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The influence of domain geometry in the boundary behavior of large solutions of degenerate elliptic problems.

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Abstract

In this paper we study the asymptotic boundary behavior of large solutions of the equation $\Delta u = d^\alpha u^p$ in a regular bounded domain Ω in \mathbb{R}^N , $N \geq 2$, where $d(x)$ denotes the distance from x to $\partial\Omega$, $p > 1$ and $\alpha > 0$. We precise the expansion which depends on the mean curvature of the boundary.

1 Introduction : notations and main results

Let Ω be a regular bounded domain in \mathbb{R}^N , $N \geq 2$, $p > 1$ and $\alpha > 0$. We denote by $d(x)$ the distance from x to $\partial\Omega$, the boundary of Ω . In this paper we consider the semilinear degenerate equation

$$\Delta u = d^\alpha u^p \quad \text{in } \Omega \quad (1)$$

and we are interesting in the large solutions of (1), that is solutions of (1) which blow up at the boundary :

$$u(x) \rightarrow +\infty \quad \text{as } d(x) \rightarrow 0. \quad (2)$$

Note already that the maximum principle implies that the solutions $u \in C^2(\Omega)$ of (1)-(2) are positive in Ω .

Equation (1) registers in problems of the form

$$\Delta u = p(x)f(u) \quad \text{in } \Omega. \quad (3)$$

Those problems were first studied by Bieberbach [4] for the case $p(x) = 1$, $f(u) = e^u$ and $N = 2$, in the context of Riemannian surfaces of constant negative curvature, and the theory of automorphic functions. The case $p(x) > 0$ for all $x \in \overline{\Omega}$ has been, largely dealt with in the literature (see [7], [12], [8], [5] for example).

Existence of solutions of (1)-(2) was established by Lair and Wood [9]. The question of the uniqueness of solutions of (1)-(2) is more delicate. When $\alpha = 0$ and $p > 1$, it is well know that problem (1)-(2) has a unique solution which satisfies

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$$\lim_{d(x) \rightarrow 0} u(x) d(x)^{\frac{2}{p-1}} = \left(\frac{2(p+1)}{(p-1)^2} \right)^{\frac{1}{p-1}}. \quad (4)$$

This was first established by Loewner and Nirenberg [10] for the case $p = (N+2)/(N-2)$. Later we can find many extensions, see for example [1], [2] and [14] and the references cited there. The case $\alpha < 0$ and $p > 0$ is studied in [6]. In the general case $\alpha \geq 0$, Marcus and Véron proved the uniqueness of the solutions of (1)-(2) under the condition $1 < p < (N+1+\alpha)/(N-1)$. Our first theorem completes this result and gives the rate of the blow-up.

Theorem 1.1 *Let $u \in C^2(\Omega)$ be a solution of (1) – (2) . Then it satisfies*

$$\lim_{d(x) \rightarrow 0} d(x)^{\frac{\alpha+2}{p-1}} u(x) = l \quad (5)$$

where l is given by

$$l = \left[\frac{(\alpha+2)(\alpha+p+1)}{(p-1)^2} \right]^{\frac{1}{p-1}}. \quad (6)$$

This theorem allows us to establish the uniqueness of solutions of (1)-(2) with no conditions on p and α .

Theorem 1.2 *Problem (1) possesses a unique large solution.*

In the second time we are interested in the influence of the geometry of Ω in the boundary behavior. When $\alpha = 0$, this problem was first studied by Bandle and Marcus [3] for the radially symmetric solutions of (1)-(2) in a ball. Later their result was extended by del Pino and Letelier [13] for general solutions. They proved that a lower-order term, still explosive, appears in the expansion of u wich depends linearly of the mean curvature of the boundary of Ω . More precisely, if $1 < p < 3$ and $\alpha = 0$, then on a sufficiently small neighborhood of $\partial\Omega$ we have the expansion

$$u(x) = \left(\frac{2(p+1)}{(p-1)^2} \right)^{\frac{1}{p-1}} d(x)^{-\frac{2}{p-1}} \left\{ 1 + \frac{N+1}{p+3} H(\bar{x}) d(x) + o(d(x)) \right\}. \quad (7)$$

Here, for all x in a neighborhood of $\partial\Omega$, \bar{x} denotes the unique point of the boundary such that $d(x) = |x - \bar{x}|$ and $H(\bar{x})$ the mean curvature of the boundary at that point. Estimate (7) generalizes to our case $\alpha \geq 0$ in the following way.

Theorem 1.3 *Let $u \in C^2(\Omega)$ a large solution of (1). Then, on a sufficiently small neighborhood of $\partial\Omega$:*

$$u(x) = l d(x)^{-\frac{2+\alpha}{p-1}} \left\{ 1 + \frac{N-1}{\alpha+p+3} H(\bar{x}) d(x) + o(d(x)) \right\}. \quad (8)$$

This theorem implies that on a sufficiently small neighborhood of $\partial\Omega$:

$$u(x) - ld(x)^{-\frac{2+\alpha}{p-1}} = \frac{N-1}{\alpha+p+3} H(\bar{x}) d(x)^{-\frac{\alpha+3-p}{p-1}} + o\left(d(x)^{-\frac{\alpha+3-p}{p-1}}\right). \quad (9)$$

Therefore, we obtain that

- if $p > \alpha + 3$, then the first member of (9) tends to 0 at the boundary,
- if $p = \alpha + 3$, then $u(x) - ld(x)^{-\frac{2+\alpha}{p-1}} = \frac{N-1}{\alpha+p+3} H(\bar{x}) + o(1)$,
- if $p < \alpha + 3$, then the first member of (9) is not bounded and the blow-up depends on the mean curvature. Roughly, the "more curved" or "sharper" towards the exterior of Ω is around a given point of $\partial\Omega$, the higher the explosion rate at that point is.

That is a generalization of the results of Bandle and Marcus [3] for the radially symmetric solutions of (1)-(2) in a ball $\Omega = B(0, R)$:

- if $p > 3$, then $u(r) - l(R-r)^{-\frac{2}{p-1}} \rightarrow 0$ when $r \rightarrow R$,
- if $p = 3$, then $u(r) - l(R-r)^{-\frac{2}{p-1}} \rightarrow \frac{C(N)}{R}$ when $r \rightarrow R$, which represents the mean curvature of the ball,
- if $p < 3$, then $u(r) - l(R-r)^{-\frac{2}{p-1}} \rightarrow \infty$ when $r \rightarrow R$.

Our paper is organized as follows :

1. Introduction
2. Asymptotic behavior and uniqueness
3. Boundary influence in the explosion rate.

2 Asymptotic behavior and uniqueness.

We begin this section by proving a classical estimate for all solution u of (1). (see [12]).

Proposition 2.1 : (*Osserman estimate*): *There exist two positive constants $a = a(\partial\Omega)$ and $C = C(\Omega, \alpha, p)$ such that for all solution $u \in C^2(\Omega)$ of equation (1), we have :*

$$u(x) \leq Cd(x)^{-\frac{2+\alpha}{p-1}} \quad (10)$$

for all $x \in \Omega$ such that $d(x) < a$.

Proof : Since Ω is regular there exist $\tilde{a} = \tilde{a}(\Omega) > 0$ and $M = M(\Omega) > 0$ such that

$$|\Delta d(x)| \leq M, \quad |\nabla d(x)| = 1 \quad (11)$$

for all $x \in \Omega$ such that $d(x) < \tilde{a}$. Set $a = \min(1, \frac{\tilde{a}}{2})$. Let $x_0 \in \Omega$ such that $d(x_0) < a$ and $r_0 = d(x_0)/2$. We denote by B_0 the ball centered at x_0 of radius r_0 and we define the function w in B_0 as follows :

$$w(x) = \lambda d(x)^{-\frac{\alpha}{p-1}} (r_0^2 - |x - x_0|^2)^{-\frac{2}{p-1}} \quad (12)$$

with $\lambda > 0$ to determine such that

$$-\Delta w + d^\alpha w^p \geq 0 \quad \text{in } B_0. \quad (13)$$

A straightforward computation gives :

$$\begin{aligned}
-\Delta w + d^\alpha w^p &= \lambda d^{-\frac{\alpha}{p-1}} (r_0^2 - |x - x_0|^2)^{-\frac{2p}{p-1}} \times \\
&\left[-\frac{\alpha(\alpha + p - 1)}{(p - 1)^2} (r_0^2 - |x - x_0|^2)^2 d^{-2} + \frac{\alpha}{p - 1} (r_0^2 - |x - x_0|^2)^2 d^{-1} \Delta d \right. \\
&\quad + \frac{8\alpha}{(p - 1)^2} (r_0^2 - |x - x_0|^2) d^{-1} \nabla d \cdot (x - x_0) - \frac{8(p + 1)}{(p - 1)^2} |x - x_0|^2 \\
&\quad \left. - \frac{4N}{p - 1} (r_0^2 - |x - x_0|^2) + \lambda^{p-1} \right].
\end{aligned}$$

Since $|x - x_0| < d(x_0) \leq 1$, $d(x) \geq d(x_0)/2$ and $r_0^3 < r_0^2$, there exists a constant $L = L(\alpha, p, M) > 0$ such that

$$-\Delta w + d^\alpha w^p \geq \lambda d^{-\frac{\alpha}{p-1}} (r_0^2 - |x - x_0|^2)^{-\frac{2p}{p-1}} (-Lr_0^2 + \lambda^{p-1})$$

in B_0 . Therefore, we choose $\lambda = L^{\frac{1}{p-1}} r_0^{\frac{2}{p-1}}$ and we obtain (13). Note that $w(x) = +\infty$ if $x \in \partial B_0$ because $-2/(p - 1) < 0$. The comparison principle implies $u \leq w$ in B_0 and in particular

$$u(x_0) \leq w(x_0) = L^{\frac{1}{p-1}} \left(\frac{d(x_0)}{2} \right)^{\frac{2}{p-1}} (d(x_0))^{-\frac{\alpha}{p-1}} \left(\frac{d(x_0)}{2} \right)^{-\frac{4}{p-1}}$$

which gives inequality (10).

We now establish an estimate from below for the solutions of (1)-(2). The results of [1] and [2] can't be used because the distance function d is not positive in $\bar{\Omega}$. Nevertheless we can adapt them as follows.

Proposition 2.2 *Let $u \in C^2(\Omega)$ be a solution of (1) – (2). Then*

$$\liminf_{d(x) \rightarrow 0} d(x)^{\frac{\alpha+2}{p-1}} u(x) \geq l \tag{14}$$

where l is defined in (6).

Proof : Let $\varepsilon > 0$, \tilde{a} be as the proof of proposition 2.1 and $\beta \in (0, 1)$. We define

$$\underline{u}(x) = \beta l ((d(x) + \varepsilon)^{-\frac{\alpha+2}{p-1}} - (\bar{a} + \varepsilon)^{-\frac{\alpha+2}{p-1}})$$

where \bar{a} will be determined such that $\bar{a} < \tilde{a}$. We have $\underline{u} > 0$ on $\partial\Omega$ and $\underline{u}(x) = 0$ for all x such that $d(x) = \bar{a}$. Moreover a straightforward computation yields

$$-\Delta \underline{u} + d^\alpha \underline{u}^p = \beta \left[\Delta d \left(\frac{\alpha + 2}{p - 1} \right) l (d + \varepsilon)^{-\frac{\alpha+p+1}{p-1}} - l^p (d + \varepsilon)^{-\frac{\alpha+2p}{p-1}} \right]$$

$$+d^\alpha \beta^{p-1} l^p \left[(d + \varepsilon)^{-\frac{\alpha+2}{p-1}} - (\bar{a} + \varepsilon)^{-\frac{\alpha+2}{p-1}} \right]^p$$

in $0 < d(x) < \bar{a}$. Using inequality (11), we obtain

$$\begin{aligned} -\Delta \underline{u} + d^\alpha \underline{u}^p &\leq \beta l^p (d + \varepsilon)^{-\frac{\alpha+2p}{p-1}} \left[M \left(\frac{\alpha+2}{p-1} \right) l^{1-p} (d + \varepsilon) \right. \\ &\quad \left. - 1 + \beta^{p-1} \left(\frac{d}{d + \varepsilon} \right)^\alpha \right] \end{aligned}$$

which implies

$$-\Delta \underline{u} + d^\alpha \underline{u}^p \leq \beta l^p (d + \varepsilon)^{-\frac{\alpha+2p}{p-1}} [\bar{M}(d + \varepsilon) - (1 - \beta^{p-1})]$$

with $\bar{M} = M \left(\frac{\alpha+2}{p-1} \right) l^{1-p}$. We now choose $\bar{a} = \frac{1}{2} \min(\tilde{a}, \frac{1-\beta^{p-1}}{M})$ and impose $\varepsilon < \frac{1}{2} \left(\frac{1-\beta^{p-1}}{M} \right)$.

Then \bar{u} is a subsolution of (1) in $0 < d(x) < \bar{a}$. By the maximum principle we derive $\underline{u} \leq u$ in $0 < d(x) < \bar{a}$. Letting ε tend to 0, this implies for all $\beta \in (0, 1)$ and x such that $d(x) < \bar{a}$:

$$\beta l \left[1 - \left(\frac{d(x)}{\bar{a}} \right)^{\frac{\alpha+2}{p-1}} \right] \leq d(x)^{\frac{\alpha+2}{p-1}} u(x).$$

Therefore for all $\beta \in (0, 1)$:

$$\beta l \leq \liminf_{d(x) \rightarrow 0} d(x)^{\frac{\alpha+2}{p-1}} u(x)$$

which ends the proof.

Because of proposition 2.2, we can describe the asymptotic behavior of radially symmetric solutions of (1)-(2).

Proposition 2.3 *Let $R > 0$ and $v \in C^2(0, R)$ a solution of*

$$-v'' - \frac{N-1}{r} v' + (R-r)^\alpha v^p = 0 \tag{15}$$

in $(0, R)$ such that

$$\lim_{r \rightarrow R} v(r) = +\infty.$$

Then

$$\lim_{r \rightarrow R} (R-r)^{\frac{\alpha+2}{p-1}} v(r) = l \tag{16}$$

where l is defined in (6).

We omit the proof of this proposition because it follows the idea of [14]: the function $w(t) = (R-r)^{\frac{\alpha+2}{p-1}}v(r)$ with $R-r = e^{-t}$ is bounded and satisfies a second order differential equation in a neighborhood of infinity and the ω -limit set of a trajectory of that equation is $\{0\}$ or $\{l\}$. Therefore proposition 2.2 implies proposition 2.3. Those results allows us to prove theorem 1.1.

Proof of theorem 1.1 : In view of (14) we must only prove that

$$\limsup_{d(x) \rightarrow 0} d(x)^{\frac{\alpha+2}{p-1}} u(x) \leq l. \quad (17)$$

Still the results of [1], [2] or [14] don't apply directly but we can adapt them. Let $y \in \partial\Omega$. Since $\partial\Omega$ is smooth, there exists a ball B_y centered at a point Y of radius R_y such that $B_y \subset \Omega$ and $\overline{B_y} \cap \partial\Omega = \{y\}$. We introduce the function V defined by $V(x) = v(|x|)$ for all $x \in B_{R_y}$ where v is a function as in proposition 2.3 with $R = R_y$. The function v exists because it is the radial solution of (1)-(2) for $\Omega = B$ (see [9]). Let $k > 1$. Finally we introduce the function V_k defined by $V_k(x) = k^{\frac{2}{p-1}}V(k(x-Y))$ for all $x \in B(Y, \frac{R_y}{k})$. Note that $B(Y, \frac{R_y}{k}) \subset B_y$ and V_k is solution of

$$-\Delta V_k + (R_y - k|x-Y|)^{\alpha} V_k^p = 0$$

in $B(y, \frac{R_y}{k})$ and satisfies

$$\lim_{|x-Y| \rightarrow \frac{R_y}{k}} V_k(x) = +\infty.$$

Since $x \in B(Y, \frac{R_y}{k})$ implies $d(x) \geq R_y - |x-Y| \geq R_y - k|x-Y|$, the comparison principle involves $u \leq V_k$ in $B(Y, \frac{R_y}{k})$. Letting k tend to 1, we obtain

$$u(x) \leq v(|x-Y|) \quad \text{in } B_y. \quad (18)$$

Because of proposition 2.3, for all $\varepsilon > 0$ there exists $\eta > 0$ such that

$$|s^{\frac{\alpha+2}{p-1}}v(R_y - s) - l| < \varepsilon \quad \forall s \in (0, \eta). \quad (19)$$

Let $\tilde{\eta} > 0$ be sufficiently small so that for all $x \in \Omega$ with $d(x) < \tilde{\eta}$ there exists a unique $y \in \partial\Omega$ such that $|x-y| = d(x)$. Then for all $x \in \Omega$ such that $d(x) < \min(\eta, \tilde{\eta})$, both inequalities (18) and (19) imply

$$d(x)^{\frac{\alpha+2}{p-1}} u(x) \leq d(x)^{\frac{\alpha+2}{p-1}} v(R_y - d(x)) < l + \varepsilon$$

and inequality (17) holds.

Proof of theorem 1.2 : Large solutions of (1) satisfy (5). Then two large solutions u_1 and u_2 of (1) are such that

$$\lim_{d(x) \rightarrow 0} \frac{u_1(x)}{u_2(x)} = 1.$$

and the result follows as in [1] or [11].

3 Boundary influence in the explosion rate.

In this section we prove theorem 1.3. As in [13] we construct suitable sub- and supersolutions of (1) in a neighborhood of $\partial\Omega$ which are inspired of the radial study that we omit here.

Since Ω is regular there exists $\bar{b} > 0$ such that d is a function of class C^2 in $\{x \in \Omega / d(x) < \bar{b}\}$, $|\nabla d(x)| = 1$ and

$$\Delta d(x) = -(N-1)H(\bar{x}) + o(1) \quad \text{as } d(x) \rightarrow 0. \quad (20)$$

Let $b_0 \in (0, \bar{b})$, $b \in (0, b_0)$ and $\varepsilon > 0$. We introduce the function Ψ defined in $E_{b,b_0} = \{x \in \Omega / b < d(x) < b_0\}$ by

$$\Psi(x) = l(d(x) - b)^{-\frac{\alpha+2}{p-1}} + \frac{l}{\alpha + p + 3} ((N-1)H(\bar{x}) + \varepsilon)(d(x) - b)^{-\frac{\alpha+3-p}{p-1}}.$$

We claim that if b_0 is chosen sufficiently small, independently of ε and b , then Ψ is a supersolution in E_{b,b_0} . Indeed, a straightforward computation using (20) gives :

$$\begin{aligned} \Delta \Psi &= l^p (d(x) - b)^{-\frac{\alpha+2p}{p-1}} + l(d(x) - b)^{-\frac{\alpha+p+1}{p-1}} \left[\frac{\alpha+2}{p-1} (N-1)H(\bar{x}) \right. \\ &\quad \left. + \frac{(\alpha+3-p)(\alpha+2)}{(\alpha+p+3)(p-1)^2} ((N-1)H(\bar{x}) + \varepsilon) + o(1) \right. \\ &\quad \left. + \frac{\alpha+3-p}{(p-1)(\alpha+p+3)} ((N-1)H(\bar{x}) + \varepsilon)((N-1)H(\bar{x}) + o(1))(d(x) - b) \right]. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} d(x)^\alpha \Psi^p &\geq (d(x) - b)^\alpha \Psi^p \\ &\geq l^p (d(x) - b)^{-\frac{\alpha+2p}{p-1}} \left[1 + \frac{p}{\alpha+p+3} ((N-1)H(\bar{x}) + \varepsilon)(d(x) - b) + o(d(x) - b) \right]. \end{aligned}$$

Then

$$\begin{aligned} -\Delta \Psi + d^\alpha \Psi^p &\geq l(d(x) - b)^{-\frac{\alpha+p+1}{p-1}} \times \\ &\quad \left[-\frac{\alpha+2}{p-1} (N-1)H(\bar{x}) - \frac{(\alpha+3-p)(\alpha+2)}{(\alpha+p+3)(p-1)^2} ((N-1)H(\bar{x}) + \varepsilon) \right. \\ &\quad \left. - \frac{\alpha+3-p}{(p-1)(\alpha+p+3)} ((N-1)H(\bar{x}) + \varepsilon)((N-1)H(\bar{x}) + o(1))(d(x) - b) \right. \\ &\quad \left. + \frac{l^{p-1}p}{\alpha+p+3} ((N-1)H(\bar{x}) + \varepsilon) + o(1) \right] \end{aligned}$$

Since

$$-\frac{\alpha+2}{p-1} - \frac{(\alpha+3-p)(\alpha+2)}{(\alpha+p+3)(p-1)^2} + \frac{l^{p-1}p}{\alpha+p+3} = 0,$$

and since the coefficient of ε is $(\alpha+2)/(p-1)$, it implies that there exists $b_0 = b_0(\varepsilon) \in (0, \bar{b})$ such that for all $0 < b < b_0$:

$$-\Delta\Psi + d^\alpha\Psi^p \geq 0 \quad \text{in } E_{b,b_0}.$$

Consider the solution u of (1)-(2). We claim that there exists a positive number K independent of $b \in (0, b_0)$ such that :

$$\Psi(x) + K \geq u(x) \tag{21}$$

for all $x \in \Omega$ with $d(x) = b_0$. In fact, if we define

$$M_0 = \max_{d(x)=b_0} u(x),$$

we can compute for all x such that $d(x) = b_0$:

$$\Psi(x) = l(b_0 - b)^{-\frac{\alpha+2}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\bar{x}) + \varepsilon)(b_0 - b)^{-\frac{\alpha+3-p}{p-1}}.$$

Since $\partial\Omega$ is regular, there exists a real $b_1 \in (0, b_0)$ such that

$$\left| \frac{1}{\alpha+p+3}((N-1)H(\bar{x}) + \varepsilon)(b_0 - b) \right| \leq \frac{1}{2}$$

for all $b \in (b_1, b_0)$, where \bar{x} is such that $d(x) = |x - \bar{x}|$. Therefore

$$1 + \frac{1}{\alpha+p+3}((N-1)H(\bar{x}) + \varepsilon)(b_0 - b) \geq \frac{1}{2}$$

and then

$$\Psi(x) \geq \frac{l}{2}(b_0 - b)^{-\frac{\alpha+2}{p-1}} \geq \frac{l}{2}(b_0 - b_1)^{-\frac{\alpha+2}{p-1}}$$

for all $b \in (b_1, b_0)$, where \bar{x} is such that $d(x) = |x - \bar{x}|$. On the other hand, for all $b \in (0, b_1]$ and $d(x) = b_0$:

$$\begin{aligned} \Psi(x) &= l(b_0 - b)^{-\frac{\alpha+2}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\bar{x}) + \varepsilon)(b_0 - b)^{-\frac{\alpha+3-p}{p-1}} \\ &\geq lb_0^{-\frac{\alpha+2}{p-1}} - C(b_0 - b_1)^{-\frac{\alpha+3-p}{p-1}} \end{aligned}$$

with $C > 0$ and because the assumption if we assume $\alpha + 3 - p > 0$ (we omit the proof in the case $\alpha + 3 - p \leq 0$ which is simpler). Finally we obtain for all $b \in (0, b_0)$:

$$\Psi(x) \geq L = \min \left(\frac{l}{2}(b_0 - b_1)^{-\frac{\alpha+2}{p-1}}, lb_0^{-\frac{\alpha+2}{p-1}} - C(b_0 - b_1)^{-\frac{\alpha+3-p}{p-1}} \right),$$

then, for all x such that $d(x) = b_0$, $u \leq M_0 \leq \max(1, M_0 - L) + L \leq \max(1, M_0 - L) + \psi$ which implies (21).

On the other hand the function $\Psi + K$ is itself a supersolution of equation (1) in E_{b,b_0} . Therefore the comparison principle implies (21) in E_{b,b_0} . Letting b tend to 0, we obtain

$$u(x) \leq ld(x)^{-\frac{\alpha+2}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\bar{x}) + \varepsilon)d(x)^{-\frac{\alpha+3-p}{p-1}} + K$$

for all $x \in \Omega$ such that $0 < d(x) < b_0$. In the same way, by considering subsolutions in the form

$$\phi(x) = l(d(x) + b)^{-\frac{2+\alpha}{p-1}} + \frac{l}{\alpha+p+3}((N-1)H(\bar{x}) - \varepsilon)(d(x) + b)^{-\frac{\alpha+3-p}{p-1}} - \bar{K}$$

we obtain expansion (8).

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