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A study of specimen thickness effects in the impact tests on polymers by numeric simulations

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Experimental data of polymer behaviour at high strain rates in the literature, mostly obtained with a split Hopkinson pressure bar (SHPB), shows a dependence on the thickness of the specimen. This paper investigates this so-called thickness effect and tries to clarify the doubts on the equilibrium assumption in SHPB tests on polymers by numerical simulations. Fictitious specimens of different thickness with rate sensitive polymer-like constitutive model (Eyring's model) are used to simulate basic data of SHPB tests. The comparison between the given behaviours and those derived from the simulated tests provides an objective appreciation of the quality of tests on polymers (with thin or thick specimen). Results of the simulation prove that the equilibrium assumptions can still be safely used in the case of tests on polymers, and indicate that the thickness effect is mainly due to the radial inertia and friction considerations.

INTRODUCTION

The mechanical behaviour of polymers at high strain rates is often involved in safety studies of engineering applications. For example, in the domain of pipelines for the distribution of fuel gas, the catastrophic failure of the polymeric pipes by a very fast crack propagation, even rare, has been observed under accidental impacts. Recent analyses of tests on real scale pipes allow for the establishment of an empirical critical pressure for the initiation of crack propagation. However, to obtain a theoretical criterion or to perform a numerical simulation, the constitutive law of the materials, of which the pipes are made, is indispensable.

A few experimental data of the polymeric behaviour at high strain rates have been reported in the literature, obtained with a split Hopkinson pressure bar (SHPB). Some authors used a very thin specimen (< 2 mm) and gave a quasi-linear relation between the yield stress with the logarithm of the strain rate so that Eyring's model (see also Section 2.2) is still valid at high strain rates. By contrast, the others used a thicker specimen and found that there is a sharp increase in the yield stress at high strain rates. Such a so-called thickness effect is observed for many polymeric materials.

Recently, Dioh and co-workers have systematically investigated this thickness effect and used specimens of different thickness (1.5 mm and 4.4 mm) but of exactly the same materials. For all the materials studied, such as medium density polyethylene (MDPE), high density polyethylene (HDPE), polycarbonate (PC), and polyether ether ketone (PEEK), tests on specimens of different thickness do lead to different rate sensitivities. Results are schematically re-drawn in Figure 1 where a sharp increase is found for tests on thick specimens.

Such results suggest not only that there exists some experimental imprecision in the classical SHPB analysis where simplifying assumptions are made, but also that one of experimental data and the corresponding behaviour modelling (why not both two), is not accurate. Dioh and co-workers supposed that the sharp increase observed using thick specimen is not a physical feature but an experimental imprecision which could be probably explained by the wave propagation effects (see Section 2) involved in the SHPB tests.

It is then of interest to examine the SHPB testing process in order to know if the classical SHPB analysis leads to acceptable results in the case of tests on polymers and to find which assumption leads to this thickness effect. For this purpose, this paper presents a numeric simulation of SHPB testing process and the author believes that it is a reliable way to appreciate objectively the two kinds of results obtained from specimens of different thickness. The basic SHPB testing data is simulated for fictitious specimens of different thickness, the rate sensitive behaviour of which is described by Eyring's model. Using the classical equilibrium assumption, the average stress–strain curve of the fictitious specimen can be calculated from those simulated basic data. The quality of the measurement in both cases (thin or thick specimen) can be then estimated by the comparison between the given and the recovered material behaviour. In addition, such a simulation allows also for an appreciation of the equilibrium assumption in SHPB tests on polymers.

SIMULATION OF SHPB TESTS ON POLYMER

Split Hopkinson pressure bar tests and its assumptions

The SHPB, or Kolsky's apparatus is a standard experimental technique in the study of the constitutive law of materials at high strain rates. A typical SHPB set-up is composed of the long input and output bars with a short specimen placed between them. The impact of the projectile
at the free end of the input bar develops a compressive longitudinal incident wave \( e_i(t) \). Once it arrives at the bar specimen interface, a reflected wave \( e_r(t) \) is developed in the input bar, whereas a transmitted wave \( e_t(t) \) is developed in the output bar (Figure 2).

From those basic experimental data (incident, reflected and transmitted waves), the classical analysis assumes homogeneous stress and strain fields in the specimen, and an average strain and an average stress are then given by equation (1).

\[
e_i(t) = \frac{2C_0}{l_e}e_f(t) \tag{1}
\]

\[
s_a(t) = \frac{E_bS_b}{S_s}e_f(t)
\]

where \( l_e \) and \( S_s \) are the length and the cross-sectional area of the specimen, whereas \( E_b \) and \( S_s \) and \( C_0 \) denote Young’s modulus, cross-sectional area and wave speed of the bar.

Nevertheless, equation (1) is correct only when the stress and strain fields in the specimen are effectively homogeneous, and this is never true because of wave propagation effects in the specimen. For example, when the input side of the specimen is loaded, the output side remains at rest until the wave goes through the specimen. Such an effect is obviously more important for a thick specimen. However, theoretical\textsuperscript{13}, experimental\textsuperscript{14} and numerical\textsuperscript{15} investigations have shown that the equilibrium assumptions, even approximate, do not introduce significant errors in the case of tests on metals. By contrast, for material such as polymeric foam, concrete, rocks, etc., where homogeneous stress and strain fields cannot be assumed, only an approximate, do not introduce significant errors in the constitutive law are written as follows:

\[
\frac{\partial \sigma(x, t)}{\partial x} = \rho \frac{\partial \varepsilon(x, t)}{\partial t} \tag{2}
\]

\[
\frac{\partial \varepsilon(x, t)}{\partial t} = \frac{\partial v(x, t)}{\partial x}
\]

\[
f(\sigma, \varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}, \ldots) = 0
\]

where \( \sigma, \varepsilon, \dot{\varepsilon} \) are the stress, the strain and the particle velocity in the specimen. \( \rho \) is the mass density.

The boundary conditions at the two faces of the specimen are given as follows:

\[
\sigma(x, t) - \frac{E_bS_b}{C_0S_s}v(x, t) = 2\frac{S_b}{S_s}E_b\varepsilon_f(t) \text{ at the input side} \tag{3}
\]

\[
\sigma(x, t) + \frac{E_bS_b}{C_0S_s}v(x, t) = 0 \text{ at the output side}
\]

The quality of the conclusion derived from the simulated tests depends on how well the constitutive law used in those simulations describes the reality. On the basis of the thermally activated mechanism, Eyring\textsuperscript{6} has proposed a model describing the yield stress \( \sigma_y \) as a function of strain rate and temperature (equation (4)).

\[
\sigma_y = A \left[ \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) + \frac{Q}{RT} \right] \text{ or } \dot{\varepsilon} = \dot{\varepsilon}_0 \exp \left( \frac{\sigma_y}{TA} - \frac{Q}{RT} \right) \tag{4}
\]

where \( A, Q, R \) are constant coefficients, and \( T \) is the absolute temperature.

It describes rather accurately the rate sensitivity of polymer at low and medium strain rate\textsuperscript{4,5,7-9}, and one of the aims in the study on thickness effects in the literature.
Numerical scheme with the method of characteristics

An elastic-plastic model with rate sensitivity of yield stress described by Eyring’s model can be constructed and expressed in the form of a Sokolovsky–Malvern type\textsuperscript{23,24} (equation (5)).

\[ \dot{\sigma} = E\dot{\varepsilon}_d \text{ if } \sigma \leq \sigma_0 \]  
\[ \dot{\sigma} = E(\dot{\varepsilon}_e + \dot{\varepsilon}_{inel}) \text{ if } \sigma > \sigma_0 \]  

where \( \sigma_0 \) is the quasi-static yield stress.

Using the Eyring model (equation (4)) to describe the non-elastic part, we have

\[ \dot{\varepsilon}_{inel} = \dot{\varepsilon}_0 \exp \left( \frac{\sigma}{\sigma_0 + B(e - a/\dot{\varepsilon}_p)} - 1 \right) \]  

The numeric problem defined by the governing equation (equation (2)) and the above constitutive law can be resolved easily by the method of characteristics, because the characteristic network is composed of families of straight lines in this case. The three families of characteristic lines and the characteristic relation that must be satisfied along those lines are defined\textsuperscript{25} in equation (7).

\[
\begin{align*}
(i) \frac{dx}{dt} &= C_0 \quad d\sigma = -\rho C_0 \frac{dv}{dr} - \rho C_0^2 \dot{\varepsilon}_{inel} dt \\
(ii) \frac{dv}{dt} &= -C_0 \quad d\sigma = \rho C_0 \frac{dv}{dr} - \rho C_0^2 \dot{\varepsilon}_{inel} dt \\
(iii) \frac{dx}{dt} &= 0 \quad d\varepsilon_{inel} = \dot{\varepsilon}_{inel} dt
\end{align*}
\]

Using a regular discretization grid, the governing equations (equation (7)) with the boundary conditions (equation (4)) are numerically integrated.

TESTS AND THEIR SIMULATIONS

In order to show the accuracy of the numerical simulation and the describing quality of this model, a set of real SHPB tests on PC at a high strain rate is performed and served as the basis for the comparison test-simulation.

Using the real measured incident wave as input data, the reflected and transmitted waves can be calculated for a fictitious specimen which has the same size as the real one and has the mechanical behaviour described by the model presented above with identified parameters. Figure 3 shows basic measured waves for a test at a strain rate of about 280 s\textsuperscript{-1} (specimen of PC, diameter 10 mm, length 10 mm), compared with the simulated ones. They are very close, and it proves the confidence of the numerical accuracy. Figure 4 shows a comparison (for a set of tests from 230 s\textsuperscript{-1} to 1650 s\textsuperscript{-1}, specimen of PC, diameter 10 mm, length 10 mm) between the stress–strain curves calculated from real measured waves (bold lines) and those from the simulated waves. The describing quality of the model is quite acceptable.

Furthermore, the stress–strain curves calculated from the simulated tests are also very close to the given model at corresponding strain rates. It suggests that the equilibrium assumption is still valid in the case of tests on polymers. It is noted that those real tests (Figure 4) are just the illustrations of the pertinence of the rate sensitive model used for simulations. This constitutive model is not limited in this range of strain rates. Simulations for tests at higher strain rates (up to 10\textsuperscript{4} s\textsuperscript{-1}, cf. Dioh et al.\textsuperscript{9}) lead to similar conclusions.

DISCUSSION ON THICKNESS EFFECTS

The confidence on the simulation and on the model allows, therefore, for an objective examination of the thickness effect. Fictitious tests are then simulated for a 15.8 mm aluminium SHPB set-up, as used by Dioh and co-workers\textsuperscript{9}, where the thickness effects are experimentally proved. The used incident wave lasts 450 \( \mu \)s with a rising time of 15 \( \mu \)s, which corresponds to a real SHPB incident time. Tests on PC with two types of specimen (diameter 12.7 mm, thickness 1.5 and 4.4 mm, as used by Dioh and co-workers\textsuperscript{9}) are simulated at several impact velocities (the strain rates range is from 10\textsuperscript{3} s\textsuperscript{-1} to 10\textsuperscript{4} s\textsuperscript{-1}). It is noted that the rising time and specimen/bar impedance ratio also have their effects on the equilibrium process in the specimen. The choice of a constant rising time (experimental observation in our tests shows very little variation) and of the same diameter for two kinds of specimen in this simulation is motivated by those considerations.

The stress–strain curves are calculated from simulated basic waves, using classical SHPB analysis. Figure 5 shows the rate sensitivities of the flow stress (at a strain of 10\%\textsuperscript{10}) derived from those two sets of simulated tests, compared with the given rate sensitivity. There is then no apparent
Experimental data reported in the literature. A rate sensitive (Eyring’s model) polymer-like constitutive relation is used in all simulations which are performed by the method of characteristics. It is concluded that the equilibrium assumption is still valid for this type of constitutive law. It can be then a priori safely used in the case of tests on many polymers. It is also shown that there are no thickness effects in the simulated tests. It may suggest that the thickness effect is mainly related to the three-dimensional effects in the SHPB test.

**REFERENCES**