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# A constitutive model for metals over a large range of strain rates Identification for mild-steel and aluminium sheets

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A phenomenological model is presented to describe the mechanical behaviour for metals and alloys over a large range of strain rates. It is an elasto-plastic model with a strain rate and temperature-sensitive yield stress. This model partially relies on physical considerations and is specially developed for an easy application in explicit finite element method (FEM) codes. The process for identifying the constants from experimental data is presented, taking into account the exact testing conditions such as temperature increase and strain rate variations during the loading. Applying the model to data obtained for mild steel and commercial aluminium sheets yields satisfactory results.

*Keywords:* Alloys; Finite element method; Metals; Strain rate

## 1. Introduction

Numerical simulation with an explicit finite element method (FEM) code is nowadays a powerful tool for design under dynamic loading conditions. For example, crash simulations in the automotive or rail industries have become commonly used tools. Recent developments in computer technology permit the use of fine meshes in real industrial numerical models, within a reasonable calculation cost, so that the accuracy of material behaviours (especially its rate-sensitive part) becomes important in those applications.

For materials such as metals and alloys, a great number of models have been proposed in past decades but only a few of them have been effectively used in commercial explicit FEM codes [1]. One reason is that most of the models are quite complicated and involve a great number of parameters. The other one is that the significance of each parameter is not so clear and its identification from some simple macroscopic tests is difficult. This is the case of so-called physical models which intend to describe the behaviour of metallic materials using microscopic parameters such as the mean dislocation density. A recent review in the domain is given by Klepaczko [2].

Models used in numerical applications are mostly of the so-called phenomenological type, based on macro-

scopic analysis of experimental data. The incremental elasto-viscoplastic model proposed by Sokolovsky [3] and Malvern [4], generalised to the three-dimensional case by Perzyna [5], and also the unified viscoplastic model of Bodner and Partom [6], have been only used in some implicit FEM codes [1].

The most simple and robust elasto-plastic models with rate-sensitive flow stresses are often used in explicit codes [1]. It is nevertheless observed that existing models of this type are too simplified or too specialised so that it is interesting to develop a better adapted model. The aim of this paper is to propose an improved elasto-plastic model which is valid over a large range of strain rates ( $10^{-4}$  to  $10^4$  s $^{-1}$ ). The simplicity, the robustness, the physical significance and also the easy identification of parameters are considered in modelling. The precision of the parameter identification is indeed as important as the theoretical quality of the model itself, especially when the dynamic testing data is involved because the loading condition is complex. An identification method of the parameters of this model from simple experimental data is then presented, taking account of the testing condition of dynamic tests such as the non-homogeneous fields of stress and strain in the specimen, variation of the strain rate during the test and the temperature increase due to quasi-adiabatic conditions.

## 2. A rate-sensitive elasto-plastic model

Elasto-plastic models such as those proposed by Johnson and Cook [7], Zerilli and Armstrong [8], and Steinburg and Lund [9] are often available in explicit codes. The models of Zerilli and Armstrong or Steinburg are more specialised in military applications in the case of very high strain rates, high temperature and high hydropressure. The most frequently used model in civil applications is that of Johnson and Cook, giving a linear increase of the flow stress with the logarithm of the strain rates (Eq. (1)).

$$\sigma = (A + B\epsilon_p^n) \left( 1 + C \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) \left( 1 - \left[ \frac{T - T_{\text{room}}}{T_{\text{melt}} - T_{\text{room}}} \right]^m \right) \quad (1)$$

where  $A$ ,  $B$ ,  $C$ ,  $n$ ,  $\dot{\epsilon}_0$  and  $m$  are constant coefficients.

Fig. 1 illustrates two sets of experimental data showing variations of flow stress at strains of respectively 0.5 and 10% for mild steel sheets used in the automotive industry. This experimental data is obtained using a split Hopkinson pressure bar set-up and a special sandwich specimen [10]. The linear approximation (for a given strain level) is quite correct within two distinct zones of strain rates (high strain rates or quasi-static;  $\dot{\epsilon} > 100 \text{ s}^{-1}$  or  $\dot{\epsilon} < 1 \text{ s}^{-1}$ ). However, strain rates involved in crash studies are mostly situated in the intermediate zone so that a linear relation is not well adapted. Furthermore, a significant decrease of the rate sensitivity with the increase of strain is observed and this is not described by the Johnson-Cook model.

In order to overcome those two disadvantages and to improve the elasto-plastic model, a Ludwig-type model with coefficients depending on strain rates is proposed in the literature [11]. The temperature term is reduced to a linear form for simplicity. It is rather acceptable because the temperature variation range is quite limited (100°C around the room temperature) in real applications of civil security problems like car crashes:

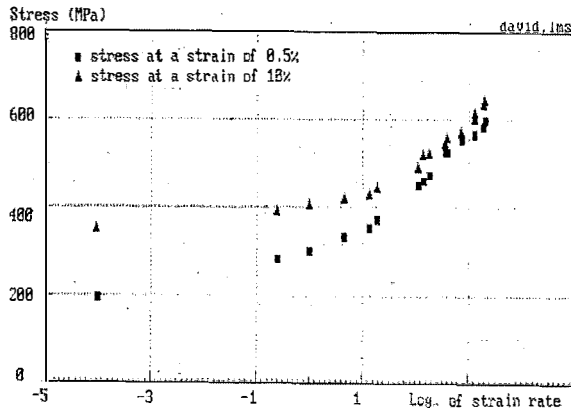


Fig. 1. Flow stresses variation with respect to strain rates from  $10^{-4}$  to  $10^4 \text{ s}^{-1}$ .

$$\sigma = (A(\dot{\epsilon}) + B(\dot{\epsilon})\epsilon_p^{n(\dot{\epsilon})})(1 - \mu\Delta T) \quad (2)$$

Coefficients  $A(\dot{\epsilon})$ ,  $B(\dot{\epsilon})$  and  $n(\dot{\epsilon})$  are functions depending on the strain rate.

The author has also used such a model with three parameters for functions  $A(\dot{\epsilon})$ ,  $B(\dot{\epsilon})$  and  $n(\dot{\epsilon})$  [10]. For instance, the function  $B(\dot{\epsilon})$  has been chosen as following:

$$B(\dot{\epsilon}) = b_1 + b_2 \log(\dot{\epsilon}/\dot{\epsilon}_0) + b_3(\dot{\epsilon})^{0.5} \quad (3)$$

where  $b_1$ ,  $b_2$ ,  $b_3$  are parameters.

The material behaviour of studied sheet metals is indeed more accurately described by this model than that of Johnson-Cook. However, such a model needs more than one parameter to describe one characteristic of material behaviours. For example, to describe the rate sensitivity of the stress associated with a strain of 0.5%, nearly all the parameters are involved and the significance of each parameter is not clearly determined. Consequently, it is difficult to identify the parameters of the model and to generalise the model to other metals and alloys without a large set of experimental data. Pure phenomenological models show here their limitation and an improvement by introducing some physical considerations is necessary.

Indeed, looking for a physical basis for the simple phenomenological model, we find that the proposed formulation in Ref. [10] could be arranged in another way so that it becomes close to the physical model proposed by Tanimura [12]. This model is based on the work of Campbell and Ferguson [13] where they indicate that rate and temperature sensitivities of flow stresses can be divided into a few zones. Similar behaviours are observed in each zone for most of metals and alloys. For example, the flow stress increases slightly with the strain rates in zone II where the thermally activated flow mechanism is observed. A sharp increase of flow stress is found in zone IV where the viscous drag is dominant. Following this concept, Tanimura [12] has proposed to model material behaviour of metals and alloys by the additive combination of a quasi-static part  $\sigma_s(\epsilon_p, T)$ , a thermally activated flow part  $\sigma_t(\epsilon_p, \dot{\epsilon}_p, T)$  and a viscous drag part  $\sigma_v(\epsilon_p, \dot{\epsilon}_p, T)$ :

$$\sigma = \sigma_s(\epsilon_p, T) + \sigma_t(\epsilon_p, \dot{\epsilon}_p, T) + \sigma_v(\epsilon_p, \dot{\epsilon}_p, T) \quad (4)$$

Constitutive models can be defined on choosing adapted form of each part [14]. It has been verified that such a concept leads to a quite acceptable description for many b.c.c. and f.c.c. metals. Nevertheless, proposed formulas are not simple and explicit enough for easy engineering uses. For example, the thermally activated flow part used in Ref. [14] depends on the quasi-static stress-strain curve.

In this paper, the author intends to combine the physical concept of the Tanimura model with the phe-

nomenological formulation used in our previous work [10] (which has given a satisfactory description of the studied material). A simple and directly usable formula for each part of the model (with several parameters to identify) is presented and it is associated with identification processes.

Considering the phenomenological form found in Ref. [10], it is noted that there are three parts in Eq. (3): a constant, a logarithmic function and a power function of the strain rate. It leads to the following formulation. The quasi-static part is a Ludwig type with constant coefficients. The thermally activated flow mechanism is represented by the linear increase with the logarithm of strain rates and viscous drag is described by the power function, commonly used for viscoplasticity. With a linear thermal sensitivity, we have:

$$\begin{aligned}\sigma_s &= (A + B\varepsilon_p^n)(1 - \mu\Delta T) \\ \sigma_t &= (C - D\varepsilon_p^m) \log\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)(1 - \mu\Delta T) \\ \sigma_v &= E\dot{\varepsilon}^k(1 - \mu\Delta T) \\ \sigma &= \left[ A + B\varepsilon_p^n + (C - D\varepsilon_p^m) \log\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) + E\dot{\varepsilon}^k \right] (1 - \mu\Delta T)\end{aligned}\quad (5)$$

where  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $k$ ,  $m$ ,  $n$  and  $\dot{\varepsilon}_0$  are coefficients of the model.

It is noted that the model presented is not limited to the uniaxial case. As with most empirical models, it can be easily generalised to three-dimensional cases (for an FEM-code application) using equivalent stress and strain in the sense of the Von Mises criterion.

### 3. Identification of parameters from experimental data, taking account of experimental condition of dynamic tests

The quasi-static experimental stress–strain curves are generally considered close to the material behaviour. On the contrary, experimental data of dynamic tests should not be simply used as the material behaviour. For instance, in dynamic tests, strain and stress fields are not homogeneous in space and they are not constant in time [15]. The identification of the parameters of a model must then take care of those effects and use with precaution average stress–strain relations obtained with experimental simplifying assumptions.

Experimental data of sheet metals used in this paper is obtained with the SHPB (split Hopkinson pressure bar) set-up, which is the most common testing method for material behaviour at high strain rates [16–18]. The testing condition of SHPB is analysed in the following.

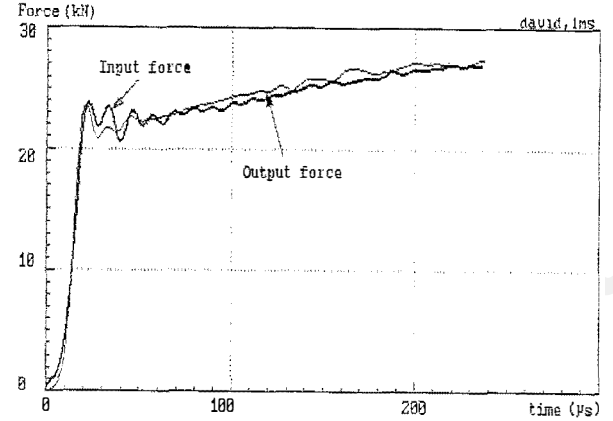


Fig. 2. Equilibrium of an SHPB test on metals.

#### 3.1. Transient effects in dynamic tests

SHPB measurements are forces and velocities at two bar–specimen interfaces and an average stress–strain curve is obtained from the assumption of homogeneous fields of stress and strain. Considering the initial phase of the loading where the specimen is loaded at one face, another face remains free until the wave induced by the loading arrives. Accordingly, before an equilibrium state is reached, the stress and strain fields in the specimen are not homogeneous. The assumption of homogeneous fields is valid only if the initial loading phase is very short. Fig. 2 shows the measured forces at the two sides of the specimen for one of our dynamic tests, which are very close. It can be then considered that the equilibrium state is rapidly achieved and that the average stress–strain curves do reflect the reality. It is emphasised that such an assumption is not automatically valid (when the specimen dimension is not appropriate, for instance) and that an inverse method should be used to obtain a stress–strain relation when this assumption is not realistic [19].

#### 3.2. Strain rate variation in dynamic tests

The strain rate during a dynamic test is not constant because the mechanical control during a very short testing duration (400 μs) is nearly impossible. A typical strain rate profile, given in Fig. 3, shows a variation of 20% around the average strain rate.

#### 3.3. Temperature rise due to quasi-adiabatic condition

Dynamic tests are not isothermal. Significant temperature increase during the dynamic test is often observed [20]. If the material is thermally sensitive, experimental data (average stress–strain curve, for example) necessarily contains a thermal softening effect. This effect must be taken into account. The present model (Eq.

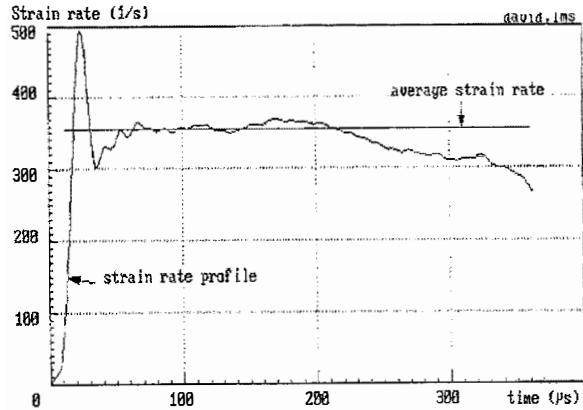


Fig. 3. Strain rate history during a test.

(4) being thermally sensitive, the thermal softening can be integrated in the identification process. For the studied mild steels, the coefficient  $\mu$  in Eq. (5) ( $0.0011\text{ }^{\circ}\text{C}^{-1}$ ) has been determined by several isothermal quasi-static tests. The temperature increase is estimated from the plastic work [10], assuming a quasi-adiabatic condition and a quasi-complete conversion of plastic work into heat [21].

In order to show the influence of thermal softening on experimental data, simulations of stress-strain curve are performed from the model (Eq. (4)) with and without thermal softening considerations. Their comparison shown in Fig. 4 indicates a gap of 5–10% for the studied mild steel.

On considering those testing conditions, a direct identification is hardly possible. The identification must be performed in an indirect way by comparing the experimental curves to the simulated ones using a set of parameters under real testing conditions (with the real strain rate history and temperature increase). It can be done numerically by a minimisation process using a curve-fitting algorithm. It can also be performed manually by a visual comparison.

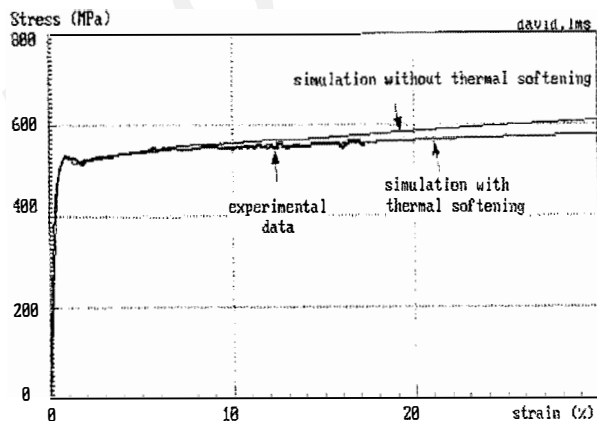
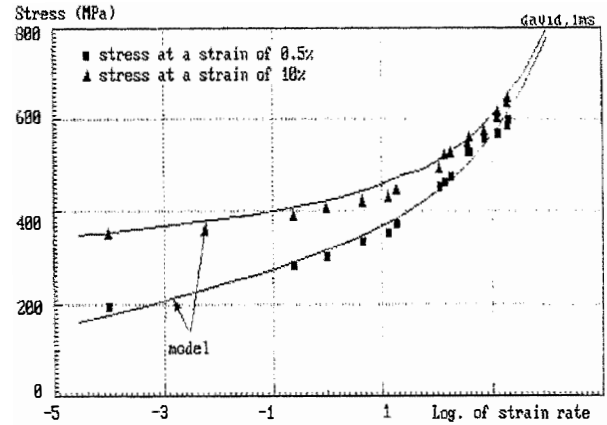
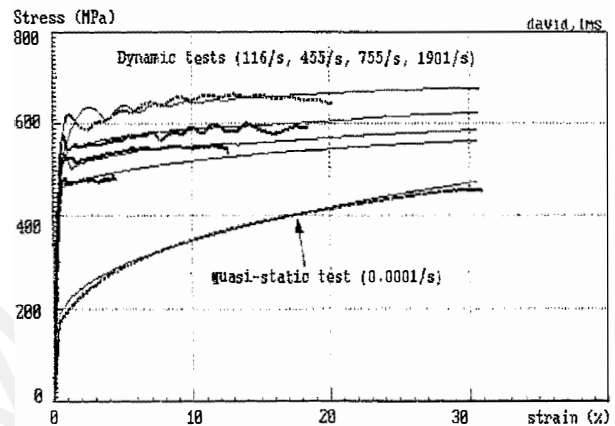


Fig. 4. Thermal softening due to adiabatic condition.



(a)



(b)

Fig. 5. (a) Modelling of the variation of flow stresses with respect to strain rates; (b) comparison between experimental and simulated curves for mild-steel.

The parameters of the model for the studied mild steels are then obtained ( $A = 145\text{ MPa}$ ,  $B = 550\text{ MPa}$ ,  $C = 35\text{ MPa}$ ,  $D = 47\text{ MPa}$ ,  $E = 185\text{ MPa}$ ;  $n = 0.42$ ,  $m = 0.3$ ,  $k = 0.3$ ). The quality of this set of parameters is shown in Fig. 5(a) and (b). The rate sensitivity of flow stresses given by the model is in good agreement with experimental data (Fig. 5(a)). Simulations of the tests from the model with identified parameters are very close to corresponding experimental curves (Fig. 5(b); the bold lines are experimental curves).

#### 4. Discussions

In this model, six parameters ( $C$ ,  $D$ ,  $E$ ,  $m$ ,  $k$ ,  $\dot{\epsilon}_0$ ) are used to describe the rate sensitivity instead of two parameters ( $C$ ,  $\dot{\epsilon}_0$ ) for the Johnson–Cook model ( $A$ ,  $B$ ,  $n$  are determined from a quasi-static test). Compared with the pure phenomenological model (Ludwig type, see Eq. (2)), the main advantage of the present model lies in the fact that each parameter plays a distinct role in the model. It is then easy to identify the parameters and to simplify the model if necessary.

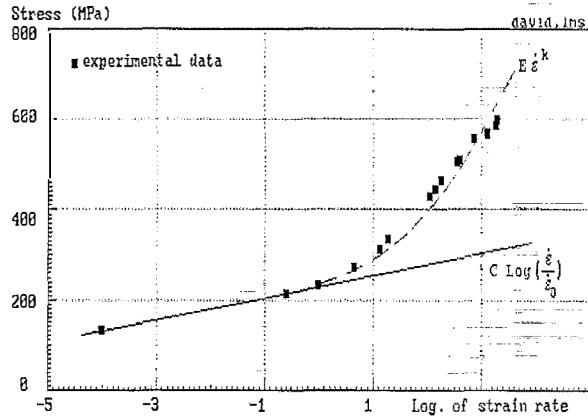


Fig. 6. Graphical explanation of parameters of the model.

Indeed, it is shown in Fig. 6 that the thermally activated flow mechanism is described by a straight line. Parameter  $C$  can be estimated by the slope of this line. The broken lines describe viscous drag and parameters  $E$  and  $k$  can be determined easily from the steep increase of the flow stress at high strain rates ( $\dot{\epsilon} > 1 \text{ s}^{-1}$ ).

Parameters  $D$  and  $m$  describe the variation of rate sensitivity with respect to the strain level. It is useful for the mild steel sheets studied and can be neglected in the case of aluminium sheets. Fig. 7 shows the comparison between prediction of the model and experimental data for the commercial aluminium sheets ( $A = 200 \text{ MPa}$ ,  $B = 250 \text{ MPa}$ ,  $C = 1 \text{ MPa}$ ,  $E = 95 \text{ MPa}$ ,  $n = 0.4$ ,  $k = 0.8$ ). In this case, parameters  $D$  and  $m$  can be neglected.

## 5. Conclusion

A elasto-plastic model for metals and alloys which is valid over a large range of strain rates is developed in this paper. Indirect method based on the simulation of the test is used for the identification of parameters from experimental data, taking account of the testing conditions complex. One advantage of the model is the easy identification of its parameters and simple physical significance of those parameters.

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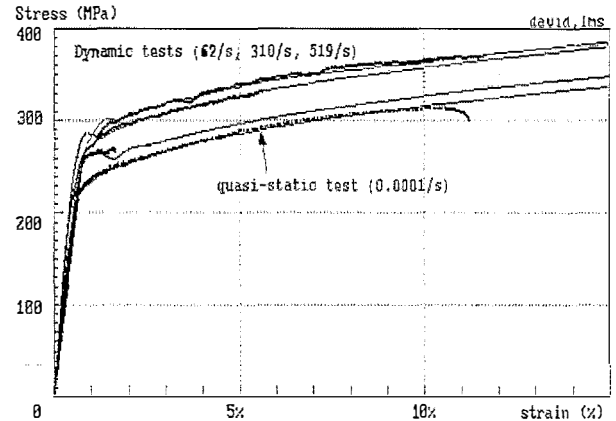


Fig. 7. Comparison between experimental and simulated curves for aluminium.

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