



**HAL**  
open science

## Joint back-analysis in mining subsidence

Andrei Constantinescu, Duc Nguyen-Minh, Desheng Deng

► **To cite this version:**

Andrei Constantinescu, Duc Nguyen-Minh, Desheng Deng. Joint back-analysis in mining subsidence. Third International Workshop on Applications of Computational Mechanics in Geotechnical Engineering, Sep 1998, Porto, Portugal. pp.177-183, 10.1201/9781003078562-22 . hal-00116189

**HAL Id: hal-00116189**

**<https://hal.science/hal-00116189>**

Submitted on 21 May 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Joint back-analysis in mining subsidence

A.Constantinescu, D. Nguyen Minh & D.Deng

*Laboratoire de Mécanique des Solides (CNRS URA 317, Polytechnique, Mines, Ponts et Chaussées) Ecole Polytechnique, 91128 Palaiseau Cedex, France*

## ABSTRACT

Subsidence due to underground mining excavation is a typical limited data problem. For a preliminary identification problem, elastic rockmass and joint behavior is supposed, and a method based on minimization error of constitutive law has been used. In this subsidence problem, joints behaviour appear to have a great importance. This inverse method is proved to be efficient and with a rapid convergence. Finally, selection and position of measuring points are analysed.

## 1 INTRODUCTION

One of the damaging effects of underground mining is subsidence. In the case of underground mining the intensity of the subsidence can reach the order of meters and a surface extension of some square kilometers and is thus larger than the effects induced by other underground projects like tunnels or deep caverns. Subsidence affects generally the safety of surface structures but has also an impact on environmental issues like surface hydrogeology. Its prediction and control are therefore an important task of mining engineering.

A typical problem of long time subsidence related to mining, can be encountered in the region of Nord Pas de Calais, between France and Belgium. A surface of  $150 \times 15$  km covers an old web of coal mines in a region with more than 1.5 million inhabitants and should be continuously monitored and managed. In order to predict the behaviour of this zone a series of informations should be gathered and processed based on using existing models. The difficulty lies on one hand on the complexity of the models relating geology, hydrology and mechanics and on the other hand on the limited amount of measured data. The purpose of this work is to show that a series of mechanical informations on the geology of a typical subsidence site can be recovered using a back-analysis technique.

In a previous geomechanical study (Constantinescu & Nguyen 1997) of a site with typical geological stratification it has been shown that the zone affected by plasticity is confined near the old mine gallery, which conducts to stress concentrations similar with those found near cracks. The subsidence in this case

is mainly driven by the elastic part of the material parameters of the layers and the joints in between. This has permitted to apply an elastic inversion technique (Constantinescu 1995) supposing known properties of the joints and to show that the elastic moduli of the layers can be recovered by back analysis from the known subsidence displacements at the surface. As a large part of the subsidence is directly related to the elastic deformation of the joints it is interesting to apply the back-analysis technique to the joints.

In this paper we shall present a complete analysis of the system involving the joints. The geological system of the layers, presented in figure 1, is a simplified model of a typical site morphology in the Nord Pas de Calais region. In a first part, a direct mechanical analysis of the system is performed using a finite element model. This analysis shows the influence of the behaviour of joints on the subsidence phenomena. In a second part a back analysis is performed in order to recover the material parameters of the joints. As a preliminary results we shall only present the case in which the rockmass and the joints behave elastically.

Previous back-analysis techniques used in geotechnics for the identification of the material parameters such as (Cividini et al. 1981; Gioda & Sakurai 1987; Sakurai et al. 1994) have been based on direct minimisation technique of a cost functional representing the difference between measured and simulated quantities. One of the drawbacks of the employed minimisations was the gradient computation obtained generally through finite difference and which drew to a computational burden.

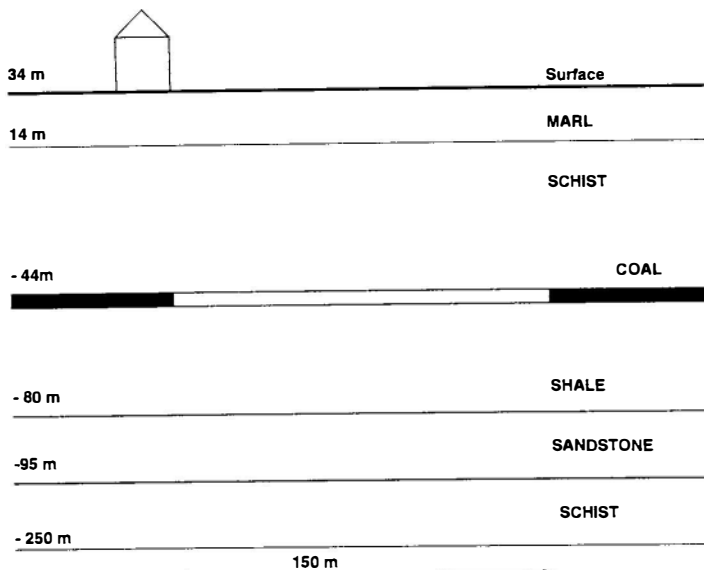


Figure 1: The reference problem of the study

The back-analysis technique used here is based on the minimisation of the error on constitutive law (Bui 1993; Constantinescu 1995). The error on constitutive law is a cost functional, the spatial integration of a distributed energy measuring the misfit in the constitutive law. The previous forms of the functional have been extended in order to include the joints. Using this error functional presents essentially two advantages: the direct spatial localization of the error and a simple gradient computation conducting naturally to an alternating direction minimization algorithm. In this framework an iteration of the descending algorithm is constituted by 2 direct computations and a simple algebraic update of the elastic moduli.

## 2 THE GEOLOGICAL CONTEXT AND THE NUMERICAL MODEL

The plane strain model shown on figure 1. is a simplified cross-section of a mine from a site near from Valenciennes. Alternation of the lithostatic layers have been chosen according to geological information provided by the geological laboratory. The rockmass was assumed to behave as a Coulomb elastoplastic material, with "reasonable" parameters. A series of material parameters are presented in figure 2. For the behaviour of the joints no data was available and as such two extreme interface hypothesis have been proposed in order to estimate the subsidence: perfectly bonded and perfectly smooth interfaces.

### 2.1 Perfectly bonded interfaces

In the case of perfectly bonded interfaces, it can be verified by numerical computations that the subsidence of the model remains practically elastic. This could be explained by the fact that compressive plastic zones are limited at the two front faces and therefore we will see that the propagation of the closure of the mine roof is practically transmitted up to the surface subsidence. On the roof and on the bottom of the mine gallery, tensile stresses prevail, but remain in a limited zone. This is in accordance with experience and site inspection where partial fracturing of the roof and the bottom have been reported. It is important to remark, that halving the compressive yield limit did not induce changes in the subsidence displacement (see figure 3.).

As a consequence of the plastic confinement and the negligible influence of plastic properties of the global subsidence we will use an elastic approach when the joints are perfectly bonded.

### 2.2 Perfectly smooth interfaces

The analysis of the perfectly smooth interface shows the importance of the material behavior of the joints. The surface settlement, which was 18 cm in the case of fully bonded interfaces, increases up to 30 cm when one joint above the opening is transformed into a smooth interface. An example for the joint corresponding to the schist/marl interface is depicted in figure 4. If both the schist/marl and the coal seam / schist interface are smooth, the surface displacement attains 45 cm (see also figure 4.). This is mainly due

	$\rho$ [kg/m <sup>3</sup> ]	$E$ [GPa]	$\nu$ [adim]	$C$ [MPa]	$\varphi$ [deg]
chalk	2000.	2.0	0.2	1.3	30.
coal		1.2	0.1	0.5	45.
sandstone		40.	0.15	22.	30.
tourtia		4.	0.2	0.001	28.
marl		6.	0.25	5.	3.
schist		10.	0.25	4.8	58.

Figure 2: Geotechnical characteristics of the materials

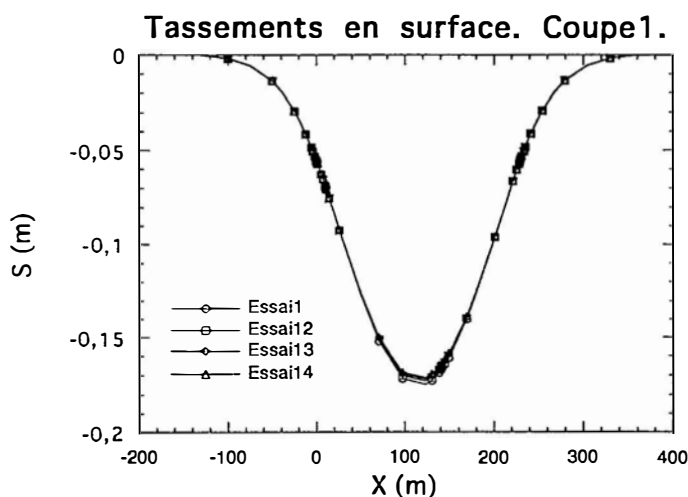


Figure 3: Vertical surface displacements from FEM computations with elastic and elastoplastic constitutive models

to bending of the overlying strata when they are liberated by joint release, inducing large development of the yielding tensile zones.

Therefore tensile strength appears to be a leading parameter of this problem in opposition to the compressive strength which did not influence the behaviour.

In the identification problem presented next, only one smooth joint (schist/marl) is considered (see figure 5).

### 3 THE IDENTIFICATION OF THE ELASTIC PARAMETERS OF A JOINT

The identification problem which will be addressed next, will be defined within an elastic rockmass and joint behavior hypothesis. The problem can be stated as follows:

*Can one find out elastic parameters of the geological joint from the knowledge of subsidence measurements at the surface ?*

It is important to remark that the subsidence measurement is surface displacement and supposes implicitly a free surface. Thus we actually dispose of two simultaneously known boundary conditions on the same boundary. It is the overspecified boundary data that permits the existence of the solution of this inverse problem as it has already been discussed in a series of mathematical papers on similar problems.

More precisely, the answer of the elastic identification question is affirmative in the case of isotropic elasticity and in some cases of anisotropic elasticity, provided *all* displacement-force boundary data are given. Ikehata (Ikehata 1990) and Nakamura & Uhlmann (Nakamura & Uhlmann 1994). A numerical reconstruction example of a continuous distribution of elastic moduli from a *finite* number of boundary measurements is described in (Constantinescu 1995).

Regarding the previous treated problem it seems that large amount of data are generally needed to recover the unknown coefficients. However, one can remark that the previous results addressed the identifica-

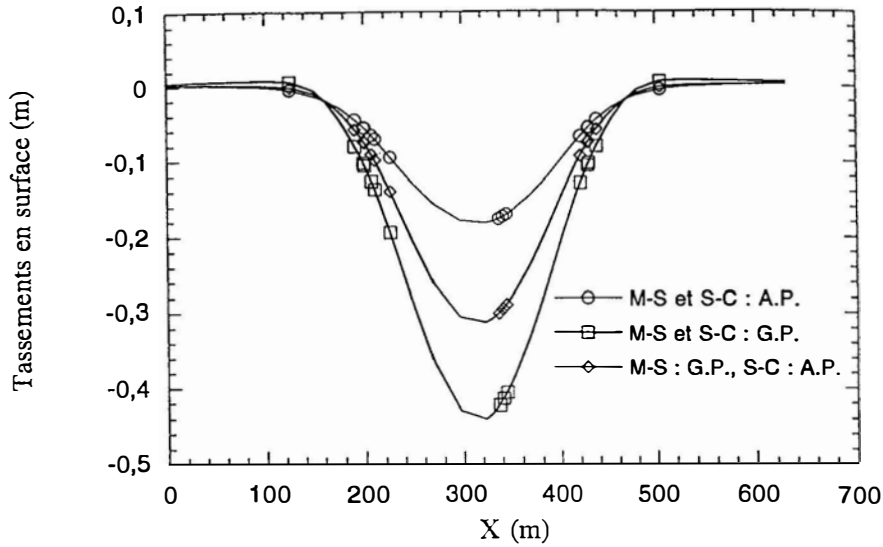


Figure 4: Vertical surface displacements computed by the FEM for perfectly bonded (AP) and perfectly smooth (GP) interfaces between different geological layers

tion of distributions of elastic moduli. As a large number of measurements are not available from a practical point of view in the subsidence problem we shall balance these deficiencies stating the following assumptions:

1. linear elasticity,
2. constant elastic parameters in the joint,

### 3.1 The identification method

In the following it will be assumed that the behaviour of the rock body is governed by the following set of equations:

$$\begin{aligned} \epsilon &= \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u}) \\ \sigma &= \mathbf{C}\epsilon \end{aligned} \quad (1)$$

$$\text{div} \sigma = \rho g \mathbf{e}_z$$

Where  $\mathbf{u}$ ,  $\epsilon$ ,  $\sigma$  denote respectively the displacement vector field, the strain and the stress tensor field.  $\mathbf{C}$  is the fourth order tensor of the elastic moduli of the rockmass.

The elastic joint  $\Gamma$  is governed (Goodman 1976) by

$$\sigma \cdot \mathbf{n} = \mathbf{K}[\mathbf{u}] \quad (2)$$

where  $\mathbf{n}$  is the unit normal vector field on the joint and  $[\mathbf{u}]$  the jump of the displacement field on the joint.  $\mathbf{K}$  is the tensor of elastic moduli of the joint, which will be supposed diagonal:

$$\mathbf{K} = \begin{pmatrix} k_n & 0 \\ 0 & k_s \end{pmatrix}$$

with  $k_n$  and  $k_s$  the normal and tangential elastic parameters of the joint.

The following boundary conditions on the surface of the rockmass will be used:

$$\mathbf{u}|_{\text{Surface}} = \mathbf{u}^m \quad (3)$$

with  $\mathbf{u}^m$  representing the subsidence measurements; or:

$$\sigma \cdot \mathbf{n}|_{\text{Surface}} = \mathbf{0} \quad (4)$$

representing the free surface.

The other faces of the rockmass where supposed to be completely blocked and therefore imposed zero displacements have been used.

The identification of  $\mathbf{K}$  is based on the minimization of the error on the constitutive law which in this case can be written as:

$$\begin{aligned} I(\mathbf{K}, \mathbf{C}, \epsilon, \sigma) &= \frac{1}{2} \int_{\Omega} |\mathbf{C}^{-\frac{1}{2}} \sigma - \mathbf{C}^{\frac{1}{2}} \epsilon|^2 dx + \\ &\frac{1}{2} \int_{\Gamma} |\mathbf{K}^{-\frac{1}{2}}(\sigma \mathbf{n}) - \mathbf{K}^{\frac{1}{2}}[\mathbf{u}]|^2 dx \end{aligned}$$

over kinematically admissible strain fields, statically admissible stress fields and admissible values of  $\mathbf{K}$ , i.e.  $k_n$  and  $k_s$ . An *alternating direction minimization method* conducts to the following algorithm:

- initialize the elastic parameters of the joint.
- with given elastic parameters do the following computations:

1. Compute the strain field  $\epsilon$  from the *Dirichlet Problem* defined by:  
Equations (1), and the boundary conditions

$$\mathbf{u}|_{Surface} = \mathbf{u}^m \quad (5)$$

2. Compute the stress field  $\sigma$  from the *Neumann Problem* defined by:  
Equations (1), (2) and the boundary conditions

$$\sigma \cdot \mathbf{n}|_{Surface} = \mathbf{0} \quad (6)$$

3. with  $\sigma$  and  $\epsilon$  calculated before update the elastic parameters of the joint:

$$k_n^2 = \frac{\int_{\Gamma} \sigma_n^2 dx}{\int_{\Gamma} u_n^2 dx}$$

$$k_s^2 = \frac{\int_{\Gamma} \sigma_s^2 dx}{\int_{\Gamma} u_s^2 dx}$$

where  $n, s$  denote the normal and tangential components of the stress on the joint and the jumps of the displacements.

### 3.2 Numerical results in the subsidence problem

The numerical results for the problem presented before were obtained on a mesh having  $\approx 1400$  linear quadratic and joint elements using the FEM object oriented language CASTEM 2000 (CEA, France, see <http://www.castem.org:8001>). A direct elastic computation took a couple of seconds on a HP 720 workstation.

The joints have been modeled using special joint elements relating directly the degrees of freedom of the two layers through a rigidity matrix. Therefore, one can consider that the joints do not have a physical thickness in the FEM model. Regarding the rigidities of the joint, direct computations with different values have been considered and finally the values  $k_n = 1 \times 10^{18} N/m^3$  and  $k_s = 2.5 \times 10^7 N/m^3$  have been chosen for the direct simulation, used to provide "measurement data" for the inverse computation. The larger normal rigidity of the joint, implies a perfect transmission of the displacement and thus does not permit a vertical jump in the displacement field. The only displacement jump corresponds to sliding along the joint and is represented by  $k_s$  rigidity in the model. Moreover, during direct computations it has been observed that  $k_n$  does not influence the subsidence phenomena but that  $k_s$  is a leading parameter.

Due to the importance of  $k_s$  on the subsidence problem, only identifications of this parameter have

been done.

The numerical computations show that the tangential parameter  $k_s$  is generally found out within 5% of the actual value after 15 – 25 iterations (see figure 6.). The starting values in the inverse algorithm have been chosen in a wide range: from one hundred thousandth to a hundred thousand times the actual parameter value.

In order to estimate the impact of the measurements position on the final identified result, an analysis using two sets of measuring points has been done. The two series of measuring points, denoted as  $A1, A2, \dots, A9$  and  $B1, B2, \dots, B9$  are plotted in fig 5. The first series  $A1, A2, \dots, A9$  is situated on the surface just above the excavation. With the measurements from these points, the convergence to the real value of sliding rigidity  $k_s$  of the joint is almost as rapid as the inversion using measurements from all the points of the subsidence surface (see figure 7 for initial value  $k_s = 2.5 \times 10^9 N/m^3$ , the converged value of  $k_s$  is  $2.5 \times 10^7 N/m^3$ ). On the contrary, with the measurements of the second series  $B1, B2, \dots, B9$  situated on the surface far from the excavation, the iteration convergence is too slow to be practically used (figure 7).

From this analysis, it can be seen that the selection of measuring points and their positions are very important in the back-analysis problem. For practical purpose, it is not necessary to measure all the displacements on the surface of this underground mining excavation. Measuring the key points above the excavation is sufficient for the identification of joint parameters. This will greatly reduce the measuring work and allow to obtain an acceptable result.

## 4 CONCLUSION

Although a rather simplistic mining model has been used, the elastoplastic finite element analysis of the mine model has clearly shown that joints between different layers play an important role on the response of the system. Two limiting cases have been considered for joints :

- fully bonded joints, which leads to an elastic behaviour of the system, as plastic zones are concentrated at the front face of long wall excavations.
- perfectly smooth interfaces, which leads to large development of plastic tensile zones. Tensile strength is then the leading mechanical parameter, due to bending of the seams.

In both cases, compressive strength showed to be of secondary importance.

Assuming, for a preliminary inverse problem study, that the system behaves elastically, one has considered the problem of identification of mechanical parameters of the joint from surface displacement mea-

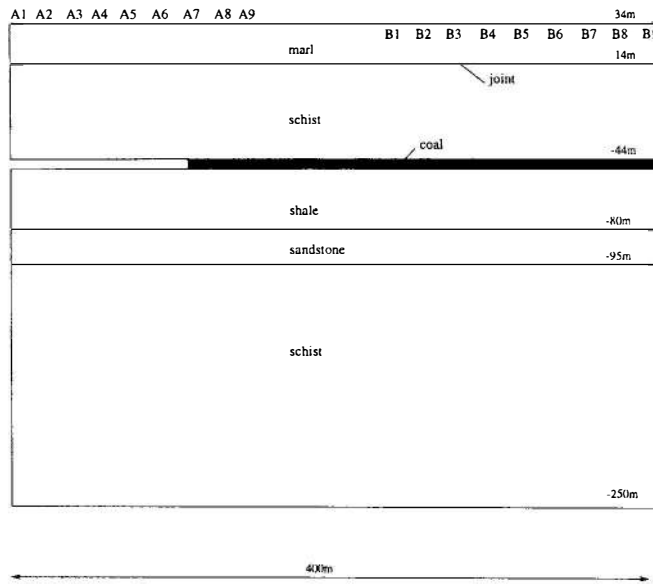


Figure 5: The contours of the different domains and layers in the inverse problem (half of the site is modelised)

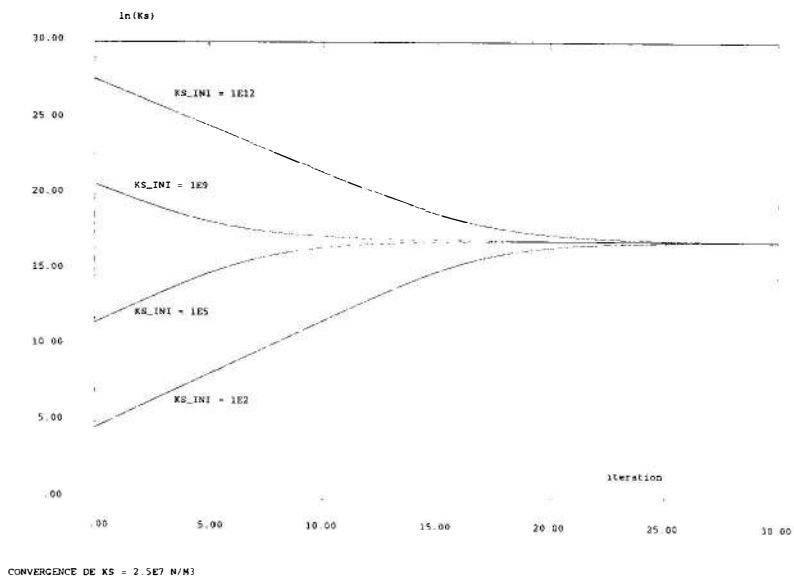


Figure 6: Convergence of the  $k_s$  for different initial values

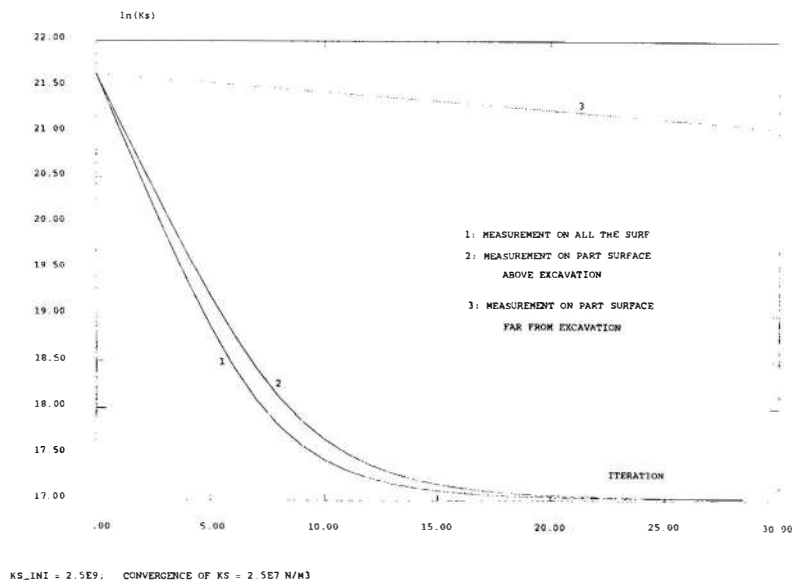


Figure 7: Convergence of the  $k_s$  for different measurements

surements. This method, based on the minimization of the error on constitutive law, has proved to give accurate results and rapid convergence. The selection of measuring points and their positions on the surface greatly affects the convergence of iteration in the identification of joint parameters. The key points which should be measured in this underground mining excavation are those just above the excavation. Extension of this method to non linear joint behaviour will be the next step of this work.

## REFERENCES

- Bui, H. D. (1993). *Introduction aux Problèmes Inverses en Mécanique des Matériaux*. Eyrolles, Paris.
- Cividini, A., Jurina, L., & Gioda, G. (1981). Some aspects of characterisation problems in geomechanics. *Int.J.Rock Mech. Min. Sci. & Geomechanical Abstr.* 18(7), 487–503.
- Constantinescu, A. (1995). On the identification of elastic moduli from displacement-force boundary measurements. *Inverse Problems in Engineering* 1, 293–315.
- Constantinescu, A. & Nguyen, M. D. (1997). An inverse problem approach in mining subsidence. In H. K. Lee, H. S. Yang, & S. K. Chung (Eds.), *1<sup>st</sup> Asian Rock Mechanics Symposium, Seoul, South Korea*, pp. 199–204. Balkema, Rotterdam.
- Gioda, G. & Sakurai, S. (1987). Back analysis procedures for the interpretation of field measurements in geomechanics. *Int.J. for Numerical and Analytical Methods in Geomechanics* 11, 555–583.
- Goodman, R. E. (1976). *Methodes of geological engineering discontinuous rocks*. West publishing company.
- Ikehata, M. (1990, dec.). Inversion formulas for the linearized problem for an inverse boundary value problem in elastic prospection. *SIAM J. Math. Anal.* 50, 1635–1644.
- Nakamura, G. & Uhlmann, G. (1994). Global uniqueness for an inverse boundary value problem arising in elasticity. *Invent.Math.* 118, 457–474.
- Sakurai, S., Akutagawa, S., & Tokudome, O. (1994). Characterisation of yield function and plastic potential function by back analysis. In Siriwardane & Zaman (Eds.), *Computer Methods and Advances in Geomechanics*. Balkema, Rotterdam.