



HAL
open science

Back analysis of loading parameters in shallow tunnels

Desheng Deng, Duc Nguyen Minh, Andrei Constantinescu

► **To cite this version:**

Desheng Deng, Duc Nguyen Minh, Andrei Constantinescu. Back analysis of loading parameters in shallow tunnels. Third International Workshop on Applications of Computational Mechanics in Geotechnical Engineering, 2001, Londres, United Kingdom. pp.177-183, 10.1201/9781003078562-21 . hal-00116188

HAL Id: hal-00116188

<https://hal.science/hal-00116188>

Submitted on 1 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Back analysis of loading parameters in shallow tunnels

Desheng Deng, Duc Nguyen Minh & Andrei Constantinescu

Laboratoire de Mécanique des Solides, Ecole Polytechnique, 91128 Palaiseau, France

ABSTRACT

Convergence Confinement (CV-CF) Method is currently used in geo-engineering practice as a plane strain approximate tunnel calculation, for example, in the scope of a NATM construction procedure. However, although improvements have been recently achieved for some simple cases, this method is impeded by undetermination of the deconfining ratio value λ_0 at the moment when support is laid. A back analysis method has been proposed on the basis of the Virtual Work Principle, in which the original CV-CF model is compared through an iterative process to a modified model in which the field measurements are prescribed. The parameter λ_0 and the lateral stress ratio k_0 of soil mass are back analysed. This method is very robust and works also for non linear rock mass behavior. For illustration of this back analysis method, 2D shallow tunnel calculation data are used as experimental data.

1 INTRODUCTION

Convergence Confinement Method (CV-CF Method) is currently used in engineering practice as a very convenient method for tunnel calculation. It is a plane strain approximation of the three dimensional tunnel problem, in which the tunnel face advance is produced by a fictitious stress vector distribution T applied over the wall of the tunnel, and varying proportionally to the initial stress as :

$$T = (1 - \lambda)\sigma^0 \mathbf{n} \quad (1)$$

Where \mathbf{n} is the normal vector to the tunnel wall, σ^0 the initial geostatic stress tensor in the rock mass. λ is called the deconfining ratio, it appears as a kinematic parameter which increases monotonically from 0 to 1 since the initial to the final stage of tunnel construction. This is a basic problem for a full face driven tunnel, with support placed on part of the wall of the tunnel at a given instant $t = t_0$, for which we have $\lambda = \lambda_0$.

In this problem, parameter λ_0 , has a determinant influence on the tunnel-support equilibrium, but it is an unknown in the framework of a plane strain approach. Although recent works, based on comparison with three dimensional calculations, have proposed semi-empirical methods to determine this parameter in some specific cases ((CORBETTA 1990), (BERNAUD & ROUSSET 1992), (NGUYEN & GUO 1993), ...), the problem remains in most

cases met in practice, and appears as one major drawback of the CV-CF Method (PANET 1995).

Another unknown is the initial geostatic stress state. For shallow horizontal tunnels, driven in sedimentary soils and rocks, under horizontal surfaces, the stress state determination comes down to the sole determination of lateral stress ratio k_0 , of the horizontal principal stress σ_h^0 to the vertical one $\sigma_v^0 = k_0 \sigma_v^0$.

For such tunnels, the parameters λ_0 and k_0 can be considered as undetermined loading parameters; the purpose of this study is to propose an inverse method to identify these loading parameters, from in situ observations during the construction phases of the tunnel. Existing back analyses methods used in geomechanics are generally direct methods (CIVIDINI et al. 1981), (SAKURAI 1993), (SOUSA et al. 1997) with a minimization of the discrepancy between measured and calculated displacements, and are generally concerned with elastic behavior. For non linear problems inverse methods are generally hard to handle. A method of minimization in constitutive law error has been proposed (CONSTANTINESCU 1994), but it is not possible for our problem to obtain explicit derivations of the objective function. The new method proposed herein to solve this problem, is based on use of the virtual work theorem applied to the CV-CF problem.

2 BACK ANALYSIS

2.1 Description of the problem

Let us call the CV-CF problem the original problem, defined as follows : the rockmass with tunnel without support is called domain Ω , with border Γ (Fig. 1). At time $t = 0$, the initial state in the rock mass is:

$$\begin{cases} \mathbf{u}(x, 0) = 0 \\ \boldsymbol{\sigma}(x, 0) = \boldsymbol{\sigma}_0(x) \end{cases} \quad x \in \Omega \quad (2)$$

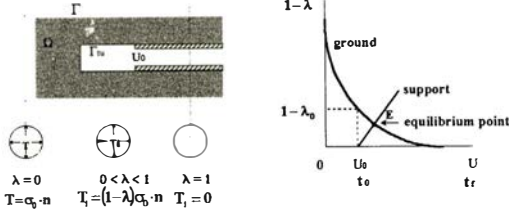


Figure 1: Tunnel advance - The Convergence Confinement Method

According to CV-CF method, tunnel excavation results in a monotonic prescribed loading on the tunnel wall according to the kinematic parameter λ (equation 1). With support placement, at time $t = t_0$ or $\lambda = \lambda_0$, the previous structure is modified with new support elements added, but the variation of fictitious forces as written here above still holds on the same border as excavation of the front face proceeds. In this problem, parameters λ_0 and k_0 can be considered as loading parameters for the final tunnel structure, and the identification will concern either one of them (the other one is supposed to be known), or simultaneously both of them.

The fictitious stress at the wall of the tunnel Γ_{tu} is given for three particular states (or instants) as follows:

$$T(x) = \begin{cases} \boldsymbol{\sigma}_0 \mathbf{n} & t = 0 \\ (1 - \lambda_0) \boldsymbol{\sigma}_0 \mathbf{n} & t = t_0 \\ 0 & t = t_f \end{cases} \quad (3)$$

t_f is the final state ($\lambda = 1$, $t_f = \infty$). Let us consider the time period $[0, t_0]$. The displacement field has to be kinematically admissible with limit condition on displacements on the border $\Gamma_u \subset \Gamma$:

$$\begin{cases} \dot{\boldsymbol{\epsilon}} = \frac{1}{2}(\nabla \dot{\mathbf{u}} + \nabla^T \dot{\mathbf{u}}) & x \in \Omega, \quad t \in [0, t_0] \\ \dot{\mathbf{u}}(x, t) = \dot{\mathbf{u}}^d(x, t) & x \in \Gamma_u, \quad t \in [0, t_0] \end{cases} \quad (4)$$

and the stress field, accounting for initial geostatic stresses (BOURGOIS 1998)

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 + \mathbf{C} : \boldsymbol{\epsilon}^e \quad (5)$$

has to be statically admissible with the limit stress conditions on the border Γ_T :

$$\begin{cases} \text{div } \dot{\boldsymbol{\sigma}} = 0 & x \in \Omega \quad t \in [0, t_0] \\ \dot{\boldsymbol{\sigma}} \mathbf{n}(x, t) = \dot{T} & x \in \Gamma_T \quad t \in [0, t_0] \end{cases} \quad (6)$$

Moreover, stress field and plastic deformations rates field are related as usual through elastoplastic constitutive laws, with plastic deformation rates as part of the total deformation rate :

$$\dot{\boldsymbol{\epsilon}} = \mathbf{S} : \dot{\boldsymbol{\sigma}} + \dot{\boldsymbol{\epsilon}}^p \quad x \in \Omega, \quad t \in [0, t_0] \quad (7)$$

Where $\boldsymbol{\epsilon}^e$ and $\boldsymbol{\epsilon}^p$ are elastic and plastic deformations respectively. \mathbf{C} is the tensor of elastic moduli, and $\mathbf{S} = \mathbf{C}^{-1}$ is the tensor of compliances.

If the unit weight of the rock mass is neglected, the virtual work principle gives the following formula (SALENÇON 1994):

$$\int_{\Omega} \boldsymbol{\sigma}(x) : \dot{\boldsymbol{\epsilon}}(x) d\Omega - \int_{\partial\Omega} \dot{\mathbf{u}} \cdot \boldsymbol{\sigma}(x) \cdot \mathbf{n}(x) ds = 0 \quad (8)$$

Where $\dot{\boldsymbol{\epsilon}}$ and $\dot{\mathbf{u}}$ are virtual deformation and virtual displacement respectively. Let \mathbf{u}^* , $\boldsymbol{\epsilon}^*$ and $\boldsymbol{\sigma}^*$ be respectively the displacement, deformation and stress fields solution to the problem at instant t_0 . Using \mathbf{u}^* as the virtual displacement $\dot{\mathbf{u}}$, these fields satisfy to the virtual work principle, and for the initial state as well. Then the following relation is obtained:

$$\int_{\Omega} (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_0) \boldsymbol{\epsilon}^* dx + \int_{\Gamma_{tu}} \lambda_0 (\boldsymbol{\sigma}_0 \mathbf{n}) \mathbf{u}^* dx = 0 \quad (9)$$

This relation involves explicitly the loading parameters of the problem.

2.2 Back analysis of one loading parameter (λ_0 or k_0)

Suppose, that one of the loading parameter is known, so only the other parameter has to be determined. Moreover, suppose that absolute displacement measurements \mathbf{u}^m are obtained on the actual structure, generally on part of border $\Gamma'_T \subset \Gamma_T$ at time t_0 when support is laid

$$\mathbf{u}(x, t_0) = \mathbf{u}^m \quad x \in \Gamma'_T \quad (10)$$

Let then consider the modified problem, in which displacements on border Γ'_T are prescribed,

$$\begin{cases} \text{div } \dot{\boldsymbol{\sigma}} = 0 & x \in \Omega \\ \mathbf{u}(x, t_0) = \mathbf{u}^m & x \in \Gamma'_T \end{cases} \quad (11)$$

Solving the modified problem for any given value of the unknown loading parameter λ_0 (resp k_0) will give

for instant t_0 , the fields \mathbf{u} , ϵ , σ .

This solution will agree with the experimental data, but not necessarily with the limit conditions of the original problem. In order to tend towards the solution of the original problem, the unknown loading parameter in the modified problem should be such that the stress and displacement fields would satisfy at best the original limit conditions. This means that equation (9) is used, with the fields, \mathbf{u} , ϵ , σ instead of the actual solution. A relationship between the loading parameter and the previous estimated one is thus obtained, which appears as a kind of fixed point problem, requiring iterations supposed to converge.

For example, if parameter λ_0 has to be back analysed (k_0 is known), we have according to equation (9)

$$\lambda_0 = -\frac{\int_{\Omega}(\sigma - \sigma_0)\epsilon dx}{\int_{\Gamma_{tu}}(\sigma_0 \mathbf{n})\mathbf{u} dx} \quad (12)$$

If parameter k_0 has to be back analysed (λ_0 is known) : Let us decompose the fictitious stress variation at the wall of the tunnel into horizontal and vertical components from time 0 to time t_0 (Fig 2),

$$\begin{cases} T_x = -k_0 \lambda_0 \sigma_y \cos \theta \\ T_y = -\lambda_0 \sigma_y \sin \theta \end{cases} \quad (13)$$

$\sigma_y = \gamma h$, γ is the unit weight of the material, and h is the depth of the center of the circular tunnel. The

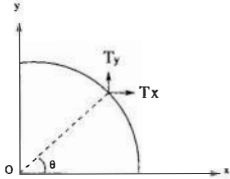


Figure 2: Decomposition of the fictitious stress at the tunnel wall

corresponding displacement components are u_x et u_y . So, the equation (9) writes as

$$\int_{\Omega}(\sigma - \sigma_0)\epsilon dx - \int_{\Gamma_{tu}}(k_0 \lambda_0 \sigma_y \cos \theta u_x + \lambda_0 \sigma_y \sin \theta u_y) dx = 0 \quad (14)$$

then

$$k_0 = \frac{\int_{\Omega}(\sigma - \sigma_0)\epsilon dx - \lambda_0 \int_{\Gamma_{tu}} \sigma_y \sin \theta u_y dx}{\lambda_0 \int_{\Gamma_{tu}} \sigma_y \cos \theta u_x dx} \quad (15)$$

2.3 Back analysis of λ_0 and k_0

If we have to back analyse simultaneously both loading parameters, we need two series of measurements at two specific instants. Suppose the displacement data \mathbf{u}^m on border $\Gamma'_T \subset \Gamma_T$ at time t_0 , and the relative displacement data $\Delta \mathbf{u}^m$ on border $\Gamma''_T \subset \Gamma_T$ during time interval $[t_0, t_f]$ are given. Note here that the two borders Γ'_T and Γ''_T are not necessarily the same, for example, final convergence measurements may be deduced from sensors placed after laying support or near the front face. In this case we have to solve two successive problems:

Problem 1, $t \in [0, t_0]$, with prescribed displacements in (eq. 10), it is the same modified problem described here above. Let the solution fields for this problem be \mathbf{u}^1 , ϵ^1 , σ^1 , equation (15) gives then

$$k_0 = \frac{\int_{\Omega}(\sigma^1 - \sigma_0)\epsilon^1 dx - \lambda_0 \int_{\Gamma_{tu}} \sigma_y^1 \sin \theta u_y^1 dx}{\lambda_0 \int_{\Gamma_{tu}} \sigma_y^1 \cos \theta u_x^1 dx} \quad (16)$$

Problem 2, $t \in [t_0, t_f]$, the support is placed, with initial state as

$$\begin{cases} \mathbf{u}(x, t_0) = \mathbf{u}^1 \\ \sigma(x, t_0) = \sigma^1 \end{cases} \quad x \in \Omega \quad (17)$$

At final time t_f , the fictitious stress on tunnel wall is $T(x) = 0$ ($\lambda = 1$). And the prescribed displacements can be chosen as

$$\mathbf{u}(x, t_f) = \mathbf{u}^1 + \Delta \mathbf{u}^m \quad x \in \Gamma''_T \quad (18)$$

The modified problem is then changed into

$$\begin{cases} \text{div} \dot{\sigma} = 0 & x \in \Omega \\ \mathbf{u}(x, t_f) = \mathbf{u}^1 + \Delta \mathbf{u}^m & x \in \Gamma''_T \end{cases} \quad (19)$$

Solving problem 2 with the initial state (17), gives the solution fields \mathbf{u}^2 , ϵ^2 , σ^2 . Then, using the virtual work principle, the following formula is obtained, similar to equation (9):

$$\int_{\Omega}(\sigma^2 - \sigma^1)(\epsilon^2 - \epsilon^1) dx + \int_{\Gamma_{tu}}(1 - \lambda_0)\sigma_0 \mathbf{n}(\mathbf{u}^2 - \mathbf{u}^1) dx = 0 \quad (20)$$

and finally

$$\lambda_0 = 1 + \frac{\int_{\Omega}(\sigma^2 - \sigma^1)(\epsilon^2 - \epsilon^1) dx}{\int_{\Gamma_{tu}} \sigma_0 \mathbf{n}(\mathbf{u}^2 - \mathbf{u}^1) dx} \quad (21)$$

Starting with initial arbitrary loading parameters, the back analysis algorithm involves iterations in which problems 1 and 2 are successively solved.

2.4 Back analysis in general cases

Absolute displacements measurements are not generally available in practice. Generally, we can have relative displacement Δl^m between two points in their direction, as by extensometers, or perpendicular to their direction, as by inclinometers. Moreover, the measurement points may be situated inside the rock-mass. Forces measurements may eventually be considered as well. These in situ observations appear as restraints in the modified problem which is solved in the same manner as with absolute displacements prescribed here above. For example, extensometer measurements will give:

$$\Delta l^m = u_2 \cos \alpha + v_2 \sin \alpha - u_1 \cos \alpha - v_1 \sin \alpha \quad (22)$$

where Δl^m is the extensometer data relative to the distance variation between two points (Fig. 3).

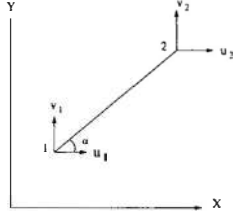


Figure 3: Displacements on two points of an extensometer

More generally, restraints result in set of equations between certain node displacements, e.g :

$$\begin{cases} u'_1 = a_{11}u_1 + \dots + a_{1n}u_n = \bar{u}_1 \\ \dots \\ u'_m = a_{m1}u_1 + \dots + a_{mn}u_n = \bar{u}_m \end{cases} \quad (23)$$

or in a condensed way :

$$[A]\{U\} = \{\bar{U}\} \quad (24)$$

where m is the number of conditions. The modified problem requires then minimizing the functional

$$\pi = \frac{1}{2}\{U\}^T[K]\{U\} - \{U\}^T\{F\} \quad (25)$$

under the conditions 24. The Lagrange multipliers method (DHATT & TOUZOT 1984) allows us to build up another functional π^* (equation 26), the stationarity of which corresponds to the solution of the problem.

$$\pi^* = \frac{1}{2}\{U\}^T[K]\{U\} - \{U\}^T\{F\} - ([A]\{U\} - \{\bar{U}\})^T\{\lambda\} \quad (26)$$

Where $\{\lambda\} = (\lambda_1 \dots \lambda_m)^T$ are the Lagrange multipliers.

The stationarity conditions for π^* lead to

$$\frac{\partial \pi^*}{\partial \{\lambda\}} = 0 \quad \rightarrow \quad [A]\{U\} = \{\bar{U}\}$$

$$\frac{\partial \pi^*}{\partial \{U\}} = 0 \quad \rightarrow \quad [K]\{U\} - \{F\} - [A]^T\{\lambda\} = 0$$

i.e.

$$[K]\{U\} = \{F\} + [A]^T\{\lambda\} \quad (27)$$

with the penalty method:

$$\{\lambda\} = G(\{\bar{U}\} - [A]\{U\}) \quad (28)$$

There are $n + m$ unknowns ($\{U\}$ et $\{\lambda\}$) and $n + m$ relations (equations 27 et 28). These unknowns are obtained by the following procedure:

- Evaluate the n components $\{U\}$ by introducing $\{\lambda\}$ (equation 28) in 27:

$$\begin{aligned} [K']\{U\} &= ([K] + G[A]^T[A])\{U\} \\ &= \{F\} + G[A]^T\{\bar{U}\} \end{aligned} \quad (29)$$

- Evaluate $\{\lambda\}$ by 28.

3 EXAMPLE

For a preliminary test of this method, we will use data from a direct non-linear tunnel calculation. The model was a shallow circular 10 m diameter tunnel (Fig. 4), 15 m deep, calculated by software CASTEM 2000 in plane strain approximation, using the convergence

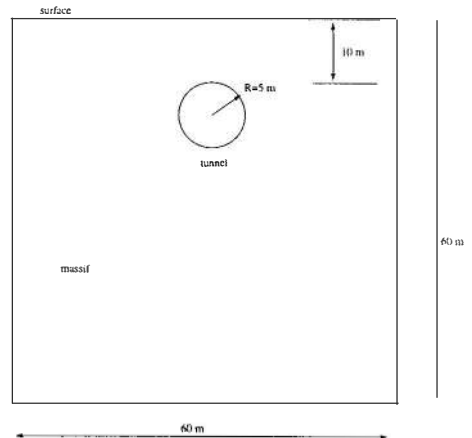


Figure 4: Calculation of a shallow tunnel

confinement method. The deconfinement ratio is supposed to be $\lambda_0 = 0.45$. The ground mass is elastic-perfect plastic (Drucker Prager). The parameters of the ground mass are: $E = 100\text{MPa}$, $\nu = 0.35$, $C = 200\text{KPa}$, $\phi = 5^\circ$, $\gamma = 2 \times 10^4\text{N/m}^3$, $k_0 = 0.8$. The 40 cm thick concrete support is elastic linear, with parameters $E = 1.5\text{GPa}$, $\nu = 0.25$.

3.1 Back analysis of one parameter (λ_0 or k_0)

In the back analysis, one half of the tunnel surface displacements ($t : 0 \rightarrow t_0$) are taken into account as measurements. Figure 5 and figure 6 illustrate the

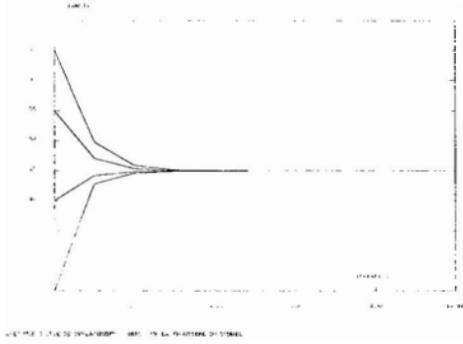


Figure 5: Convergence of λ_0 with different initial values

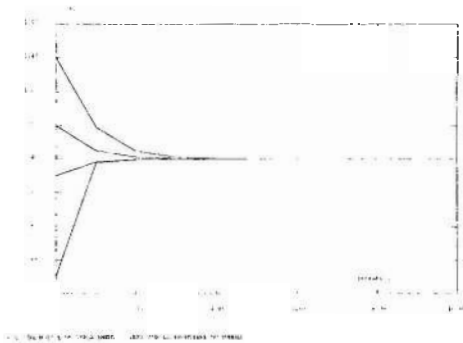


Figure 6: Convergence of k_0 with different initial values

convergence of λ_0 and k_0 for different initial values. It can be noticed that iteration convergence is very fast. Indeed, only 3 – 4 iterations are necessary on λ_0 (or k_0) to obtain less than 5% discrepancy, with initial λ_0 values taken in interval 0.25 to 0.65, and k_0 in interval 0.1 to 1.4.

3.2 Back analysis of λ_0 and k_0

Here λ_0 and k_0 are simultaneously back analysed. Direct calculation data for two instants, e.g. on two tunnel sections ($t = t_0$ and $t = t_f$) are used in this analysis, with one half of the tunnel wall displacements taken into account for input data. Figure 7 shows the convergence of λ_0 and k_0 , when these two parameters are back analysed simultaneously, with the following initial values: $\lambda_0 = 0.5$, $k_0 = 0.1$. We can notice in this case that iteration convergence is slower than when a sole parameter is back analysed. 15 – 20 iterations are needed to obtain less than 5% accuracy for loading parameters.

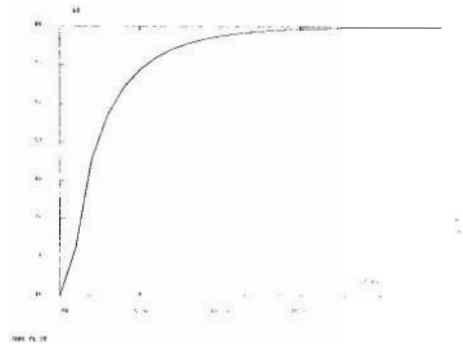
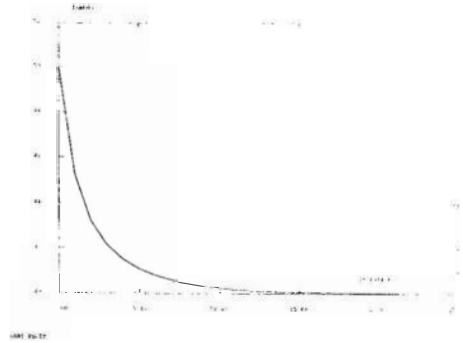


Figure 7: Convergence of λ_0 and k_0

3.3 Back analysis λ_0 using the relative measurements between measuring points

To be nearer from practical applications, let us now consider relative measurements between points. Using the direct calculation data, the relative measurements between point A and B, $\Delta l_{AB} = -0.012365$, B and C, $\Delta l_{BC} = -0.0106653$, A and C, $\Delta l_{CA} = -0.01979$ (Fig. 8) are given for $t = t_0 \rightarrow t_f$,

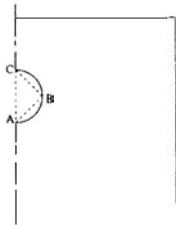


Figure 8: *Distribution of measuring points*

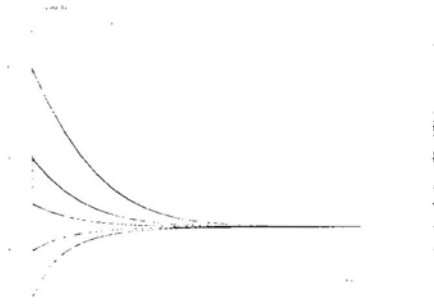


Figure 9: *Convergence of λ_0 for different initial values using relative measurements*

standing for convergence meters data. Inclinations of the different convergences are $\alpha_{AB} = 45^\circ$, $\alpha_{BC} = 135^\circ$, et $\alpha_{CA} = 270^\circ$.

The restraint conditions for the modified problem write as follows:

$$\begin{cases} u_B \cos \alpha_{AB} + v_B \sin \alpha_{AB} - v_A \sin \alpha_{AB} = \Delta l_{AB} \\ v_C \sin \alpha_{BC} - u_B \cos \alpha_{BC} - v_B \sin \alpha_{BC} = \Delta l_{BC} \\ v_C - v_A = \Delta l_{CA} \end{cases} \quad (30)$$

The Lagrange multiplier method with penalty, allows us to back analyse λ_0 with 5% accuracy within 15 iterations (figure 9).

3.4 Influence of measuring point location, and noise analysis

In the tunnelling problem, the excavation of the tunnel results in maximum displacements on the wall of the tunnel, which also corresponds to the border where loading forces are applied. This is the reason why these displacements are easier to back analyse by our method, due to a better sensitivity. A minimum of three measurement points, including one on the floor of the tunnel section is needed for back analysis of λ_0 , which will require 20 - 25 iterations for some 5% accuracy.

To appreciate the noise effects, a back analysis of λ_0 has also been performed, with convergence given

on half the wall with uniform 10% noise, while vertical surface displacements was given without noise, which led to 9% error on λ_0 determination. On the contrary, if the convergence is given without noise in the preceding case, and only surface displacements are subject to 10% uniform noise, the determination of λ_0 would only suffer 0.8% error. When discrete wall displacements are given, for example on eight points, including two lateral ones, with 20% uniform noise, the resulting relative error on λ_0 would be around 4%. These results show that the back analysis method presented herein is efficient and stable.

4 CONCLUSIONS

In this paper, we have proposed a new back analysis method for geotechnical engineering, based on the Virtual Work Principle, by comparing the original calculation model to a modified model in which the field measurements are prescribed. The deconfining ratio λ_0 in the convergence confinement method for tunnel calculation, and the lateral stress coefficient of soil k_0 have then been back analysed. Application of that method to data obtained from a plane strain numerical model, has shown its efficiency and robustness, even for nonlinear ground behavior. Provided convergence measurements be available on the tunnel wall, iteration of the back analysis has shown to converge rapidly, whether the ground be modeled by linear elasticity or elastoplasticity. Realistic cases have been envisaged as well, e.g. when relative measurements and point measurements inside the rockmass are involved. In this latter case, the iteration convergence is slower, more specially as the measurement points are farther from the wall of the gallery. The iteration convergence also allows to appreciate the pertinence of the locations and the number of measurement points. Further validation of that method needs its application to 3D numerical models, and finally, to real case histories. It is also expected an extension of that method to identify mechanical properties of the rockmass.

REFERENCES

- RNAUD, D. & ROUSSET, G. (1992). La nouvelle "méthode implicite" pour l'étude du dimensionnement des tunnels. *Revue française de géotechnique* 60, 5-26.
- BOURGOIS, L. (1998). *Contrôle optimal et problèmes inverses en plasticité*. Ph. D. thesis, Ecole polytechnique.
- CIVIDINI, A., JURINA, L., & GIODA, G. (1981). Some aspects of 'characterization' problems in geomechanics. *Int. J. Rock mech. min. sci. & geomech. abstr.* 18, 487-503.
- CONSTANTINESCU, A. (1994). *Sur l'identification des modules élastiques*. Ph. D. thesis, Ecole polytechnique.
- CORBETTA, F. (1990). *Calculs analytiques et numériques de tunnels profonds*. Ph. D. thesis, Ecole polytechnique.
- DHATT, G. & TOUZOT, G. (1984). *Une présentation de la méthode des éléments finis*. Paris: Maloine S.A. éditeur.

- NGUYEN, M. D. & GUO, C. (1993). Sur un principe d'interaction massif-soutènement des tunnels en avancement stationnaire. In SOUSA & GROSSEMAN (Eds.), *Eurock'93*, Rotterdam. Balkema.
- PANET, M. (1995). *Le calcul des tunnel par la méthode convergence-confinement*. Paris: Ecole Nationale de Ponts et Chaussées.
- SAKURAI, S. (1993). Back analysis in rock engrg. In J. A. HUDSON (Ed.), *Comprehensive rock engrg.*, pp. 543–569. Pergamon.
- SALENÇON, J. (1994). *Mécanique des milieux continus*. Paris: Ecole Polytechnique.
- SOUSA, L. R., CASTRO, A. T., LEITAO, N. S., & SOUSA, J. A. (1997). Interpretation of the monitored behaviour of underground structures using computational methods. In R. F. AZEVEDO, E. A. VARGAS, L. R. SOUSA, & M. M. FERNANDES (Eds.), *Applications of computational mechanics in geotechnical engrg.*, Rotterdam, pp. 401–418. Balkema.