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# IDENTIFICATION STRATEGIES FOR RECOVERING MATERIAL PARAMETERS FROM INDENTATION EXPERIMENTS <sup>1</sup>

Andrei CONSTANTINESCU, Nicolas TARDIEU

*Laboratoire de Mécanique des Solides (CNRS UMR 7649)  
Ecole Polytechnique - 91128 Palaiseau cedex  
e-mail: constant;tardieu@lms.polytechnique.fr*

## ABSTRACT

This paper reports the progress obtained by the authors on the development of a numerical method for the identification of material parameters from indentation experiments. The proposed technique minimizes a least squares cost functional using gradient descent methods. It has previously been shown that the adjoint state method can be extended to variational inequalities and this permits to compute the gradient of the cost functional with low numerical cost. Using this technique a series of identifications from indentation experiments have been done and their results are discussed in the sequel.

## KEYWORDS

indentation / elastoviscoplasticity / identification / contact problem / adjoint state method

## INTRODUCTION

An indentation experiment is performed by pressing a punch on a material sample and measuring the applied force and the indentation depth. The underlying inverse problem treated in this paper is:

*The identification of the parameters of the constitutive law of the sample from the simultaneous knowledge of the displacement and the resultant force on the indenter.*

This inverse problem is of practical interest as the indentation test does not demand the creation of complicated specimens and is simple to perform. However the applications are restricted by the difficult mechanical interpretation of the test.

One can differentiate two types of indentation experiments:

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- In the *Hardness Test* the punch is pressed with a given force on the sample. The punch is then lifted and the vertical displacement is deduced from geometrical measurements on the residual imprint. This test is also known as the Brinell, Rockwell or Vickers hardness test, the name denoting just the shape of the indenter : three face pyramid, sphere or respectively a four face pyramid.
- In the *Continuous Indentation Test* the punch is pressed into the sample and then lifted under continuous force or displacement control and the dual quantity is measured. An example of measured indentation curve is given in Figure 1.

The drawback of the hardness test is the final displacement measurement, which does integrate in one global answer loading, unloading as well as the rate effects during the indentation. However, due to its experimental simplicity compared to the continuous indentation test it is wide used in the industry. The results presented in this paper are focused on the interpretation of the continuous indentation test but they can easily be extended to hardness tests.

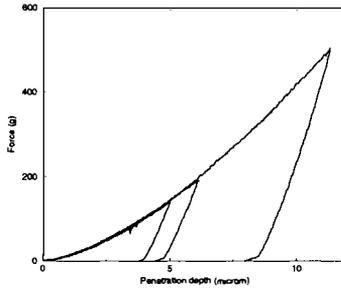


Figure 1: Experimental displacement-force indentation curve

The state of the art in mechanical interpretation of the indentation test is based on a series of closed form solutions and flow rules for the elasto-plastic constitutive behaviour. The monographs of Tabor [16] and Johnson [6] give an overview of the techniques used and the analytical solutions implied. A series of more recent papers like [4, 7, 13, 15] have introduced new semi-empirical formulas based a series of constitutive assumptions and an interpretation of the deformation process under the indenter. One of the less covered areas is the one of viscoelastic and elastoviscoplastic constitutive laws, where the rate dependence of the behaviour is important. Unfortunately even for rate dependent laws, the classical methods do not provide a systematic tool to identify constitutive parameters to be finally used in industrial structural computations.

A state of the art of the mathematical formulation of the direct problem is presented in the monograph by Kikuchi and Oden [8]. One can remark a series of difficulties related to the contact conditions inherent to the indentation problem.

From the point of view of the inverse problem we dispose of a small number of results. The existing studies formulate the inverse problem as the minimization of a cost functional. The cost functional is generally a least squares distance between computed and measured indentation forces or displacements.

As theoretical results we can mention the works of Hasanov, who showed in a series of papers [9, 10, 11, 12] that the inverse problem admits a solution. He assumed that the material has a nonlinear elastic behaviour. The numerical identification method proposed was based on a step wise trial an error search of the tangent elastic moduli with increasing vertical displacement

of the punch. The disadvantage of his reconstruction approach is that it can not be applied to rate dependent behaviours or to inhomogeneous sample (for example functionally graded materials).

A numerical approach, also based on the minimization of a least squares functional, is that of Koguchi [14]. He identified the elastic moduli of a layered material. His minimization was based on a simplex method and a closed form solution of the direct problem.

In the same global framework of the minimization of a least squares cost functional the authors have obtained a series of theoretical and numerical results. They have shown that the adjoint state method can be extended to variational inequalities in order to compute the gradient of the cost functional [18] and used this technique to identify elastic [18] and elastoviscoplastic [2] coefficients from simulated indentation experiments. This paper presents an overview of these identification techniques and discusses a series of identifications from experimental indentation curves.

## THE INVERSE INDENTATION PROBLEM

The inverse problem formulated as:

*Identify the parameters  $\mathbf{c}$  of the constitutive law of the sample from the knowledge of the indentation curve, given by the displacement-force measurements pairs:  $(U_i^{exp}, F_i^{exp})$ ,  $i = 0 \dots I$ .*

The framework proposed here to solve this inverse problem was the minimization of the least squares functional:

$$\mathcal{J}(\mathbf{c}) = \frac{1}{2} \sum_{i=0}^I (F_i(\mathbf{c}) - F_i^{exp})^2 = \frac{1}{2} \sum_{i=0}^I \left( \int_{\Gamma_C} p_i(\mathbf{c}) d\Gamma - F_i^{exp} \right)^2 \quad (1)$$

where  $p$  denotes the contact pressure on the contact surface  $\Gamma_C$ .

The direct dependence of the computed forces  $F_i(\mathbf{c})$  of a direct solution of the indentation problem, with material parameters  $\mathbf{c}$  and imposed punch displacements given by  $U_i^{exp}$ ,  $i = 0 \dots I$ , leads to a constrained minimization problem.

In order to compute the gradient of the cost functional  $\frac{\partial \mathcal{J}}{\partial \mathbf{c}}$  a Lagrangian  $\mathcal{L}$  functional has been constructed by adjoining to  $\mathcal{J}$  the variational formulation of the direct problem. Supposing the existence of a saddle point of the Lagrangian  $\mathcal{L}$  it has been shown that the gradient of  $\mathcal{J}$  can be computed from the solution of the direct contact problem and the solution of an adjoint problem.

For the sake of simplicity we shall not present here the complete deduction and the equations of these problems. Two classes of constitutive behavior have been treated under the assumption of small displacements and rotations: *linear elasticity* in [17] and in *elastoviscoplasticity* [19, 2]. The passage to large displacements and small rotations for these behaviours in the formalism of standard generalized materials is presented in [20].

A schematic view of the direct and the adjoint problem are presented in Figure 2. Some important properties of the adjoint problem:

- It is *not* a contact problem. For the cost functional (1), its loading is given by a Dirichlet condition (imposed displacement) on the effective contact surface of the direct problem.
- It is a time dependent system of partial differential equations and a final condition given by the solution of a well-posed elasticity problem.

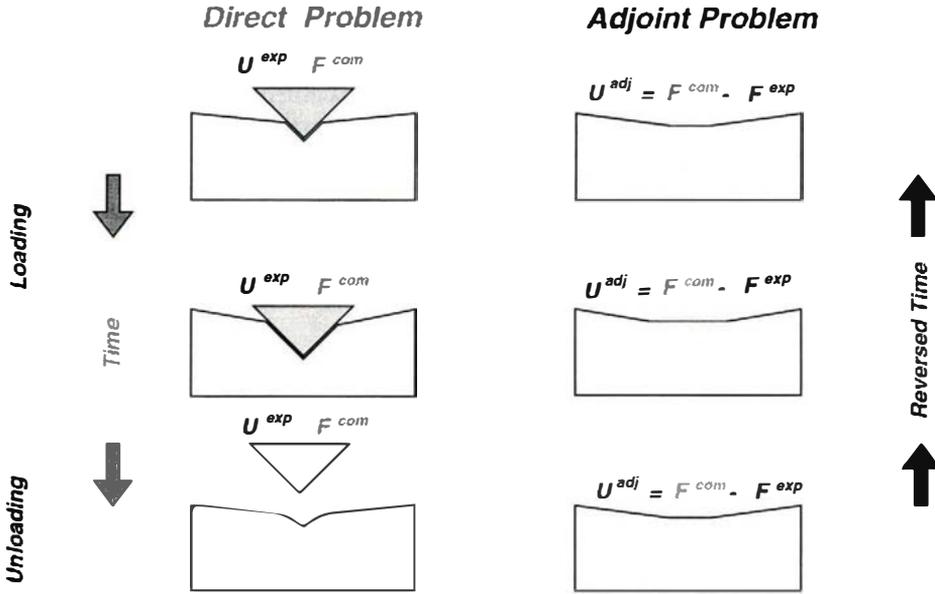


Figure 2: Indentation test: The Direct problem

- Interpreted in the reversed time, the adjoint problem is a linear elastic problem if the direct problem is elastic [17], and a viscoelastic problem if the direct problem is elastoviscoplastic [2].
- The parameters of the adjoint constitutive law are determined by the coefficients of the direct constitutive law and the solution of the direct problem.

Under the assumptions of small strains and an elastic constitutive law:

$$\boldsymbol{\sigma}(\mathbf{u}) = \mathbf{A}(\mathbf{c}) : \boldsymbol{\varepsilon}(\mathbf{u}) \quad (2)$$

where  $\mathbf{u}$ ,  $\boldsymbol{\varepsilon}$ ,  $\boldsymbol{\sigma}$ ,  $\mathbf{A}$  denote respectively the displacement vector the strain and the stress and the elasticity tensor; the gradient of the cost functional takes the following form:

$$\nabla_{\mathbf{c}} \mathcal{J} = \frac{\partial \mathcal{L}}{\partial \mathbf{c}} = \int_{\Omega} \boldsymbol{\varepsilon}(\mathbf{u}) : \frac{\partial \mathbf{A}}{\partial \mathbf{c}} : \boldsymbol{\varepsilon}(\mathbf{u}^*) \, d\Omega \quad (3)$$

where  $\mathbf{u}^*$  is the solution of the adjoint equation.

In the computations, the direct and the adjoint problem have been integrated using the finite element method and coupled with a gradient descent algorithm as presented in the next section.

## THE NUMERICAL COMPUTATIONS

The direct and the adjoint problem have been programmed in the finite element oriented language Castem 2000 [1]. The adjoint viscoelastic problem has been integrated using an implicit (Forward Euler) integration scheme. The commonly used gradient descent method was the BFGS algorithm with an Armijo line search rule [3].

The finite element meshes were refined in the contact area in order to keep results precise in a reasonable computational time. From a series of meshes, the one with the largest density and not affecting the resultant forces on the indentation curves has finally been chosen (see Figure 3).

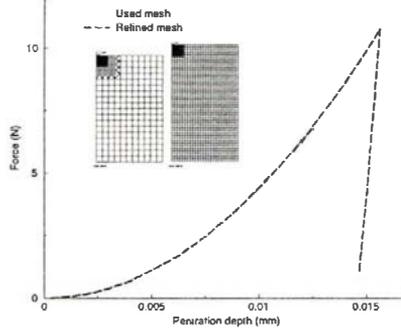


Figure 3: Comparison of computed indentation curves for a coarse and a fine mesh

A series of identifications using simulated measurements and the previous technique have already been presented for elastic materials in [18] and for Maxwell and Norton-Hoff viscoelastic materials in [2]. In the sequel we shall present a series of identifications using the same technique and real experiments.

## DISCUSSION OF EXPERIMENTAL RESULTS

A series of indentation experiments were performed on three different materials: nylon, duraluminium and polyethylene. The choice of the materials was justified by their behaviour varying from elastoplastic to viscoelastoplastic at normal room temperature.

The material samples were cylinders with  $\approx 2.5 \times 2.5 \text{ cm}$ , diameter  $\times$  height. As the maximum indentation depth was about  $25 \mu\text{m}$  the faces of the samples have been polished and assured to be parallel. The indentation was done with a Vickers pyramid by a displacement controlled machine.

The identification were done supposing that the material behaves under the elastoviscoplastic Norton-Hoff law and under the assumption of large displacements and small rotations.

The Norton-Hoff law is a standard generalized constitutive law [5] without work hardening. The flow rule can be written as:

$$\varepsilon(\Delta \mathbf{u}_i) = \mathbf{S} : \Delta \boldsymbol{\sigma}_i + \Delta \boldsymbol{\varepsilon}_i^p \quad \Delta \boldsymbol{\varepsilon}_i^p = \frac{\partial \Phi(\boldsymbol{\sigma}_i, \mathbf{c})}{\partial \boldsymbol{\sigma}} \Delta t \quad (4)$$

where the pseudo potential  $\Phi$  is given by

$$\Phi(\boldsymbol{\sigma}_i, \mathbf{c}) = \frac{K}{m+1} \left\langle \frac{(\boldsymbol{\sigma}_i)_{eq} - \sigma^Y}{K} \right\rangle_+^{m+1}$$

where  $\sigma^Y$  is the elasticity limit,  $\langle \cdot \rangle_+$  is the positive part operator and  $(\cdot)_{eq}$  is the equivalent Mises stress. Finally the plastic strain increment is determined by :

$$\Delta \boldsymbol{\varepsilon}_i^p = \frac{3}{2} \left\langle \frac{(\boldsymbol{\sigma}_i)_{eq} - \sigma^Y}{K} \right\rangle_+^m \frac{\bar{\boldsymbol{\sigma}}_i}{(\boldsymbol{\sigma}_i)_{eq}} \Delta t$$

$\bar{\sigma}_i$  is the deviatoric part of  $\sigma_i$ .

The Young modulus, the yield limit, the viscosity and the power coefficient have been identified. The Poisson coefficient has been fixed at the a priori value  $\nu = 0.3$ .

The row data from the indentation machine has been used for the identification. The last part of the unloading has been neglected, due to a convergence problems in the direct algorithm in unloading at large step sizes.

The indentations on the *Nylon* sample have been performed at  $3\mu\text{m}/\text{min}$ . The experimental and identified indentation curves are presented in Figure 4. The identified parameters (after 53 iterations) are given in Table 1

It is difficult to judge the validity of the identified parameters as we did not dispose of other types of experimental curves in order to determine classically the material parameters. The Young modulus and the yield limit are in the intervals commonly reported in the literature for this material.

Table 1: Identified parameters for the Nylon

	Initial Values	Final Values
	Iteration 56	
$E$ (MPa)	1000.	1930.
$K$ (MPa.s $^{1/n}$ )	100.	83.95
$n$	4.	7.34
$\sigma^y$ (MPa)	30.	15.59
$\mathcal{J}$	2.08	0.020

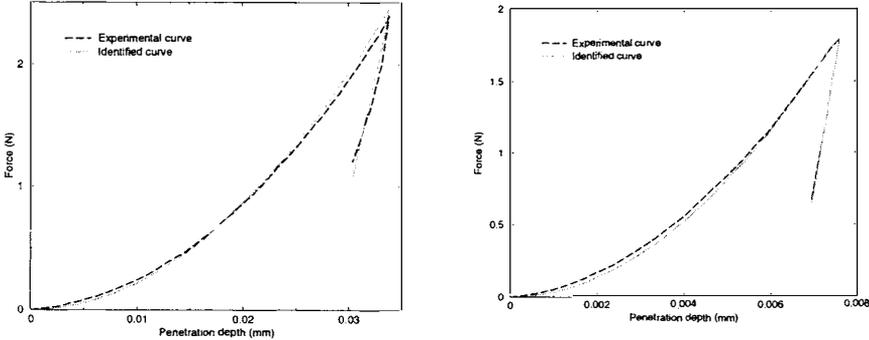


Figure 4: Experimental and identified indentation curve for the Nylon and the Duralumin

The second sample was a *duraluminium (Au4G)* and the penetration rate was  $1\mu\text{m}/\text{min}$ . The experimental and the identified curve are in good agreement (see Figure 4). However a direct inspection shows for example that the identified Young modulus of  $\approx 35$  GPa (see Table 2) is at half the common sense value of  $\approx 70$  GPa for this material. In making this comparison one should also keep in mind that the duraluminium is almost elastoplastic at room temperature and that it does not correspond exactly to the chosen material law. Another difference comes from

the role played by the Young modulus in different constitutive laws which can lead to different interpretations for this parameter.

Table 2: Identified parameters for the duraluminium

	Initial Values	Final Values
	Iteration 43	
$E$ (MPa)	50000.	37471.8
$K$ (MPa.s <sup>1/n</sup> )	1500.	2750.76
$n$	5.	4.60
$\sigma^y$ (MPa)	100.	52.09
$\mathcal{J}$	0.22	0.0034

The third material, the *Polyethylene* was indented with a rate was  $6\mu\text{m}/\text{min}$ . The identified indentation curve is good agreement with the experimental one even if the loading and the unloading are under and respectively overevaluated (Figure 5).

In this case, we did dispose of an experimental traction curve and the identified values (see Table 3) were used to simulate the same traction experiment. The traction curves are pretty different even if the shape and the order of magnitude are similar.

A possible explanation justifying the difference could be the order of magnitude of difference in the global strain rates of the experiments:  $\dot{\epsilon} = 3 \cdot 10^{-4} \text{s}^{-1}$  for the traction and  $\dot{\epsilon} = 2 \cdot 10^{-3} \text{s}^{-1}$  for the indentation.

It is also possible that the friction between the diamond indenter and the polyethylene sample has not been negligible.

Table 3: Identified parameters for the polyethylene

	Initial Values	Final Values
	Iteration 53	
$E$ (MPa)	200.	520.
$K$ (MPa.s <sup>1/n</sup> )	10.	7.90
$n$	4.	7.93
$\sigma^Y$ (MPa)	6.	9.28
$\mathcal{J}$	3.85	0.039

The performed identification can be qualified as encouraging taking into account that no a priori information has been used and that a rate dependent constitutive law has been identified from a single experiment at practically constant loading rate.

It is important to notice that the indentation curves have always been well represented in spite of the differences between identified and expected material parameters.

A global drawback comes also directly from the Norton-Hoff constitutive law itself. As shown in a previous work [2], the cost functional presents a long valley if expressed in well

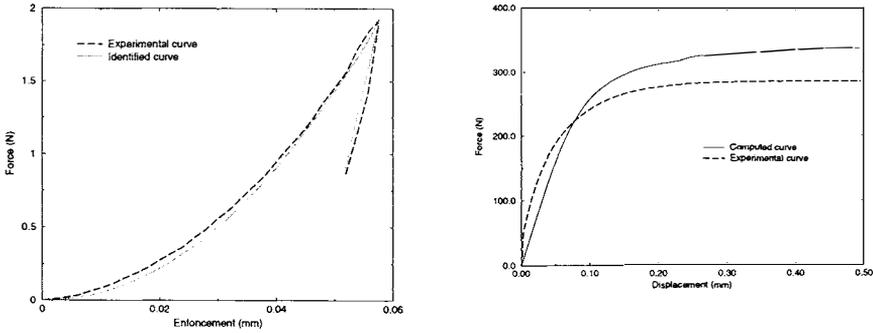


Figure 5: *Polyethylene*. Left: Experimental and identified indentation curves for the Polyethylene. Right: Experimental traction and computed traction with the parameters identified from the indentation

definite parameter plane. This would suggest that the material parameters are slightly correlated. As one can remark, a decrease in the yield limit could be compensated by an increase of the viscosity and its power coefficient.

## OTHER IDENTIFICATION STRATEGIES

Besides the identification problem reported in the last section, there are two essential drawbacks of the identification strategy used before. Both are related with the incomplete usage of the available informations. Their resolution could conduct to computationally faster identifications.

The first drawback comes from the evaluation of the cost functional itself. The cost functional and its gradient are integrated on the whole time interval and therefore the descent algorithm can not take into account different sensitivities of the material parameters in the different sections of the loading-unloading curve.

A typical method taking into account the "local" sensitivity of a nonlinear function in a least squares distance is the Levenberg-Marquardt method. A direct application of this method is not possible in our case due to the current gradient computation using the adjoint state method translated in the difficulty of computing directly  $\frac{\partial F_i(\mathbf{c})}{\partial \mathbf{c}}$  instead of  $\frac{\partial \mathcal{J}(\mathbf{c})}{\partial \mathbf{c}}$ . This can be overcome by changing the expression of cost functional in order to obtain the desired result. The first tests done using a Levenberg-Marquardt descent algorithm are promising.

The second drawback is related to the number of evaluation of the cost functional in the line search of the gradient descent algorithm. For a descent step, one needs a first evaluation of the functional and its gradient ; several evaluations of the functional are necessary in order to determine the optimal step size. In our case, we have remarked that the evaluation of the cost functional is  $\approx 3$  more expensive as the one of the gradient. It is therefore interesting to compute *also* the gradient at each evaluation of the cost functional and then to use the obtained information more effectively.

In order to solve this problem, we proposed to estimate a model function for our cost functional from a small number of cost functional and gradient computation and then minimize this model function.

An example of the computed cost functional and its model are represented in Figure 6. The

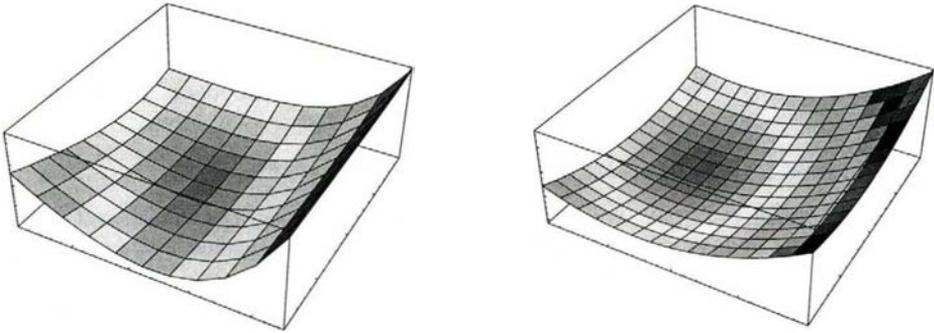


Figure 6: The cost functional and its polynomial model in the case of a Maxwell constitutive law

constitutive model was a Maxwell viscoelasticity law and the model function was obtained by fitting the cost functional over a quadratic polynomial with material parameters as variables. With 9 random evaluations of the cost functional and its gradient, the identified parameters were 5% different of the real values.

Both strategies, the Levenberg-Marquardt descent algorithm and the model functions are currently under development for the inverse indentation problem and more results will be provided in the near future.

## CONCLUSION

This paper described an original method for identifying material characteristics from indentation tests. Based on efficient mathematical methods (adjoint state methods and optimization) the method applied to different behaviours ranging from elasticity to elastoviscoplasticity (sufficiently smooth standard generalized constitutive laws). The efficiency of this technique has been proven through numerically and experimentally based studies.

The perspectives are the combination of the existing results with more efficient identification strategies like Levenberg-Marquardt algorithm or polynomial models and the generalization of the adjoint state method to elastoplastic constitutive laws.

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- [1] Castem2000 Finite Element Code <http://www.castem.org>:8001
- [2] Constantinescu A Tardieu N 1999 On the identification of nonlinear constitutive laws from indentation tests *Submitted to Inverse Problems in Engineering*
- [3] CULIOLI J.C. - *Introduction à l'optimisation* Ellipses, 1994

- [4] Doener M F, Nix WD 1986 A method of interpreting the data from depth-sensing indentation measurements *J. Mater. Res.*, **1** (4), 601–609
- [5] B. Halphen, Q.S. Nguyen, Sur les matériaux standards généralisés, *Journal de Mécanique*, 1975.
- [6] Johnson K L 1985 *Contact Mechanics* Cambridge University Press
- [7] S. Jayaraman, G.T. Hahn, W.C. Oliver, C.A. Rubin, P.C. Bastias, Determination of monotonic stress-strain curve of hard metals from ultra-low-load indentation tests, *Int. J. Solids Structures*, Vol. 35, Nos. 5-6, 1998.
- [8] Kikuchi N , Oden J T 1988 *Contact Problems in Elasticity: A Study of Variational Inequalities and Finite Element Methods* SIAM, Philadelphia
- [9] Hasanov A, Seyidmamedov Z 1995 The solution of an axisymmetric inverse elasto-plastic problem using penetration diagrams *Int. J. Non-Linear Mechanics*, **30** (4), 465–477
- [10] Hasanov A 1995 An inverse problem for an elastoplastic medium *SIAM J. Appl. Math.*, **55**(5), 1736–1752
- [11] Hasanov A 1997 Inverse coefficient problems for monotone potentials operators *Inverse Problems*, **13** (5), 1265–1278
- [12] Hasanov A 1998 Inverse coefficient problems for elliptic variational inequalities with a nonlinear monotone operator *Inverse Problems*, **14** (5), 1151–1170
- [13] Loubet J.L, Georges J.M, Marchesini O., Meille G., Vickers indentation curve for a magnesium oxyde (MgO) *J. of Tribology*, 106, 1984
- [14] Koguchi H 1999 Determination of mechanical properties of thin films and functional gradient materials using an inverse technique *Conference on Inverse Problems in Engineering: Theory and Practice*, ASME
- [15] Oliver W.C., Pharr G.M., An improved technique for determining hardness and elastic moduli used load and displacement sensing indentation experiments, *J.Mater.Res.*, **7**(6), 1992
- [16] Tabor D 1951 *Hardness of metals* Oxford University Press
- [17] Tardieu N Constantinescu A 1999 On the determination of the elastic coefficients from indentation tests accepted for publication in *Inverse Problems*,
- [18] Tardieu N Constantinescu A 1999 On the identification of nonlinear constitutive laws from indentation tests *Conference on Inverse Problems in Engineering: Theory and Practice*, ASME
- [19] Tardieu N Constantinescu A Taheri S 1999 Identification inverse de lois de comportement standard généralisées sans écrouissage par indentation *Colloque National en Calcul des Structures*, Giens, France
- [20] Tardieu N 2000 *Identification des lois de comportement élastoviscoplastique par indentation*, Thèse de Doctorat, Ecole Polytechnique, France