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Stability analysis of a jointed rock excavation through a homogenization method

Analyse de stabilité d'une excavation creusée dans un massif rocheux fracturé par une méthode d'homogénéisation

Standsicherheit einer geklüfteten Böschung durch eine Homogenisierungsmethode

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ABSTRACT: The stability of an excavation cut in a rock mass comprising a regular network of joints is analysed within the framework of a homogenization method. This method relies upon an explicit formulation of a macroscopic strength condition for the jointed rock mass considered as a homogeneous anisotropic material. Making use of two different kinds of failure mechanisms on the homogenized structure, upper bound estimates are then derived for the stability factor, which prove better than those obtained from a direct approach in which the intact rock material and the joints are considered separately.

RESUME: On s'intéresse à la stabilité d'un talus excavé dans un massif rocheux traversé par un réseau régulier de fractures, analysée par une méthode d'homogénéisation. Celle-ci repose sur une formulation explicite du critère de résistance macroscopique de la roche fracturée considérée comme un milieu homogène anisotrope. Mettant en œuvre deux différents types de mécanismes de ruine de l'ouvrage homogénéisé, on obtient alors des bornes supérieures du facteur de stabilité, qui se révèlent meilleures que celles provenant d'un calcul direct dans lequel la roche intacte et les joints de fracture sont considérés séparément.

ZUSAMMENFASSUNG: Man interessiert sich an die Standsicherheit einer Böschung in einem Felskörper, der von einem regelmässigen Kluftnetz durchgequert ist. Diese Arbeit stützt sich auf die Homogenisierungstheorie, die von der Bestimmung der anisotropischen und homogenen makroskopischen Grenzbedingung des geklüfteten Felsmaterials ausgeht. Zwei verschiedenen Bruchmechanismen der Böschung werden berücksichtigt. Sie führen zu Berechnungen des Standsicherheitsfaktors, die sich besser erweisen als diejenigen, die man aus einer unmittelbaren Methode erhaltet, in der das felsmaterial und die Klüften getrennt betrachtet werden.

1 INTRODUCTION

Taking into account the mechanical behaviour of joints in jointed rock masses has always been of paramount importance to civil or mining engineers involved in the failure design of structures built in such materials. Most usual approaches either refer to the well known "block theory" (Goodman and Shi (1985) ; Goodman (1995)) which attempts to identify the possible failure patterns along the joints through geometrical and kinematical con-

siderations (concept of key-block), or to the "distinct element method" (Cundall (1988) ; Hart *et al.* (1988)) which regards the jointed rock mass as an assemblage of rigid or deformable blocks of intact rock in mutual interaction through discontinuities. However, for a dense network of joints, the numerical treatment generated by the application of such methods is getting rapidly untractable as the number of block elements increases, so that a homogenization approach seems far more advisable for dealing with such a situation.

The aim of the present contribution is to put the implementation of such an approach into practice on the particular example of a jointed rock excavation whose stability analysis is carried out by means of the upper bound kinematic method of yield design. As a preliminary, it requires the determination of the global strength properties of the jointed rock mass regarded as a homogeneous medium, which has been recently performed by Bekaert and Maghous (1996). Results of this analysis are discussed and compared with those derived from a direct approach in which joint surfaces and blocks of intact rock are treated separately.

2 AN ILLUSTRATIVE EXAMPLE

The stability of an excavation cut in a jointed rock mass such as that shown in Fig. 1 is investigated. The constituent rock mass is intersected by two orthogonal arrays of joints normal to the plane of the figure. Handling this problem as a two dimensional (plane strain) problem in the Oxy-frame, it turns out that the stability of such a structure is assessed through the following relationship:

$$h^+ = \frac{c_r}{\gamma_r} K^+(\beta, \frac{c_j}{c_r}, \varphi_j, \varphi_r, \theta) \quad (1)$$

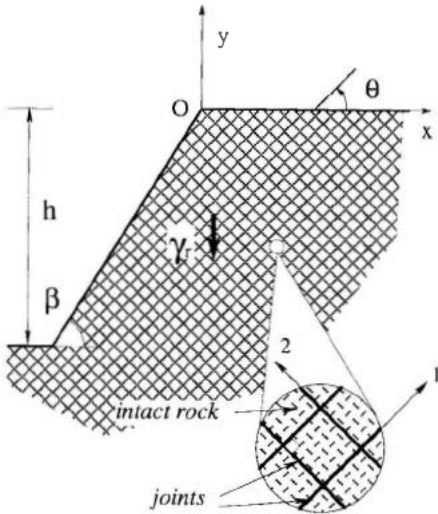


Fig. 1. Stability analysis of a jointed rock mass excavation.

where K^+ is the critical height beyond which failure of the excavation will occur, β the facing angle, γ_r the specific weight of the rock mass, c_r and φ_r (resp. c_j and φ_j) the cohesion and friction angle of the intact rock matrix (resp. of the joints, modelled as interfaces), and θ the inclination of the joints with respect to the horizontal plane. Therefore, analysing the stability of this structure amounts to evaluating the non dimensional factor K^+ .

Assuming that the rock mass is densely jointed, which means for instance that the spacing between two successive joints is small enough when compared with the excavation height h , it seems quite natural, from an engineering standpoint, to treat the composite jointed rock mass as a *homogeneous but anisotropic* continuum whose strength properties will now be specified.

3 MACROSCOPIC STRENGTH CONDITION FOR JOINTED ROCK MASS

A general formulation of the macroscopic strength criterion for a homogeneous rock material cut by several sets of densely and regularly distributed joints has been recently given by Bekaert and Maghous (1996) by applying the yield design homogenization theory, already used in the context of reinforced soil mechanics (de Buhan *et al.* (1989) ; de Buhan and Salençon (1990)). This criterion is now made explicit in the particular case of two orthogonal families of joints.

Denoting by $\sigma_{\alpha\beta}$ ($\alpha, \beta = 1, 2$) the stress components in a frame whose axes are taken parallel to the joint directions, the macroscopic failure condition writes:

$$\begin{cases} \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2} - (\sigma_{11} + \sigma_{22}) \sin \varphi_r \\ \leq 2c_r \cos \varphi_r \end{cases} \quad (2a)$$

$$|\sigma_{12}| \leq c_j + \min\{\sigma_{11}, \sigma_{22}\} \tan \varphi_j \quad (2b)$$

The first condition represents the Mohr-Coulomb strength criterion for the intact rock expressed in the context of "plane strain" conditions, while the second inequation corresponds to the joints failure condition, since σ_{12} and σ_{22} (resp. σ_{11}) are simply the shear and normal stresses acting upon the joint interface placed along direction 1 (resp. 2). Introducing the following change of variables:

$$p = \frac{\sigma_{11} + \sigma_{22}}{\sqrt{2}}, \quad s = \frac{\sigma_{22} - \sigma_{11}}{\sqrt{2}}, \quad t = \sqrt{2}\sigma_{12} \quad (3)$$

the above conditions may be expressed in the form:

$$\begin{cases} \sqrt{s^2 + t^2} \leq (p + \sqrt{2} H_r) \sin \varphi_r & (4a) \\ |t| \leq (\sqrt{2} H_j + p \pm s) \tan \varphi_j & (4b) \end{cases}$$

where $H_r = c_r / \tan \varphi_r$ and $H_j = c_j / \tan \varphi_j$.

Referring to the three-dimensional space of coordinates (p, s, t) , Ineq. (4a) defines a set of points bounded by a circular cone whose axis coincides with the line of equation $s = t = 0$ (line of isotropic stresses), while its apex is located at point $(p = -\sqrt{2} H_r, s = t = 0)$ (Fig. 2). Its aperture angle ψ_r is such that $\tan \psi_r = \sin \varphi_r$, hence $0 \leq \psi_r \leq \pi/4$.

Similarly, Ineqs. (4b) define a polyhedral domain, the boundary surface of which is made of four planes intersecting the p -axis at point $p = -\sqrt{2} H_j$. Since the strength properties of the joints are generally considerably smaller than those of the intact rock matrix, the latter point lies within the above mentioned cone ($H_j < H_r$), so that the representative domain of the macroscopic failure condition (4) is constructed as the intersection of the cone by the polyhedral domain, as shown in Fig. 2, where it is represented in the quarter of space ($s \leq 0, t \geq 0$). Note that it is symmetric with respect to planes of equations $s = 0$ and $t = 0$.

A Mohr-plane-type representation of the macroscopic criterion may also be derived. Let \underline{n} be

the inner unit normal to any elementary oriented facet of homogenized material, inclined at angle α with respect to either of the joints directions ($0 \leq \alpha \leq \pi/4$), and (σ_n, τ_n) the normal and shear stress components acting upon this facet (compressive stresses are counted positive). A suitable geometrical representation of the macroscopic strength condition consists in drawing for any value of α , the domain of admissible stresses (σ_n, τ_n) , as it has been done in Fig. 3 in the case when $\tan \varphi_j \leq \sin \varphi_r$ for ($\varphi_r \leq \alpha \leq \pi/4$).

As is quite apparent from this figure, the domain of admissible stresses is drawn by truncating the usual strength domain of the intact rock material (bounded by the dotted line) by two orthogonal segments intersecting each other at point $(\sigma_n = -c_j / \tan \varphi_j, \tau_n = 0)$. The overall reduction in strength due to the presence of joints is thus clearly revealed from such a representation, as well as the anisotropy of the homogenized material since the strength domain is obviously dependent on the facet orientation.

As regards the latter aspect, it should be emphasized that the classical notion of "intrinsic" curve is no more relevant for formulating such a strength criterion. Even the concept of "anisotropic" cohesion and friction angle tentatively proposed by Jaeger (1960) or Mc Lamore and Gray (1967) remains inappropriate.

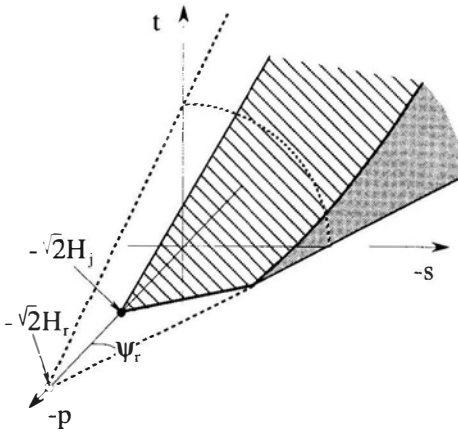


Fig. 2. View of the macroscopic strength domain in the (p, s, t) space.

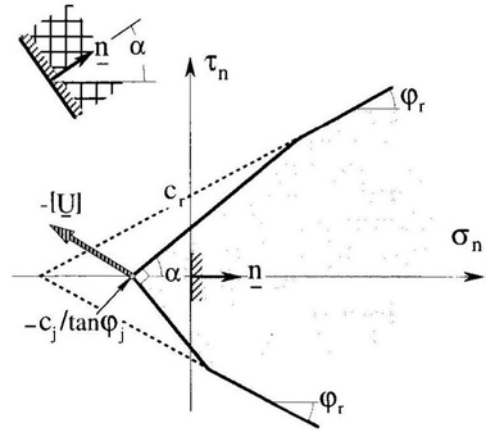


Fig. 3. Representation of the macroscopic strength condition for jointed rock mass in the (σ_n, τ_n) plane.

The explicit formulation of the macroscopic failure condition for the jointed rock mass makes it then possible to perform the stability analysis of the above excavation, notably by means of the upper bound kinematic approach of yield design (Salençon (1990, 1992) ; Chen and Liu (1990)). It is based on the statement that for the structure to remain safe, it is necessary that, given any (virtual) kinematically admissible velocity field \underline{U} ("failure mechanism"), the following inequation must be satisfied:

$$W_e(\gamma_r, \underline{U}) \leq W_{mr}(\underline{U}) \quad (5)$$

where $W_e(\gamma_r, \underline{U})$ is the work done by the external forces (reduced here to the weight of the rock material) in such a mechanism, that is:

$$W_e(\gamma_r, \underline{U}) = - \int_{\Omega} \gamma_r U_y dx dy \quad (6)$$

(U_y denotes the Oy-component of \underline{U}) and $W_{mr}(\underline{U})$ is the maximum resisting work expanded in the same mechanism. Since no deformation will be involved in the mechanisms to be considered in the sequel, its expression reduces to:

$$W_{mr}(\underline{U}) = \int_{\Sigma} \Pi^{hom}(\underline{n}, [\underline{U}]) d\Sigma \quad (7)$$

where $[\underline{U}]$ denotes the velocity jump across a discontinuity surface (line) Σ when following its unit normal \underline{n} and function Π^{hom} is calculated as the maximum value of the scalar product of the stress vector (σ_n, τ_n) acting upon the facet of inner normal \underline{n} by $-\underline{U}$, where (σ_n, τ_n) is subject to remain within the corresponding strength domain:

$$\Pi^{hom}(\underline{n}, [\underline{U}]) = \max \{ -(\sigma_n \underline{n} + \tau_n \underline{t}) [\underline{U}] ; (\sigma_n, \tau_n) \in \mathcal{G}^{hom}(\underline{n}) \} \quad (8)$$

where $\mathcal{G}^{hom}(\underline{n})$ is the domain of allowable stresses in the (σ_n, τ_n) plane represented in Fig. 3, and $(\underline{n}, \underline{t}) = \pi/2$.

As a consequence, this maximum value is obtained at points (σ_n, τ_n) of the boundary line of $\mathcal{G}^{hom}(\underline{n})$ such that $-\underline{U}$ is an outer normal to the strength domain at those same points. In the particular situation of Fig. 3, such a calculation is

possible as far as $[\underline{U}]$ is inclined to the jump line Σ by an angle at least equal to φ_r .

5 PRACTICAL IMPLEMENTATION

The upper bound kinematic approach has been performed by making use of two kinds of failure mechanisms depicted in Fig. 4:

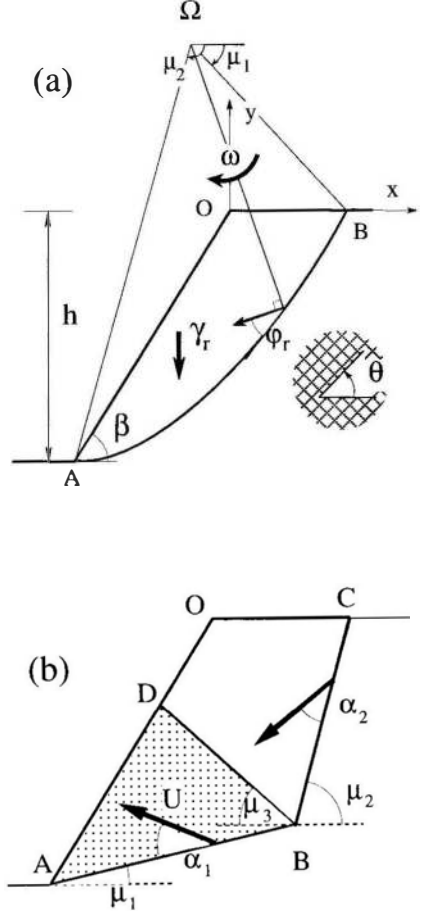


Fig. 4. (a) Rotational failure mechanism used in the upper bound kinematic approach.

(b) Piecewise rigid-block failure mechanism.

- A classical rotational failure mechanism (Fig. 4a) in which a volume of jointed rock rotates about point Ω with an angular velocity ω , the velocity

jump line AB being an arc of logspiral of angle φ_r and focus Ω . It thus comes out:

$$W_e(\gamma_r, \underline{U}) = \omega \gamma_r h^3 \tilde{w}_e(\beta; \mu_1, \mu_2) \quad (9a)$$

and

$$W_{mr}(\underline{U}) = \omega c_r h^2 \tilde{w}_{mr}\left(\frac{c_j}{c_r}, \varphi_j, \varphi_r, \theta, \beta; \mu_1, \mu_2\right) \quad (9b)$$

where \tilde{w}_e and \tilde{w}_{mr} are non dimensional functions of the above non dimensional arguments, and (μ_1, μ_2) are angles specifying the position of the center of rotation Ω . Introducing these expressions into Ineq. (5) for $h = h^+$ and taking Eq. (1) into account leads to :

$$K^+ \leq \frac{\tilde{w}_{mr}}{\tilde{w}_e}\left(\frac{c_j}{c_r}, \varphi_j, \varphi_r, \theta, \beta; \mu_1, \mu_2\right) \quad (10)$$

since both members can be divided by ω taken clockwise positive, that is finally

$$K^+ \leq K_1^u = \min_{\mu_1, \mu_2} \left\{ \frac{\tilde{w}_{mr}}{\tilde{w}_e} \right\} \quad (11)$$

where K_1^u , which is computed by a numerical minimization procedure, represents the best *upper bound estimate* for K^+ which can be expected from exploring such rotational failure mechanisms.

- A second class of *piecewise rigid-block*^(*) *failure mechanisms* has been also considered (Fig. 4b), where a triangular wedge topped by a quadrilateral volume are given rigid body translations separately. Once the velocity magnitude U of the lower block has been fixed, such a mechanism is entirely determined through three angles μ_i ($i = 1, 2, 3$) which specify the orientations of the different jump lines, along with two more angles α_j ($j = 1, 2$) giving the inclination of the velocities. The corresponding expressions of W_e and W_{mr} take the following forms

$$W_e(\gamma_r, \underline{U}) = U \gamma_r h^2 \hat{w}_e(\beta; \mu_i, \alpha_j) \quad (12a)$$

$$W_{mr}(\underline{U}) = U c_r h \hat{w}_{mr}\left(\frac{c_j}{c_r}, \varphi_j, \varphi_r, \theta, \beta; \mu_i, \alpha_j\right) \quad (12b)$$

hence from (5) with $h = h^+$, taking Eq. (1) into account:

$$K^+ \leq K_2^u = \min_{\mu_i, \alpha_j} \left\{ \frac{\hat{w}_{mr}}{\hat{w}_e} \right\} \quad (13)$$

the minimization being also achieved numerically.

(*) In order to avoid any misunderstanding, it should be mentioned that the term "block" does not refer here to pieces of intact rock in the initial structure, but merely means that a volume of homogenized jointed rock mass is given a (virtual) rigid body kinematics.

6 RESULTS AND COMPARISON WITH A DIRECT APPROACH

For the sake of convenience, a stability reduction factor defined as the ratio between $K^u = \min \{K_1^u, K_2^u\}$, that is the optimal upper bound estimate of K^+ derived from the exploration of both kinds of failure mechanisms, and the corresponding value K_0^u calculated for a non fractured rock excavation, is introduced. The variations of this factor with the joint orientation θ have been plotted in Figs. 5 for the following data:

$$c_j/c_r = 0.5, \varphi_j = 20^\circ, \varphi_r = 30^\circ \text{ and } \beta = 75^\circ, 90^\circ$$

It is quite apparent from these figures that the joint orientation has a decisive influence on the stability of the structure. While this stability is little affected by joints placed along the horizontal and vertical directions ($\theta = 0^\circ$ or 90°), the stability factor is reduced by almost 60% for joints inclined at $\beta/2 + \varphi_j/2$ (approximate values for which the solid curves reach their minimum). It should be mentioned that the best (that is the lower) prediction is obtained from the piecewise rigid-block failure mechanism of Fig. 4b, except in the case of a vertical cut ($\beta = 90^\circ$) for either small or large values of θ , where it turns out that the rotational failure mechanism is prevailing.

Those results are worth being compared with a direct approach where intact rock matrix and joint interfaces are treated as geometrically distinct constituent materials. Making use of a triangular wedge failure mechanism in which the velocity jump line perfectly coincides with the joint of orientation θ passing through the toe of the excavation (Fig. 6), and whose velocity is taken inclined at angle φ_j to this line, the following analytical expressions are easily obtained:

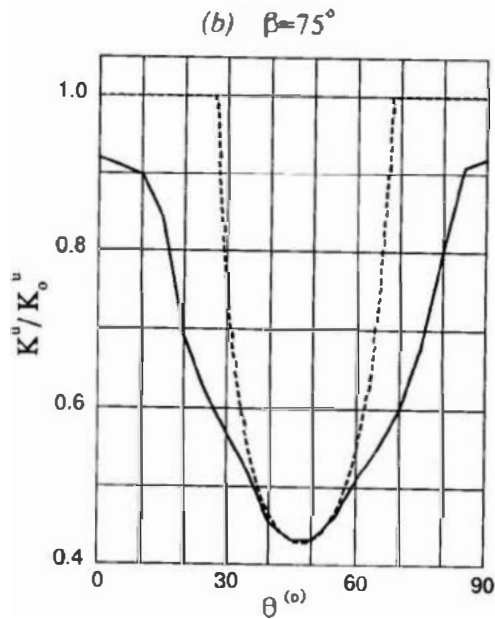
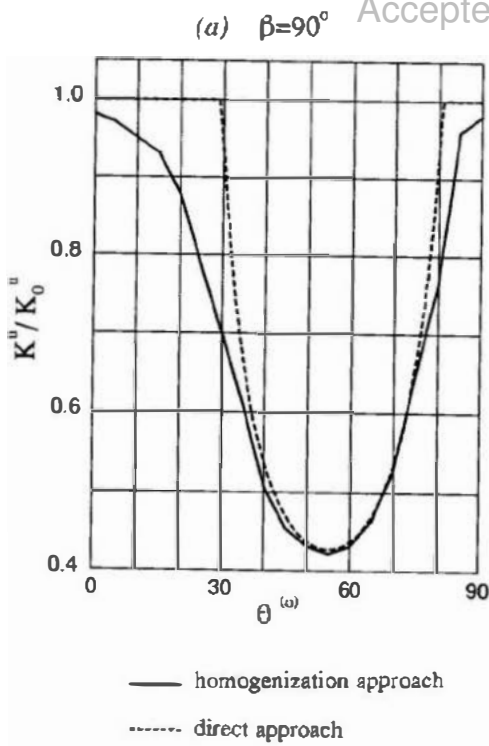


Fig. 5. Stability reduction factor vs. joints inclination. (a) $\beta = 90^\circ$ (vertical cut) (b) $\beta = 75^\circ$.

$$W_c(\gamma_r, \underline{U}) = \frac{1}{2} \gamma_r U h^2 \sin(\theta - \varphi_j) (\cot \theta - \cot \beta)$$

$$W_{mr}(\underline{U}) = c_j \cos \varphi_j U h / \sin \theta$$

It is worth noting that the calculation of the maximum resisting work has been derived from (7) with $\Sigma=AB$, where function $\Pi^{hom}(\cdot)$ has been replaced by that relative to the joint strength condition:

$$\Pi(\underline{n}, [\underline{U}]) = \max \{ -(\sigma_n \underline{n} + \tau_n \underline{t}) [\underline{U}] ; |\tau_n| \leq c_j + \sigma_n \tan \varphi_j \}$$

or in the present case:

$$\Pi(\underline{n}, [\underline{U}]) = c_j \cos \varphi_j U$$

It follows from (5) through (1) that:

$$K^+ \leq K_{dir}^u = 2 \frac{c_j}{c_r} \frac{\cos \varphi_j \sin \beta}{\sin(\theta - \varphi_j) \sin(\beta - \theta)} \quad (14)$$

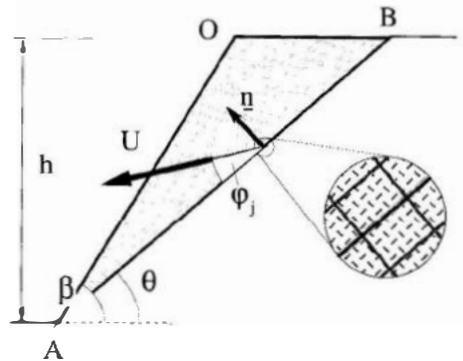


Fig. 6. Direct implementation of the kinematic approach on the initial structure by means of a triangular wedge mechanism.

The corresponding estimates of the stability reduction factor, given by K_{dir}^u/K_0^u , are represented by the dashed curves of Fig. 5 which show the same trend as the solid curves, that is a sharp decrease of the stability factor when θ is approaching its critical value $\beta/2 + \varphi_j/2$, which realizes

the minimum of K_{dir}^u . When K_{dir}^u is greater than K_0^u (that is the reduction factor exceeds the unit value), the triangular wedge failure mechanism becomes irrelevant. It should therefore be replaced by a rotational failure mechanism which produces no better estimate than K_0^u , since the velocity discontinuity line does not involve any joint. Hence the horizontal dashed lines.

For θ ranging from 40° to 75° the estimates obtained from the direct and homogenization approaches are very close in the case of a vertical excavation, this interval being reduced to $35^\circ - 55^\circ$ for a slightly inclined facing ($\beta = 75^\circ$). On the other hand, outside of these intervals, the homogenization approach quite significantly improves the predictions of the direct analysis. By way of example, it can be seen in Fig. 5b that, for a joint inclination equal to 67° and $\beta = 75^\circ$, the relative improvement amounts to more than 40%.

7 CONCLUDING REMARKS

The feasibility of a homogenization technique for designing jointed rock structures, based on the preliminary formulation of a macroscopic failure condition of the jointed rock mass, has been demonstrated on the example of an excavation, taken for illustrative purpose. Its main advantage when compared with a direct approach, lies in the fact that the use of geometrically complex mechanisms, aimed at capturing the weakness zones formed by the joints, in order to produce reliable estimates, is no more required. However, one must remain cautious when applying the results of this method, since it is to be kept in mind that its validity is subject to the condition that the network of joints is sufficiently dense. Should the latter condition be not satisfied, like in the case of a rock mass structure comprising relatively few joints, resorting to a direct analysis seems quite preferable. It thus appears that the direct and homogenization approaches, should be regarded as complementary rather than opposite design methods.

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