Vlasak and Vasquez, 2003), the two other teams challenging on this specific problem. In order to make a fair comparison, all of the tests have been performed following the same operating mode (1 hour of execution time, same hardware). Table 4 gives the results on the CELAR instances for all approaches in terms of interference and span value. Columns LNS, SA and TS give the results of the Large Neighborhood Search, Simulated Annealing and Tabu Search methods, respectively. For each approach, we have reported the results obtained for both criterions: if the interference value is greater than 0, the latter is then reported in column MI-FAP and the associated span value is indicated in parenthesis; on the contrary, if all electromagnetic constraints have been satisfied, the span value is reported in column MS-FAP. Best solutions are displayed in bold.

The LNS method is in general superior to the SA and TS methods, with the notable exception of instance 19. This particular instance is the only one that involves different weights in the expression of the sum of violated constraints, which could explain the bad performance of our LNS approach on it. Indeed, the sub-problems selection process may be less adequate to that particular case, since it does not take weights into account for the selection of the freed links. Another remark that can be raised from the results presented above concerns the good behavior of the LNS method on highly decomposable instances when minimizing the span criterion (interference free instances 8, 9, 10, 12 and 29). In this case, it literally outperforms the results obtained by the two others approaches. This tends to prove the effectiveness of the neighborhood reduction rules involved during the second phase of the process. An interesting outcome could then consist in extending this methodology to non-decomposable instances. These ones could be divided into several components regarding connectivity criteria in the constraint graph, allowing then the LNS method to work on smaller problems during the second phase.

instance	LNS		SA (Sarzeaud *et al*)		TS (Vlasak *et al*)	
	MI-FAP	MS-FAP	MI-FAP	MS-FAP	MI-FAP	MS-FAP
01	-	**548**	-	549	-	**548**
02	-	**629**	-	**629**	-	**629**
03	2 (599)	-	**2** (**580**)	-	2 (623)	-
04	-	520	-	520	-	**519**
05	-	**599**	-	623	-	676
06	-	**718**	-	**718**	-	758
07	6 (666)	-	**4** (**698**)	-	8 (687)	-
08	-	**620**	-	646	-	642
09	-	**544**	-	656	-	860
10	-	**412**	-	692	-	849
11	-	604	-	656	-	**601**
12	-	**572**	1 (639)	-	2 (634)	-
13	9 (399)	-	**6** (**399**)	-	8 (380)	-
14	-	398	-	360	-	**354**
15	**31** (**399**)	-	73 (399)	-	44 (399)	-
16	**46** (**146**)	-	**46** (**146**)	-	**46** (**146**)	-
17	46 (99)	-	**45** (**98**)	-	**45** (**98**)	-
18	-	**404**	1 (476)	-	-	408
19	4385 (492)	-	4375 (496)	-	**3998** (**496**)	-
20	**150** (**492**)	-	193 (496)	-	152 (496)	-
21	-	**982**	2 (964)	-	4 (994)	-
22	-	**788**	-	818	1	894
23	-	**380**	-	**380**	-	**380**
24	-	**410**	-	430	-	**410**
25	2* (540)	-	**2** (**490**)	-	2* (540)	-
26	-	**480**	1 (492)	-	-	**480**
27	**2** (**490**)	-	4 (490)	-	**2** (**490**)	-
28	-	**610**	-	646	-	638
29	-	**542**	-	852	-	866
30	**23** (**912**)	-	27 (912)	-	32 (912)	-

Table 4: Computational results on the FAPPG CELAR instances

7 Comparison of the models

Finally, the last goal of the study was to establish a comparison between the model including the cumulative interference constraints and the model which replaces these constraints by more constrained binary interference constraints. Recall that the classical representation of the interference constraints can be obtained by replacing all CEM_2 constraints (2) by CEM_1 constraints (1), through a uniform distribution of the "right to disrupt":

$$\lambda_{ij}|f_i - f_j| \geq \frac{\Lambda_i}{|P_i|} \quad \forall j \in P_i \tag{35}$$

There exists in theory, because of the construction of the constraints, a solution for the problem with the cumulative constraints as least as good as the best solution of the classical model. However, the objective of the current study is to determine whether the methods are able to find these solutions in a reasonable amount of time. In other words, is the increase in complexity of the constraints balanced by the quality of the obtained solutions?

From the CELAR instances, we then generated the set of corresponding instances for the classical binary model. The results displayed in Table 5 are obtained with the LNS method. If a solution satisfies all electromagnetic constraints, the span value is indicated in column MS-FAP. Otherwise, the interference value is displayed on column MI-FAP. Values in parenthesis indicate the results obtained on the model with cumulative constraints. The experiments have been conduced as described in the previous section.

Instance	MI-FAP	MS-FAP	Instance	MI-FAP	MS-FAP
01	-	571 (548)	16	57 (46)	-
02	1 (0)	-	17	55 (46)	-
03	3 (2)	-	18	-	488 (404)
04	-	718 (520)	19	5629 (4385)	-
05	1 (0)	-	20	178 (150)	-
06	-	(761) 718	21	24 (0)	-
07	17 (6)	-	22	10 (0)	-
08	-	620 (620)	23	-	380 (380)
09	-	650 (544)	24	-	440 (410)
10	-	500 (412)	25	2 (2)	-
11	-	617 (604)	26	-	480 (480)
12	-	600 (572)	27	2 (2)	-
13	13 (9)	-	28	-	634 (610)
14	1 (0)	-	29	-	548 (542)
15	31 (31)	-	30	27 (23)	-

Table 5: Benefits of using the cumulative interference constraints

The results show that the model with cumulative constraints obtains a larger number of interference free solutions than the model with only binary interference constraints. This shows clearly the benefit of introducing the cumulative interference constraints to solve practical frequency allocation problems.

8 Conclusion

We have performed an experimental comparison of two models and several methods to solve frequency assignment problems with cumulative interferences.

The large neighborhood search method we propose is superior to the existing heuristics, although it does not dominate them on all instances.

Last, our study establishes that good heuristic methods can take advantage of a direct representation of the cumulative constraints, despite their complexity. Such a result is of practical importance to solve real assignment problems since practitioners would benefit from switching, at least partially, to the new model.

As suggested by our preliminary encouraging results in this way, a future direction of research may consist in designing efficient constraint propagation techniques for the cumulative interference constraints.

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