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# A NEW APPROACH TO CONVERGENCE CONFINEMENT METHOD

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Determining the instant for support activation in plane strain tunnel calculations is an essential step, as for Convergence Confinement Method. Recent researches on that subject lead to an original development of that Method which allows for an accurate determination of that instant. The proposed methodology results from "experimental numerical laws" learned from comparison between plane strain approach and direct three dimensional one. For this purpose, a reference three dimensional geometry is considered: a constant rate advancing circular tunnel in anisotropic geostatic stress state, with one principal direction parallel to tunnel axis. The isotropic state of stress is first considered. It is shown that two fundamental principles allow to treat completely the plane strain approach. It is then shown how these principles can be extended to the anisotropic stress state, using a "mean convergence curve concept. Finally, it is proposed to apply such a method to three dimensional tunnels in steady state advancement. With such improvements, the Convergence Confinement Method appears as a self-contained plane strain approach to the three dimensional advancing tunnel problem, without much additional intricacy to classical approaches.

## 1 Introduction

Support is fundamental to the stability of deep tunnels. The three dimensional nature of this problem involves an intricate interplay between the construction phases, rock mass and support behaviour. The framework of continuum mechanics is considered herein. Although three dimensional calculation has achieved constant progress in tunnel modelling, it remains heavy to carry out, and approximate plane strain analyses are still preferred and currently used in practice.

Among other methods, Convergence Confinement (CV-CF) is the most popular, using a variable fictitious stress state at the tunnel wall in order to reproduce the effects of face advance on convergence of a tunnel section.. However, up to now, efficiency of these approaches have been seriously impeded by the drawback of undetermined instant for support activation, an essential parameter which controls rock mass-support interaction.

This paper presents recent researches on this subject which allow to eliminate now satisfactorily such undetermination, without much modifying the nature of calculations. The method presented herein is intended to be self contained, independent of the particular behaviour of the rock mass and the support system.

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## 2 Defining a reference tunnel problem

The method for determining the support activation instant is based on general rules learned from comparison between the direct three-dimensional problem and the corresponding plane strain approach. To demonstrate this, let us consider a simplified "reference tunnel problem", which contains the main ingredients of a three dimensional tunnel problem: a circular tunnel of unit radius (Figure 1), with anisotropic geostatic stress state, one principal stress direction being parallel to tunnel axis. This tunnel and its support placement are advancing at constant strain rate. Remark that steady state hypothesis is understood in CV-CF plane strain approach.

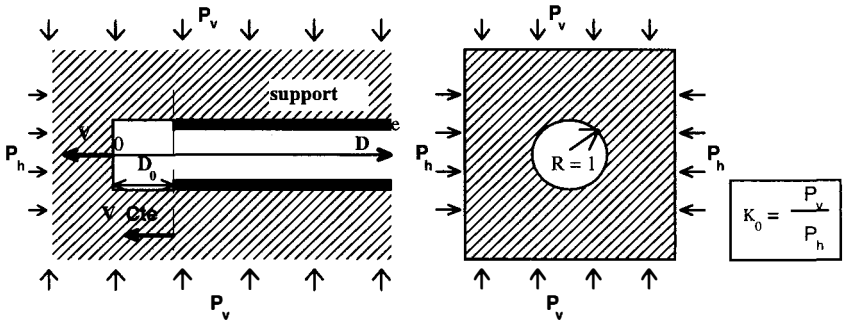


Figure 1: Reference tunnel problem

As intrinsic rules are expected to be found out, details of constitutive laws are less important, so a simple elastoplastic incompressible Mises model has been mainly used:

$$\sqrt{J_2} - C = 0$$

This problem is defined by the following parameters

Stress anisotropy factor .....	$K_0 = P_h / P_v$	
Mean geostatic pressure	$P = (\sigma_h + \sigma_v) / 2$ :	
Loading factor .....	$N_s = P / C$	
Support laying distance to the face .....	$D_0$	(1)
Relative support stiffness .....	$K^* = K_n / K_m$	
$K_m$ is the elastic ground stiffness, and $K_c$ support normal stiffness		
Let us also define $K_b$ the support bending stiffness		

For direct calculations (excavation, support placement), specific softwares have been built up, taking advantage of the peculiar hypotheses of the reference problem. These results in a versatile Finite Element programs, based on the steady state algorithm (Nguyen Q.S & al. <sup>8</sup>, Maitournam <sup>12</sup>, Corbetta <sup>4</sup>, Guo <sup>3</sup>), which allows to treat the steady state problem on a fixed geometry, instead of using a classical sequential procedure. In case of anisotropic stress field, Fourier's development of solutions in the angular direction  $\theta$  has been used, along with the steady state algorithm (Braham et al. <sup>13</sup>, Maitournam et al.<sup>11</sup>), which replaces the three dimensional problem by a succession of two dimensional problems.

### 3 Statement of the problem

The CV-CF Method is illustrated on the generic case of a circular tunnel in rotational symmetry  $K_0 = 1$  (Figure 2), with a fictitious pressure  $P_i$  applied at the wall, or conversely, a deconfining ratio  $\lambda$  by relationship  $P_i = 1 - \lambda$  ( $P_i$  is normalized by  $P$ ).

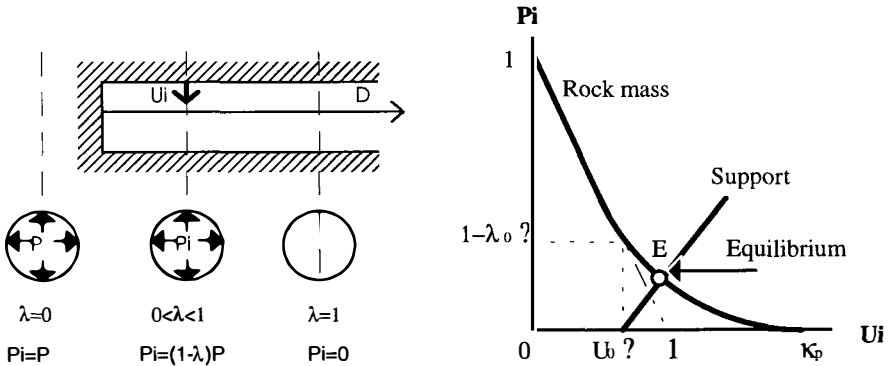


Figure 2: Fictitious pressure concept in plane strain approach and CV-CF diagram

Equilibrium of the supported tunnel appears on the non dimensional diagram  $P_i$  vs  $U_i$  (convergence normalised by free elastic one) as intersection between rock mass convergence curve, and support confinement reaction curve. This latter curve is easily obtained, but its abscissa at the origin,  $U_0$  (or, conversely, the deconfining ratio  $\lambda_0$ ), is undetermined. Indeed, the actual time for activation is more likely represented by distance to tunnel face  $D_0$  for support placement, so the basic problem is to find out the relationship between this parameter  $D_0$  and  $U_0$ .

This CV-CF method is currently extended to three dimensional cases, using plane strain FEM analyses, but of course, the undetermination of the support activation instant still remains.

Up to now, no systematic method allowed to resolve such undetermination (AFTES<sup>1</sup>). Different studies on this subject, initiated since years 1990's (Corbetta<sup>4</sup>, Bernaud<sup>6</sup>, Guo<sup>3</sup>), have allowed now to eliminate satisfactorily this drawback, giving practically the same results as by direct three dimensional approach.

#### 4 Circular symmetric case ( $K_0 = 1$ )

The question of Figure 2 is equivalent to the following set of equations :

$$\begin{cases} P_i = M(U_i) & \text{ground} \\ P_i = K^*(U_i - U_0) & \text{sup port} \\ U_0 = ? \end{cases} \quad (2)$$

Which give support pressure  $P_i = P_\infty$  and final convergence  $U_i = U_\infty$  of tunnel, provided a supplementary equation for  $U_0$  is given in terms of  $D_0$ , and eventually, of  $U_i$ . Indeed, the problem could be resolved if the relationship  $U_i$  vs distance to the face  $D$  could be determined (Figure 3).

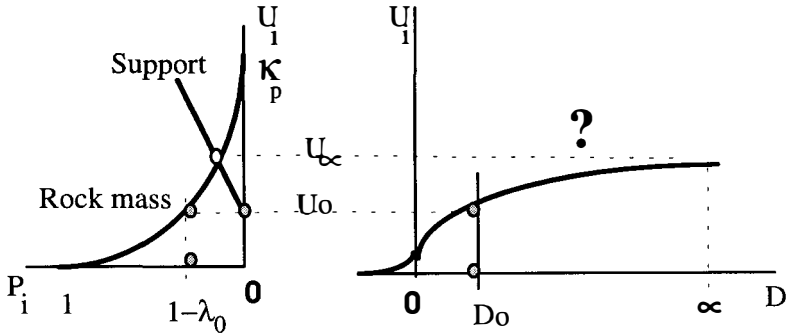


Figure 3: Scheme for determining  $U_0$

#### 4.1 Self Similarity Principle and approximate solution

An approximate determination of  $U_0$  refers to the free convergence curve of the tunnel,  $U_i^f(D)$ , instead of  $U_i(D)$  curve so that :

$$U_0 \cong U_i^f(D_0) \tag{3}$$

Historically, this solution was effectively proposed for the elastic case. An analytical expression can be given for the free elastic convergence  $U_i^f(D) = F(D)$ , for instance (Panet <sup>2</sup>) :

$$F(D) = 1 - 0,71 \left( \frac{0,84}{0,84 + D/R} \right)^2 \tag{4}$$

remark that  $\lambda_0 = U_0$  only for linear elasticity.

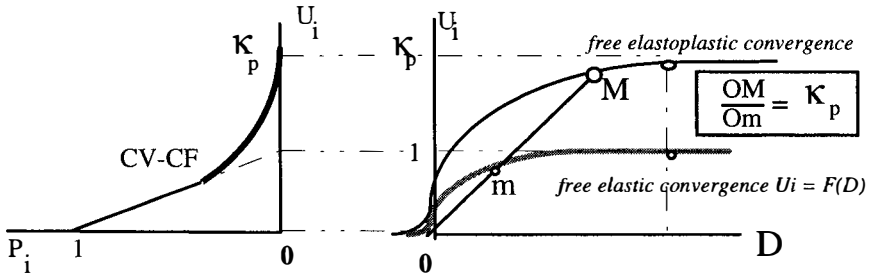


Figure 4: Schematic diagram of self similarity principle

In elastoplasticity, a Principle, called the Self Similarity Principle, based on numerical data evidence for various elastoplastic behaviours (Corbetta <sup>4</sup>, Corbetta & Nguyen-Minh <sup>5</sup>), allows us to derive the free elastoplastic convergence curve  $U_i^f(D)$  from the normalised elastic one  $U_i = F(D)$ , by self similarity from the origin (Figure 4). The equation of this curve expresses in terms of elastic function  $F(D)$  and the self similarity ratio  $\kappa_p$  obtained from a CV-CF calculation:

$$U_i^f(D) = \kappa_p F(D / \kappa_p) \tag{5}$$

Thus, by a quite procedure to the elastic case, we can easily obtain an approximate value for  $U_0$ , much more convenient than tables for  $\lambda_0$  proposed by AFTES<sup>1</sup>.

#### 4.2 Interaction Principle

However, the approximation based on the free convergence curve  $U_i^f(D)$  may lead to important errors up to 40% on support pressure (Bernaud<sup>6</sup>). This can be termed as the face support interaction effect, illustrated on Figure 5.

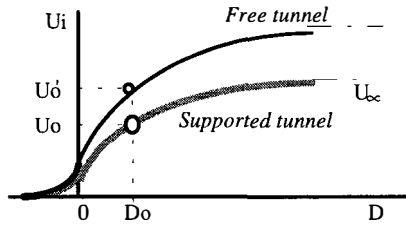


Figure 5: Face support interaction effect

In order to analyse that effect, extensive parametric analyses were carried out on the direct numerical model, including different stability numbers, relative support stiffness  $K^*$ , and support distances  $D_0$  (Figure 6).

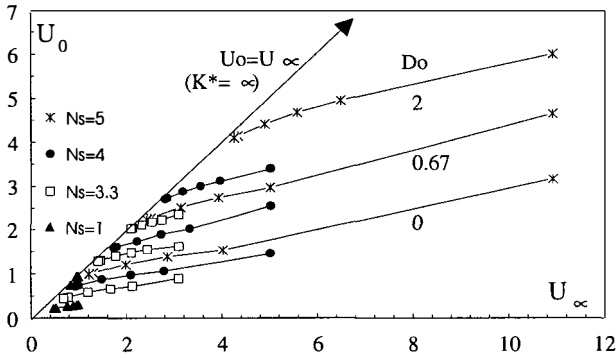


Figure 6: Numerical data for  $U_0$  vs  $U_\infty$  (Mises model)

When plotted in an adimensional diagram  $U_0^*$  vs  $U_\infty^*$ , by dividing  $U_0$  and  $U_\infty$  by their respective free convergence values :

$$U_0^* = \frac{U_0}{U_0^f} \quad ; \quad U_\infty^* = \frac{U_\infty}{\kappa_p} \quad (6)$$

the results present a definite trend, called the Interaction Principle expressed as follows :

$$U_0^* = \Phi(U_\infty^*)$$

$$\Phi(x) = 0.55 + 0.45x - 0.42(1-x)^3 \quad (7)$$

Where  $0,23 \leq \Phi(x) \leq 1$  since  $0 \leq x \leq 1$ .

This curve does not depend on any parameters of the model, e.g. support stiffness  $K^*$ , distance for setting support  $D_0$ , stability number  $N_s$ . We shall admit its universality, i.e., its validity for any other rock mass constitutive law. This has been effectively verified by internal and external validations, using other constitutive laws or alternative theories; moreover, interaction principle was proved to work in viscoplasticity (Guo<sup>3</sup>).

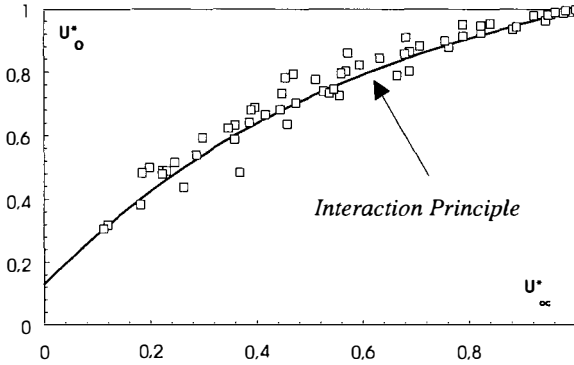


Figure 7: Interaction Principle  $U_0^* = \Phi(U_\infty^*)$

### 4.3 Conclusion

The missing equation for  $U_0$  in system (2) is finally obtained :

- approximately, with  $U_0 \cong U_1^f(D_0)$  , using Self Similarity Principle (eq (5)),
- with taking account of Interaction Principle (6), combined with the Self Similarity Principle (5), which gives  $U_1^f(D_0)$  .



## 5 Examples

### 5.1 Elastic tunnel

The final convergence of a supported elastic tunnel writes as :

$$U_i = \frac{1 + K^* U_0}{1 + K^*} \quad (8)$$

Remark  $0 \leq U_i \leq 1$ ; two different expressions for  $U_0$  can be given, depending on neglecting or not the support interaction effect (eq. (5) , (6) and (7) for  $U_0$ ). There results a final tunnel closure  $U_i = U'_\infty$  or  $U_i = U_\infty$ , as illustrated graphically, in a  $(Y, U_i)$  diagram, by setting  $Y$  equal to either member of eq. (8) (Figure 8).

If  $K^*$  were set to infinity, the second member of eq (8), taking account of interaction, is  $Y = F(D_0) \Phi(U_\infty)$ , which is function  $\Phi(U_\infty)$  reduced by affinity  $F(D_0)$ .

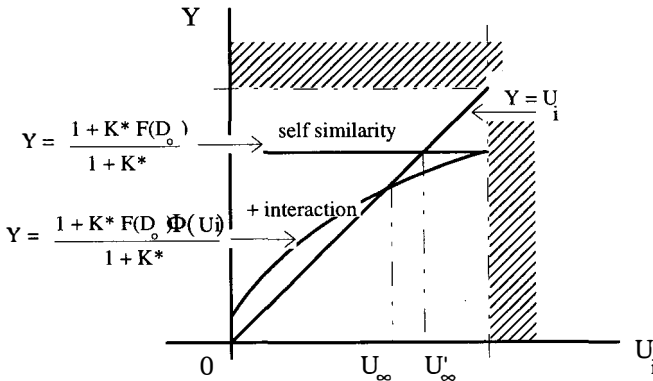


Figure 8: Graphical illustration of the two kinds of solution for elastic tunnel

### 5.2 Elastoplastic tunnel (Tresca model, cohesion C)

The elastoplastic convergence curve writes :

$$P_i = 1 - \lambda = 1 - \frac{1}{N_s} (1 + \text{Ln}(N_s U_i)) \quad (9)$$

Elastoplastic behaviour for the supported tunnel implies conditions on  $U_i$  and  $U_0$ :

$$\frac{1}{N_s} \leq U_i \leq \kappa_p \quad ; \quad \frac{1 + K^* U_0}{1 + K^*} \geq \frac{1}{N_s} \quad (10)$$

where  $N_s = P/C$  is the stability number.

Taking into account support reaction gives :

$$\frac{1 + K^* U_0}{1 + K^*} = \frac{K^* U_i + \frac{1}{N_s} (1 + \text{Ln}(N_s U_i))}{1 + K^*} \quad (11)$$

which is the analogous of equation (8). Let us again represent eq (11) in  $(U_i, Y)$  diagram, with  $Y=Y1$  (first member) and  $Y=Y2$  (second member) (Figure 9). Keeping in mind inequality (10) and  $F(D_0 / \kappa_p) < 1$ , this diagram clearly shows that a unique solution exists for  $U'_\infty$  and  $U_\infty$ , as well as their difference.

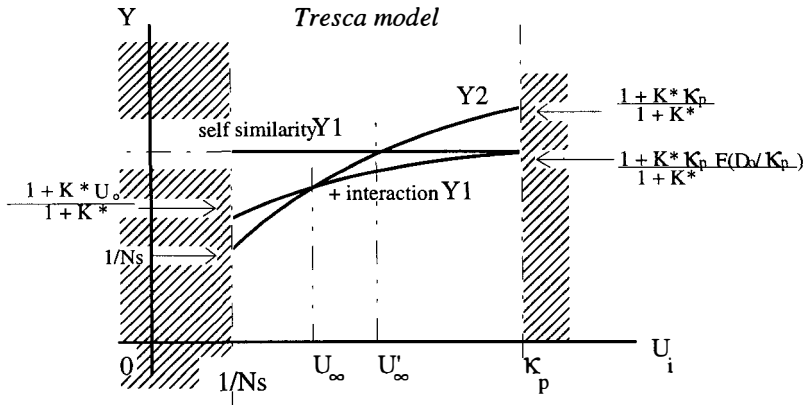


Figure 9: Graphical illustration of the two kinds of solution for elastoplastic tunnel

## 6 Circular tunnel in anisotropic geostatic stress state ( $K_0 \neq 1$ )

When rotational symmetry is lost, the fictitious pressure concept can be naturally extended to a fictitious stress, applied at the wall of the gallery. It is currently admitted that this fictitious stress be proportional to the initial stress state, via the deconfining ratio  $\lambda$  :

$$\underline{\underline{\sigma}}^R = (1-\lambda)\underline{\underline{\sigma}}_0 \quad (12)$$

Note that stresses and pressures are now normalised to the mean geostatic pressure  $P$ , and convergences and displacements to the elastic convergence of the same tunnel under isotropic geostatic pressure  $P$ .

### 6.1 Validity of fictitious stress concept and deconfining ratio

This concept has however to be verified, and, effectively, we checked that (Guo <sup>3</sup>, Nguyen Minh and Guo <sup>10</sup>) :

- For a free tunnel, a  $\lambda(D)$  relationship can be exhibited, which relates the deformed section in 2D approach to a tunnel section at a given distance  $D$  in the direct 3D calculation. Remind, according to what has been discussed here above, relation  $\lambda(D)$  is not sufficient to determine the support activation instant.

- There exists  $\lambda=\lambda_0$ , for activating support, allowing to find back the final deformed section of the direct 3D approach.

### 6.2 Mean convergence curve concept

Let us define a mean fictitious pressure  $\bar{P}_i$  applied at the wall of tunnel, as the mean flux of the fictitious applied stress, and the mean convergence  $\bar{U}_i$  of the tunnel as the mean flux of displacement at the wall (e.g. half surface variation of section).

It is interesting to note, for an elastic tunnel, that the mean convergence curve  $\bar{U}_i, \bar{P}_i$  :

$$\begin{aligned} \bar{P}_i &= (1-\lambda) \\ \bar{U}_i &= \lambda \quad (\text{in elasticity}) \end{aligned} \quad (13)$$

is independent on the stress anisotropy ratio  $K_0$ , and is identical to that of the tunnel under isotropic geostatic pressure  $P$ . This shows that defining mean values according to (13) is adequate to express an intrinsic behaviour of the elastic tunnel under anisotropic stress state. This is no more true in elastoplasticity: the  $\bar{U}_i, \bar{P}_i$  curve then characterises the intrinsic influence of the anisotropic stress ratio  $K_0$  (Figure 10).

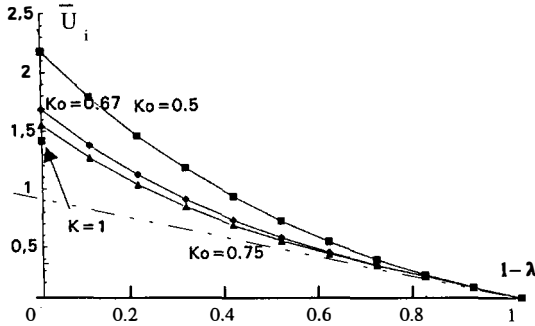


Figure 10: Effect of stress anisotropy on mean convergence curve in elastoplasticity

The "mean convergence curve" concept is all the more interesting, that the bending support stiffness has a negligible influence on the deformation of a tunnel section. This can be understood, since the ratio of normal stiffness to bending stiffness is  $\frac{K_n}{K_b} = \frac{4}{3} \left( \frac{R}{e} \right)^2 \gg 1$  ( $e$ , support thickness).

Tunnel closure appears thus to be controlled uniquely by the normal support stiffness.

### 6.3 Method for determining $\lambda_0$

The interesting properties of the mean convergence curves allow us to expect determining  $\lambda_0$  by postulating that Interaction Principle still works on it for the anisotropic loading case. However, it must be checked that Self Similarity is still valid, which is actually the case as shown on Figure 11.

Finally, determining  $\lambda_0$  for the anisotropic stress state is reduced to that of a rotational symmetry problem with a "mean rock mass convergence" curve. Let us remind again, that, except in elasticity, this "mean" curve is not the convergence of the circular tunnel submitted to mean the geostatic stress  $P$ , but is sensitive to  $K_0$ .

The solution thus obtained in CV-CF method has been proved to agree quite well with the direct tunnel calculation (Guo <sup>3</sup>), which validates at the same time the applicability of Interaction Principle to the non isotropic geostatic stress state.

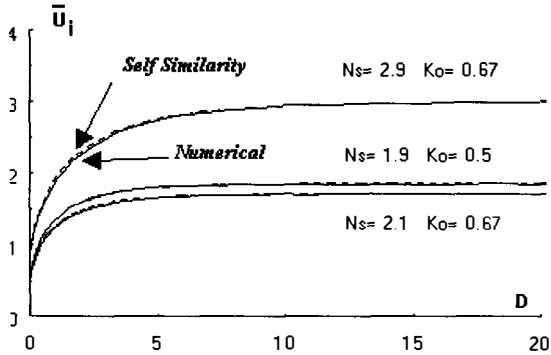


Figure 11: Self Similarity for free tunnel in anisotropic geostatic stresses

## 7 Conclusion

For plain strain approaches, it has been proved, on a reference three dimensional geometry, that two universal Principles take into account satisfactorily the three dimensional influence of the front face:

- The Self Similarity Principle describes the free convergence behind the face
- The Interaction Principle describes the relationship between convergence of two particular points of the supported tunnel, one on the leading edge of the support, and the other far behind the face.

In the general case, when geostatic stress is anisotropic, these Principles have to be associated with the "mean free convergence" curve to determine accurately the support activation instant.

It is proposed to extend this "mean convergence" concept to analyses of more general tunnel geometries, rock mass structure and rock mass anisotropy, provided the "steady state" conditions prevail for the tunnel under study. This results in a new plane strain approach, which appears as a self-contained CV-CF Method, for treating three dimensional deep tunnel problems, by using practically the same classical software procedures with a minimum added intricacy.

Naturally, if the problem is too far from the reference problem, it may be necessary to treat it by a direct approach, namely, using the steady state method which is well adapted. It may then be expected that the methodology presented herein be extended, provided some modification to the specific cases encountered.

## Acknowledgements

The authors are grateful to Dr Habibou Maitournam for his help in steady state calculations

## Nomenclature

- $D, D_0$  distance to the face of a unit radius tunnel, and distance to tunnel face for support laying.
- $P$  mean geostatic stress (isotropic geostatic stress)
- $P_i$  fictitious pressure at the tunnel wall
- $\lambda$  deconfining ratio
- $U_i$  radial displacement at the tunnel wall
- $U_0, U_0^\ell$  convergence when laying the support and convergence of the free tunnel at distance  $D_0$
- $U_\infty, U_\infty^\ell$  stabilised convergence of supported tunnel and stabilised convergence of free tunnel
- $\sigma_v, \sigma_h = K_0 \sigma_v$  vertical and horizontal principal geostatic stresses (anisotropic geostatic stress state)
- $K_0$  ratio of vertical horizontal principal initial stress
- $P = \frac{\sigma_v + \sigma_h}{2}$  mean geostatic stress
- $E, \nu$  elastic parameters of the rock mass
- $K_n, K_b$  normal and bending support stiffness
- $K_n^* = K_n / K_m$  normal relative support stiffness where  $K_m = E / (1 + \nu)$  is rock mass stiffness

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