



Location of additional sensors for FDI

Christian Commault, Jean-Michel Dion, Sameh Yacoub Agha

► To cite this version:

Christian Commault, Jean-Michel Dion, Sameh Yacoub Agha. Location of additional sensors for FDI. ACD 2006 - 21th Workshop Advanced Control and Diagnosis ACD'06, Nov 2006, Nancy, France. <hal-00113032>

HAL Id: hal-00113032

<https://hal.science/hal-00113032v1>

Submitted on 10 Nov 2006

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

LOCATION OF ADDITIONAL SENSORS FOR FDI

Christian Commault* Jean-Michel Dion*
Sameh Yacoub Agha*

* *Laboratoire d'Automatique de Grenoble. LAG-CNRS,
ENSIEG-INPG BP 46
38402 Saint Martin d'Hères,
France.*

*Email: {Christian.Commault, Jean-
Michel.Dion, Sameh.Yacoub-Agha}@inpg.fr*

Abstract: We present a structural analysis for sensor location in the Fault Detection and Isolation problem. We deal with this problem when the system under consideration is structured, that is, the entries of the system matrices are either fixed zeros or free parameters. With such structured systems one can associate a graph. We define in this graph a set of separators (Input Separators) which allows us to get sets of system variables in which additional sensors must be implemented to solve the considered FDI Problem. It is shown that these separators parameterize all the solutions, and allow to choose among a set of potential additional sensors to solve the *FDI* problem.

Keywords: Linear systems, Structured systems, Fault detection and isolation, Sensor location selection.

1. INTRODUCTION

The Fault Detection and Isolation (FDI) problem has received considerable attention in the past ten years (Chen and Patton, 1999; Frank, 1996). It consists of building residuals from the available data and isolating, whenever possible, the faults using the residuals. When this FDI problem is not solvable with the existing sensors we look for a solution using additional sensors, see for example (Raghuraj *et al.*, 1999).

We consider the FDI problem in the framework of structured linear systems which represent a large class of parameter dependent systems (Lin, 1974; Dion *et al.*, 2003). Structured systems can be described by graphs and simple properties of these graphs will allow us to get interesting structural information on the system. The FDI problem has been tackled in this context and a reduced sys-

tem where the fault free additional sensors should be implemented has been determined (Commault and Dion, 2007).

In this paper we prove that a set of separators (Input Separators) in the graph of the system characterizes fully the considered FDI problem and give a necessary and sufficient solvability conditions in terms of number of additional sensors measuring variables between faults and separators in the associated graph. We show that these results allow to choose among a set of potential additional sensors to solve the *FDI* problem.

This paper is an extension of preliminary results (Commault *et al.*, 2006) where only necessary conditions were given.

The paper is structured as follows. The problem is formulated in Section 2. Structured systems are presented in Section 3. In Section 4, we define Input Separators and give their properties. Appli-

cation to the sensor location problem is performed in Section 5. Concluding remarks end the paper.

2. PROBLEM FORMULATION

2.1 Observer-based FDI problem

Let us consider the following linear time-invariant system :

$$\Sigma \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f(t) \in \mathbb{R}^r$ the fault vector and $y(t) \in \mathbb{R}^p$ the measured output vector. A, C, L and M are matrices of appropriate dimensions.

A dedicated residual set is designed using a bank of r observers for system (1), according to the dedicated observer scheme (Chen and Patton, 1999).

The i th observer of this bank of r observers is designed for a system of type (1) as follows:

$$\dot{\hat{x}}^i(t) = A\hat{x}^i(t) + K^i(y(t) - C\hat{x}^i(t)), \quad (2)$$

where $\hat{x}^i(t) \in \mathbb{R}^n$ is the state of the i th observer, K^i is the observer gain to be designed such that $\hat{x}^i(t)$ asymptotically converges to $x(t)$, when $f(t) = 0$.

The residuals are defined as :

$$r_i(t) = Q^i(y(t) - C\hat{x}^i(t)), \text{ for } i = 1, \dots, r, \quad (3)$$

where Q^i is a $1 \times p$ matrix.

Let us state now our FDI diagnosis problem.

FDI problem The bank of observer-based FDI problem consists in finding, if possible, matrices K^i and Q^i , such that, for $i = 1, 2, \dots, r$, $A - K^i C$ is stable, and the fault to residual transfer matrix is non zero, proper and diagonal.

2.2 Sensor location for FDI

Consider again the system (1). Often the above defined FDI problem has no solution using only the existing sensors on the system. In this case we consider new sensors which could be implemented on the system. We assume that these new sensors are fault free.

Consider the composite system denoted by Σ^c .

$$\Sigma^c \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t), \\ z(t) = Hx(t) + Pf(t) \end{cases} \quad (4)$$

where the output vector z collects the new measurements.

If additional sensors are necessary we look for an implementation which minimizes their number, and give results concerning the additional sensors location. Our study will be achieved in the framework of structured systems that we introduce now.

3. LINEAR STRUCTURED SYSTEMS

We will consider models based on the available physical knowledge on the system. These models capture the relations between internal variables but without fixing the precise value of the parameters. Frequently, starting from a nonlinear dynamical model of a system one gets linearized models around different set points which share the same structure but with different parameter values. A distillation column example is worked out in (Commault and Dion, 2006). This is illustrated in Figure 1 where a nonlinear model is linearized around two different set points.

Such models which incorporate prior knowledge

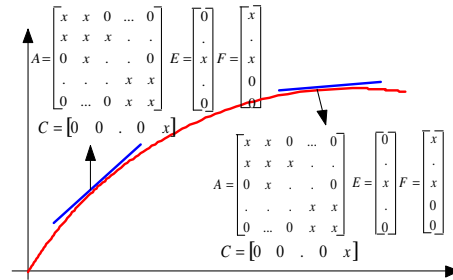


Fig. 1. Nonlinear system

on the structure have been often used in the literature see e.g (Lin, 1974; Meyer *et al.*, 1994; Blanke *et al.*, 2003)

In this paper we will consider linear structured systems as in (Lin, 1974). We consider linear systems as described in (1), but with parameterized entries and denoted by Σ_Λ

$$\Sigma_\Lambda \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \end{cases} \quad (5)$$

This system is called a linear structured systems if

the entries of the composite matrix $J = \begin{bmatrix} A & L \\ C & M \end{bmatrix}$ are either fixed zeros or independent parameters (not related by algebraic equations). $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_k\}$ denotes the set of independent parameters of the composite matrix J . More details can be found in (Dion *et al.*, 2003).

For such systems one can study generic properties i.e. properties which are true for almost all values of the parameters collected in Λ (Murota, 1987).

A directed graph $G(\Sigma_\Lambda) = (V, W)$ can be easily associated with the structured system Σ_Λ of type

(5) where the matrix $\begin{bmatrix} A & L \\ C & M \end{bmatrix}$ is structured:

- the vertex set is $V = F \cup X \cup Y$ where F , X and Y are the fault, state and output sets given by $\{f_1, f_2, \dots, f_r\}$, $\{x_1, x_2, \dots, x_n\}$ and $\{y_1, y_2, \dots, y_p\}$ respectively,

- the arc set is $W = \{(f_i, x_j) | L_{ji} \neq 0\} \cup \{(x_i, x_j) | A_{ji} \neq 0\} \cup \{(x_i, y_j) | C_{ji} \neq 0\} \cup \{(f_i, y_j) | M_{ji} \neq 0\}$, where A_{ji} (resp. C_{ji}, L_{ji}, M_{ji}) denotes the entry (j, i) of the matrix A (resp. C, L, M).

Recall that a directed path in $G(\Sigma_\Lambda)$ from a vertex $i_{\mu 0}$ to a vertex $i_{\mu q}$ is a sequence of arcs $(i_{\mu 0}, i_{\mu 1}), (i_{\mu 1}, i_{\mu 2}), \dots, (i_{\mu q-1}, i_{\mu q})$ such that $i_{\mu t} \in V$ for $t = 0, 1, \dots, q$ and $(i_{\mu t-1}, i_{\mu t}) \in W$ for $t = 1, 2, \dots, q$. If $i_{\mu 0} \in F$ and $i_{\mu q} \in Y$, P is called a fault-output path. If $i_0 \in V_1$ and $i_l \in V_2$, where V_1 and V_2 are two subsets of V , P is called a V_1 - V_2 path. Moreover, if i_0 is the only vertex of P which belongs to V_1 and $i_l \neq i_0$ is the only vertex of P which belongs to V_2 , P is called a *direct* V_1 - V_2 path.

A set of paths with no common vertex is said to be vertex disjoint. A V_1 - V_2 linking of size k is a set of k vertex disjoint V_1 - V_2 paths. A linking is maximal when k is maximal.

Consider now composite system as defined in (4). All the previous definitions can be extended to the composite structured system Σ_Λ^c with associated graph $G(\Sigma_\Lambda^c)$ where Σ_Λ^c is defined as

$$\Sigma_\Lambda^c \begin{cases} \dot{x}(t) = Ax(t) + Lf(t) \\ y(t) = Cx(t) + Mf(t) \\ z(t) = Hx(t) + Pf(t) \end{cases} \quad (6)$$

In this paper we will consider that Σ_Λ is structurally observable (Lin, 1974; Murota, 1987).

Give now the result concerning the diagonal FDI problem by using a bank of observers, which was stated first in (Commault *et al.*, 2002).

Theorem 1. Consider the structurally observable system with r faults Σ_Λ as defined in (5) and the associated graph $G(\Sigma_\Lambda)$. The bank of observer-based diagonal FDI problem, is generically solvable if and only if:

$$k = r, \quad (7)$$

where k is the size of a maximal linking in $G(\Sigma_\Lambda)$.

4. INPUT SEPARATORS

Consider the graph $G(\Sigma_\Lambda) = (V, W)$ of a structured system of type (5) with vertex set V and edge set W .

Definition 2. (van der Woude, 2000) A separator S is a set of vertices such that any fault-output path has at least one vertex in S . The dimension of a separator is the number of vertices in S .

Define now a particular set of separators which will be useful in the sequel.

Definition 3. A separator S of dimension d is an input separator if for any separator S' of dimension d' , such that any direct path from faults to S contains a vertex in S' , we have $d' > d$.

This means that a separator is an input separator if there is no separator of lower or equal dimension between faults and this separator.

Among all the input separators, the input separator which has the minimal dimension can be proved to be unique. It is called the minimal input separator and denoted S^* (van der Woude, 2000). S^* can be obtained using standard maximum flow algorithms as the Ford and Fulkerson algorithm (Hu, 1982). The dimension of S^* is equal to the maximal size of a fault-output linking in $G(\Sigma_\Lambda)$. S^* is indeed the first bottleneck between faults and outputs. S^* may contain fault, state and output vertices.

We will give now a characterization of the input separators.

Proposition 4. Consider the structured system Σ_Λ and its associated graph $G(\Sigma_\Lambda)$.

A separator S of dimension d is an input separator if and only if:

- There exists a F - S linking of size d in $G(\Sigma_\Lambda)$.
- For any separator S' such that any direct path from F to S contains a vertex in S' , the maximal size of a $F - S'$ linking in $G(\Sigma_\Lambda)$ is of dimension $d' > d$.

Example 1 Consider the structured system Σ_Λ whose associated graph is depicted in Figure 2

From this graph we can remark that:

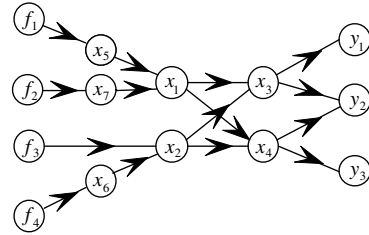


Fig. 2. Graph $G(\Sigma_\Lambda)$ of Example 1

- $\{y_1, y_2, y_3\}$ is a separator of dimension 3. but it is not an input separator because there are other separators of dimension 2 and 3 between the faults and this separator.

- $\{x_3, x_4\}, \{x_1, x_2\}$, are two separators of dimension 2, and $\{x_1, x_2\}$ is an input separator of minimal dimension and it is then the minimal input separator S^* of the system.

- $S_3^1 = \{f_1, f_2, x_2\}$, $S_3^2 = \{f_3, f_4, x_1\}$, are input separators of dimension 3.

Definition 5. Consider the structured system Σ_Λ with its associated graph $G(\Sigma_\Lambda)$. Consider two separators S and S' of $G(\Sigma_\Lambda)$. Let T_s (resp. $T_{s'}$) be the set of all vertices in any direct path from F to S (resp. S') in $G(\Sigma_\Lambda)$ except for the vertices of S (resp. S'). We will say that S is greater than or equal to S' , $S \succ S'$ if $T_s \subset T_{s'}$.

For example in Figure 2, we can get:

- $\{f_1, f_2, f_3, f_4, x_6\}$ is the set associated with the separator $\{x_2, x_5, x_7\}$.
- $\{f_1, f_2, f_3, f_4, x_5, x_6, x_7\}$ is the set associated with the separator $\{x_1, x_2, x_3, x_4\}$.
- $\{f_1, f_2, x_5, x_7\}$ is the set associated with the separator $\{x_1, f_3, f_4\}$.
- $\{x_2, x_5, x_7\} \succ \{x_1, x_2, x_3, x_4\}$.
- $\{x_1, f_3, f_4\} \succ \{x_1, x_2, x_3, x_4\}$.
- $\{x_2, x_5, x_7\}$ and $\{x_1, f_3, f_4\}$ are not comparable.

5. APPLICATION TO THE SENSOR LOCATION PROBLEM IN FDI

5.1 Structural analysis for the sensor location problem

We will now use the properties of separators to tackle the sensor location problem and give a necessary condition expressed in terms of number of additional sensors (Commault *et al.*, 2006).

Theorem 6. Consider the linear structurally observable system Σ_Λ defined by (5) with its associated graph $G(\Sigma_\Lambda)$. Consider a separator S of $G(\Sigma_\Lambda)$ and its associated set T_s defined in Definition 5. Consider the composite system Σ_Λ^c with additional measurements $z(t)$ defined in (6) and its associated graph $G(\Sigma_\Lambda^c)$. The FDI problem on Σ_Λ^c has a solution only if there are at least $r - d$ additional sensors which measure vertices of T_s , where d is the dimension of the separator S and r the number of faults.

Applying Theorem 6 to S^* the minimal input separator, we get the following: (Commault and Dion, 2007).

Corollary 7. The minimal number of additional sensors we have to add to solve our FDI problem is equal to $r - k$, where k is the dimension of S^* . These sensors must measure variables in T_{s^*} .

Definition 8. Consider the structured system Σ_Λ with its associated graph $G(\Sigma_\Lambda)$. Consider a separator S of $G(\Sigma_\Lambda)$ and its associated set T_s . Define the constraint associated with S as the minimal number of sensors that should measure vertices in T_s to solve the FDI problem.

From Theorem 6 this minimal number of sensors is equal to $r - d$ where r is the dimension of F and d is the dimension of S .

Proposition 9. Let S_1 and S_2 be two separators of dimensions d_1 and d_2 respectively. The constraint associated with the separator S_1 implies the constraint associated with the separator S_2 if: $S_1 \succ S_2$ and $d_1 \leq d_2$.

Proof

Let T_{S_1} and T_{S_2} be the sets associated with S_1 and S_2 . Assume that $T_{S_1} \subseteq T_{S_2}$ and $d_1 \leq d_2$. Compare the two constraints associated with S_1 and S_2 . The constraint associated with S_1 tells us that at least $\nu_1 = r - d_1$ new sensors should measure vertices of T_{S_1} . The constraint associated with S_2 tells us that at least $\nu_2 = r - d_2$ new sensors should measure vertices of T_{S_2} . As $d_1 \leq d_2$ then $\nu_1 \geq \nu_2$ and as T_{S_1} is a subset of T_{S_2} , it follows that when the constraint associated with S_1 is respected, the constraint associated with S_2 will be satisfied too. \square

For example in Figure 2, we have:

- The constraint associated with the separator $\{x_2, x_5, x_7\}$ says that at least one new sensor should measure vertices in $\{f_1, f_2, f_3, f_4, x_6\}$ to solve the FDI problem.
- The constraint associated with the separator $\{x_1, f_3, f_4\}$ says that at least one new sensor should measure vertices in $\{f_1, f_2, x_5, x_7\}$ to solve the FDI problem.
- The constraint associated with the separator $\{x_1, x_2, x_3, x_4\}$ says that at least zero new sensor should measure vertices in $\{f_1, f_2, f_3, f_4, x_5, x_6, x_7\}$ to solve the FDI problem.

Corollary 10. The set of constraints associated with all the possible separators of $G(\Sigma_\Lambda)$, is implied by the set of constraints associated with input separators.

Proof

Consider a separator S of $G(\Sigma_\Lambda)$ of dimension d . If this separator is not an input separator, from Definition 3 there is an input separator S' of dimension $d' \leq d$ between F and S . From Proposition 9 the constraint associated with S' implies the constraint associated with S . \square

For example in Figure 2:

- The constraint associated with the separator $S_1 = \{x_3, x_4\}$ is implied by the constraint associated with the separator $S_2 = \{x_1, x_2\}$ because $T_{S_1} = \{f_1, f_2, f_3, f_4, x_1, x_2, x_5, x_6, x_7\}$, $T_{S_2} = \{f_1, f_2, f_3, f_4, x_5, x_6, x_7\}$ and $T_{S_2} \subseteq T_{S_1}$.
- The constraints associated with the two separators of dimension 3, $S_1 = \{x_1, f_3, f_4\}$, $S_2 =$

$\{x_2, f_1, f_2\}$ are not comparable with our relation order because neither $T_{S_2} \not\subseteq T_{S_1}$ nor $T_{S_1} \not\subseteq T_{S_2}$. Now state our main result which parameterizes all the solutions of the sensor location problem in *FDI* in terms of input separators.

Theorem 11. Consider the linear structurally observable system Σ_Λ with r faults defined by (5) with its associated graph $G(\Sigma_\Lambda)$. Consider the composite system Σ_Λ^c with additional measurements $z(t)$ defined in (6) with corresponding vertex set Z and its associated graph $G(\Sigma_\Lambda^c)$. The *FDI* problem on Σ_Λ^c has a solution if and only if for any input separator S of $G(\Sigma_\Lambda)$ with associated set T_S defined in Definition 5 there are at least $r - d$ additional sensors which measure vertices of T_S , where d is the dimension of S .

Proof

Necessity follows from Theorem 6.

Sufficiency: Assume that after adding $r - k$ additional sensors we do not have a solution for the *FDI* problem, where k is the dimension of the minimal input separator S^* of $G(\Sigma_\Lambda)$. This means that we do not have a linking of dimension r between F and $Y \cup Z$. The minimal input separator S_c^* of the graph $G(\Sigma_\Lambda^c)$ of the extended system has dimension $k_c < r$.

Denote by μ the dimension of $Z \cap S_c^*$. Define the set $\tilde{S}_c^* = S_c^* \setminus (Z \cap S_c^*)$ of dimension $k_c - \mu$. Prove now that the set \tilde{S}_c^* is a separator of $G(\Sigma_\Lambda)$.

Assume that there exists a $F - Y$ path with no vertex in \tilde{S}_c^* . This path can not have a vertex in Z then this path has no vertex in S_c^* . This contradicts the fact that S_c^* is a separator of $G(\Sigma_\Lambda^c)$ and then any $F - Y$ path has a vertex in \tilde{S}_c^* which is then a separator of $G(\Sigma_\Lambda)$.

Consider now an additional sensor z in $Z \setminus (Z \cap S_c^*)$. Assume that such a sensor measures a vertex in $T_{\tilde{S}_c^*}$. There would exist a path from F to z with no vertex in S_c^* contradicting the fact that S_c^* is a separator of $G(\Sigma_\Lambda^c)$. Then the additional sensors in $Z \setminus (Z \cap S_c^*)$ measure no vertex in $T_{\tilde{S}_c^*}$ and the maximal number of sensors measuring vertices in $T_{\tilde{S}_c^*}$ is μ . The constraint associated with the separator \tilde{S}_c^* tells us that the minimal number of additional sensors which measure vertices in $T_{\tilde{S}_c^*}$ is $r - \dim(\tilde{S}_c^*) = r - (k_c - \mu)$. Since $k_c < r$ this number is greater than μ which leads to a contradiction.

Using Corollary 10 it follows that there exists an input separator for which the constraint is also violated. \square

Example 2: This is a very simple academic example which allows to illustrate the basic concepts and results of the paper. Consider the structured system with 6 faults and 3 outputs whose graph is depicted in Figure 3. One has:

- The separator $\{f_6, y_1, y_2\}$ is the minimal input separator and it is of dimension 3 so the minimal number of additional sensors needed for any solution of the *FDI* problem is $6 - 3 = 3$. From Corollary 7 additional sensors measuring vertices only in the set $\{y_1, y_2, y_3, x_4, f_6\}$ are useless for the sensor location problem.

From now on we consider only minimal solutions. i.e solutions with three additional sensors.

- The input separators of dimension 4 are:

$\{y_1, y_2, f_1, f_6\}$, $\{y_1, y_2, f_2, f_6\}$, $\{y_1, y_2, f_3, f_6\}$, $\{y_1, y_2, f_4, f_6\}$, $\{y_1, y_2, f_5, f_6\}$, $\{y_1, y_2, x_1, f_6\}$. The constraints associated with these separators tell us that two different sensors can not measure only $\{f_1\}$ or $\{f_2\}$ or $\{f_3\}$ or $\{f_4\}$ or $\{f_5\}$ or $\{x_1\}$ or $\{f_4$ and $x_2\}$ or $\{f_5$ and $x_3\}$.

- The input separators of dimension 5 are:

$\{f_1, f_2, f_3, f_6, y_2\}$, $\{f_3, f_4, f_5, f_6, y_1\}$, $\{y_1, y_2, f_1, f_4, f_6\}$, $\{y_1, y_2, f_1, f_5, f_6\}$, $\{y_1, y_2, f_2, f_4, f_6\}$, $\{y_1, y_2, f_2, f_5, f_6\}$. The constraints associated with $\{f_1, f_2, f_3, f_6, y_2\}$, (resp. $\{f_3, f_4, f_5, f_6, y_1\}$) tell us that at least one additional sensor should measure vertices in $\{f_4, f_5, x_2, x_3\}$, (resp. $\{f_1, f_2, x_1\}$).

- The input separator of dimension 6 is:

$\{f_1, f_2, f_3, f_4, f_5, f_6\}$.

We remark also that the constraints associated with the separators: $\{y_1, y_2, f_1, f_4, f_6\}$, $\{y_1, y_2, f_1, f_5, f_6\}$, $\{y_1, y_2, f_2, f_4, f_6\}$, $\{y_1, y_2, f_2, f_5, f_6\}$, are implied by the constraints associated with the previous input separators, and they do not provide more information for the sensor location problem.

5.2 Choice of additional sensors

In practice only some variables (or linear combination of variables) are available for measurement. It follows that for a given system only a limited number of additional sensors is implementable. In this part we will start with a given set of available additional sensors $z(t)$ as in (6). In this case the sensor location problem amounts to choose in this set the sensors to be implemented for solving the *FDI* problem. We will illustrate this on the previous example with a set of potential additional sensors $Z = \{z_1, z_2, z_3, z_4, z_5\}$ that measure vertices in $G(\Sigma_\Lambda)$ as illustrated in Figure 4.

Using the previous analysis we have the following properties:

- First we can check if the *FDI* problem is solvable using Theorem 1 on the graph $G(\Sigma_\Lambda^c)$. The proposed set of additional sensors is a solution for the *FDI* problem, since with these additional sensors we have a $F - \{Y \cup Z\}$ linking of size $r = 6$.

- From Corollary 7 we need at least $r - k$ additional sensors which should measure variables in T_{S^*} . In our example this condition is satisfied since four additional sensors measure variables in T_{S^*} .

- For the *FDI* problem the additional sensors which measure variables only outside T_{S^*} are use-

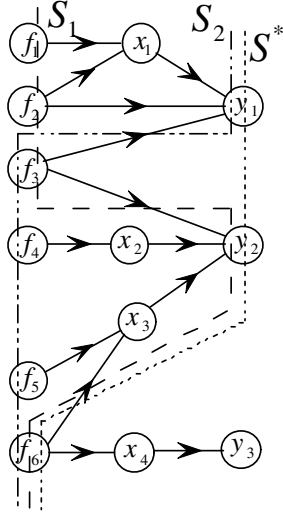


Fig. 3. Graph $G(\Sigma_A)$ of Example 2

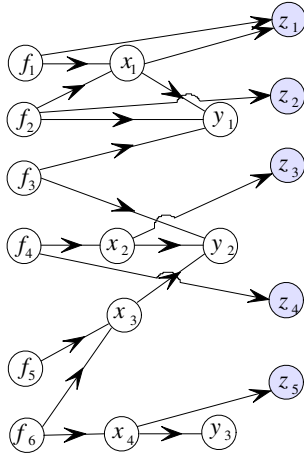


Fig. 4. Graph $G(\Sigma_A^c)$ of Example 2 with 5 additional sensors.

less. The additional sensor z_5 measures only x_4 which does not belong to T_{S^*} , z_5 is then a useless sensor for the *FDI* problem.

- Among the useful additional sensors it turns out that some belong to any solution, they are called essential while others not. In our example the two sensors z_3 and z_4 can not belong to a minimal solution at the same time. This follows from the constraint associated with the separator $\{y_1, y_2, f_4, f_6\}$. It follows that the sensors z_1 and z_2 are essentials.

-The only two minimal solutions with three additional sensors are: $\{z_1, z_2, z_3\}$ and $\{z_1, z_2, z_4\}$.

6. CONCLUDING REMARKS

In this paper we have presented a structural analysis which is well suited for the sensor location in the Fault Detection and Isolation problem (FDI). The proposed analysis is based on the associated system graph and on specific separators. This prior structural analysis allows to determine

the minimal number of needed additional sensors and the sets of internal variables which will be of interest to measure for solving our *FDI* problem. This analysis provides with a complete parametrization of all the solutions for the considered sensor location problem, and allows to choose among a set of potential additional sensors to solve the *FDI* problem.

REFERENCES

- Blanke, M., M. Kinnaert, J. Lunze and M. Staroswiecki (2003). *Diagnosis and fault-tolerant control*. Springer-Verlag.
- Chen, J. and R.J. Patton (1999). *Robust model-based fault diagnosis for dynamic systems*. Kluwer Academic Publishers.
- Commault, C. and J.M. Dion (2006). Approche structurale du diagnostic, application à un modèle de colonne à distiller. *CIFA Bordeaux*.
- Commault, C. and J.M. Dion (2007). Sensor location for diagnosis in linear systems: a structural analysis. *IEEE Transactions on Automatic Control*, To appear.
- Commault, C., J.M. Dion and S. Yacoub Agha (2006). Structural analysis for sensor location in fault detection and isolation. IFAC Safe-Process. Pekin.
- Commault, C., J.M. Dion, O. Sename and R. Morteian (2002). Observer-based fault detection and isolation for structured systems. *IEEE Trans. Automat. Control* **47**(12), 2074–2079.
- Dion, J.M., C. Commault and J. van der Woude (2003). Generic properties and control of linear structured systems: a survey. *Automatica* **39**(7), 1125–1144.
- Frank, P.M. (1996). Analytical and qualitative model-based fault diagnosis - a survey and some new results. *European Journal of Control* **2**, 6–28.
- Hu, T.C. (1982). *Combinatorial Algorithms*. Addison-Wesley publishing company.
- Lin, C.T. (1974). Structural controllability. *IEEE Trans. Automat. Control* **19**, 201–208.
- Meyer, M., J.M. Le Lann, B. Koehret and Enjalbert (1994). Optimal sensor location on a complex plant, using a graph oriented approach. *Computers Chem. Eng.* **18**, 535–540.
- Murota, K. (1987). *Systems Analysis by Graphs and Matroids*. Vol. 3 of *Algorithms and Combinatorics*. Springer-Verlag New-York, Inc.
- Raghuraj, R., M. Bhushan and R. Rengaswamy (1999). Locating sensors in complex chemical plants based on fault diagnosis observability criteria. *AIChE Journal* **45**(2), 310–322.
- van der Woude, J.W. (2000). The generic number of invariant zeros of a structured linear system. *SIAM Journal. of Control* **38**, 1–21.